Linking Research & Practice

The NCTM Research Agenda Conference Report

Fran Arbaugh

Beth Herbel-Eisenmann

Nora Ramirez

Eric Knuth

Henry Kranendonk

Judith Reed Quander

NCTM®
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Acknowledgments

The work of the NCTM Research Agenda Project has truly been collaborative, bringing together a unique combination of school-based leaders in mathematics education, a diverse group of prominent educational researchers, researchers from fields outside of mathematics education, and representatives from the various organizations. We believe that the final document has been strengthened by the participation of such a diverse group of mathematics education stakeholders.

Space limitation prohibits the listing of all the people who contributed to the production of this document. To properly recognize contributors, we have included a full list of NCTM Research Agenda Planning Committee members and August 2008 Conference participants in Appendices B and C. The contributions of all these people enhanced this project and resulting documents.

We wish to extend special thanks to a number of people. First, to the initial Research Agenda Planning Committee (Chair Jim Fey, Ed Silver, Matt Larson, Randy Philipp, Nora Ramirez, Kathy Heid, Ken Krehbiel, and Judith Reed Quander), who proposed this conference to the NCTM Board of Directors and sought funding for the conference. We would also like to thank Jim Rubillo and Skip Fennell, who championed this project from the beginning, recognized its importance, and followed it through to fruition. We thank Henry (Hank) Kepner for his leadership throughout the final stages of this project.

Special thanks go to Judith Reed Quander, who has been integral in this work from the time she began as NCTM Director of Research. She has worked with all the related committees, has sent many e-mails, has organized all of us to get the job done, and has contributed greatly to the writing team. We also thank Fran Arbaugh, who facilitated the work of the August 2008 Conference and led the writing team. We are grateful to Kathe Richardson, Andrea Christie, Kelley Buchheister, and David Barnes for their support before, during, and after the August 2008 Conference. We thank Diana Lambdin, Kathy Essex, and Paula Lahann for their evaluation efforts. Finally, we thank the members of the NCTM Board of Directors who served from 2004 through the completion of this project for first making Linking Research and Practice a strategic priority and then continuing to support the work. Without their vision and support, this project would not exist.

This material is based upon work supported by the National Science Foundation under Grant No. 0736558. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

1
Linking Research and Practice

> At the same time that [educational] practitioners are unique consumers of research, they must also be the well-springs of research.
>
> — (*Lloyd, Weintraub, & Safer 1997, p. 536*)

IN AMERICAN education today, widespread agreement exists about the critical need to improve the mathematical proficiency of all students in elementary and secondary schools. Toward that end, numerous reform initiatives have been undertaken over the past 15 years: specifying high expectations for student learning; designing assessments aligned to the expectations; developing new curriculum materials to support learning and teaching; and improving the recruitment, preparation, and continuing education of mathematics teachers. Although research underlies the development of these efforts, the research base is sometimes lacking or uneven. Moreover, research is often not considered in practice; practitioners often make and enact decisions about curriculum, instruction, and assessment without regard to research. As a result, educational researchers can be heard to lament, "Teachers and districts don't pay any attention to our research." Teachers and district personnel, at the same time, often reply, "Research has very little to do with the decisions I make on a daily basis." Very little common ground appears to exist between researchers and practitioners in this scenario.

Even for those school and district-based practitioners who seek to make decisions that are informed by educational research, the research they seek is often published in research journals that are difficult for practitioners to access and/or the research is written in an academic style that makes findings difficult to interpret. Too often, the practical implications of academically written research reports are not apparent to practitioners. Further, once practitioners identify and locate appropriate research that could help them solve a problem of practice, they may have difficulty finding time to read and digest research findings—a practice that is not a part of the educational system in most U.S. schools (Burkhardt & Schoenfeld, 2003).

On the other hand, academia (as a whole) rewards educational researchers for publishing studies in research journals. Few institutions of higher education, particularly where faculty are required to have an active research agenda, value "practitioner pieces"—articles or book chapters written for a practitioner audience. Much educational research is written for and read by researchers within a small community; rarely is it shared with the community that needs to understand research findings the most—practitioners.

When considering these very broad and generalized descriptions of the relationship between educational research and practice, the gap between research and practice appears, at times, to be insurmountable. Scholars in a number of content areas, however, have begun to argue that a targeted focus on better *linking research and practice* is necessary to improve the landscape of educational research, the ways that it is used (or not used) in day-to-day decision-making in schools and districts, and, ultimately, to improve student learning. For example, Donovan, Bransford, and Pellegrino (1999, p. 7) describe five "paths through which research influences practices": (a) directly (teachers read and then implement research findings in their classrooms); (b) educational materials; (c) preservice and in-service education; (d) policy; and (e) the public (including the media). Carnine (1997) argues that "influence producers" (e.g., educational professional organizations, teachers unions, businesses, advocates) have an effect on both "knowledge producers" (researchers and effective practitioners) and "regulation producers" (legislatures and boards), who, in turn, influence "knowledge consumers" (publishers/developers and practitioners) (p. 518). Both sets of authors cited above treat the relationship as unidirectional—that educational research benefits practice when a pathway or dissemination route is secured between researcher and practitioner in order to facilitate the flow of research findings to consumers (practitioners) who then implement the research in their schools and districts. A different way of considering this relationship has evolved in the mathematics education community over the past few years and grounds the content of this document.

NCTM's Linking Research and Practice Strategic Priority

NCTM has a long history of involvement in and support of mathematics education research, including, but certainly not limited to, the sponsorship of research agenda conferences in the 1980s and research catalyst conferences in 1991 and 2003; sponsorship of the annual Research Presession; copublication of the first and second *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992; Lester 2007); and publication of a prestigious research journal in the field of mathematics education (the *Journal for Research in Mathematics Education*). Although these efforts have greatly supported the mathematics education research community, none of these efforts have focused explicitly on the relationship between research and practice.

The call for a better linking of research and practice has been echoed in the mathematics education community for some time. Take, for example, the appeal made by Judith Sowder (then editor of the *Journal for Research in Mathematics Education* [*JRME*]) to the mathematics education research community in a plenary at the 2000 NCTM Annual Conference Research Presession:

> This is my plea to all of you—a research report in *JRME* should not stop there. If you want your work to be read beyond a small circle of researchers, if you want to affect practice, then rewrite your research for a teacher audience, or an administrator audience, or for a mathematician audience. (Sowder, 2000, pp. 17–18)

Silver (2003), in a *JRME* editorial, presented a perspective for considering the relationship between research and practice through the use of the metaphor he titled "border crossing." Silver argued that researchers and practitioners live in different spaces that are separated by a border (much like two countries), and that there is a need for residents in both spaces to "cross the border" from time to time. He extends the metaphor by considering notions of currency exchange and speaks to the ways that each community can contribute to the exchange:

> Thinking about currency exchange highlights one of the challenges faced by those who seek to traverse the border between research and practice in mathematics education. In the research community, the valued currency is theory. Theoretical perspectives are central. Work that contributes to the development or refinement of theory is highly valued. In contrast, across the border in the land of educational practice, the valued currency is practical application. Work has value in this community to the extent that it can be directly applied to the improvement of some important domain of practice—such as curriculum design, assessment development, or classroom practice.
>
> Although the residents on each side of the border between research and practice have different currency valuation schemes, they can productively engage in exchange. Researchers have much to offer, including theoretical perspectives that might be useful in framing and describing practical issues and problems, research methods that might illustrate data-collection practices with practical utility, and findings that possess sufficient generalizability to support

appropriate use in applied settings. Practitioners also have much to offer, including a set of important issues and concerns that could and should be addressed in research, a collection of insights gained in and through practice, and a passionate concern for the improvement of education. The two groups have much to gain from collaboration in the borderlands between research and practice. (Silver, 2003, p. 183)

In 2004, the NCTM Board of Directors identified Linking Research and Practice as a strategic priority. In an effort to better support this strategic priority and the relationship between research and practice, NCTM has undertaken several initiatives in the past few years. One initiative has been the establishment of several committees, each charged with focusing attention on the issues of linking research and practice—from committees who conceptualized a series of activities that NCTM could undertake under the auspices of this priority to committees that carried out the work of the initiative. One of the first groups established was the Linking Research and Practice Task Force [LRPTF], which presented its advisory report to the NCTM Board of Directors in January of 2005 (LRPTF, 2005). In that report, the task force proposed a conceptual framework that could guide efforts toward linking practice and research (see **Figure 1**).

The members of this task force conceptualized a pathway in which research findings are disseminated to teacher leaders, found at (1). Then, teacher leaders share research findings with practitioners through the use of a set of tools (2), practitioners implement those findings and document results (3), and then report their classroom-based results back to the teacher leaders (4), who feed those results back to researchers (5). In this framework, open lines of communication exist between researchers and practitioners. The NCTM Research Committee (Battista, et al., 2007) wrote an excellent accounting of NCTM's responses to and plans for the recommendations made in the LRPTF report.

In the intervening years, perspectives on linking research and practice have evolved to consider the relationship between research and practice to be bidirectional. In other words, and

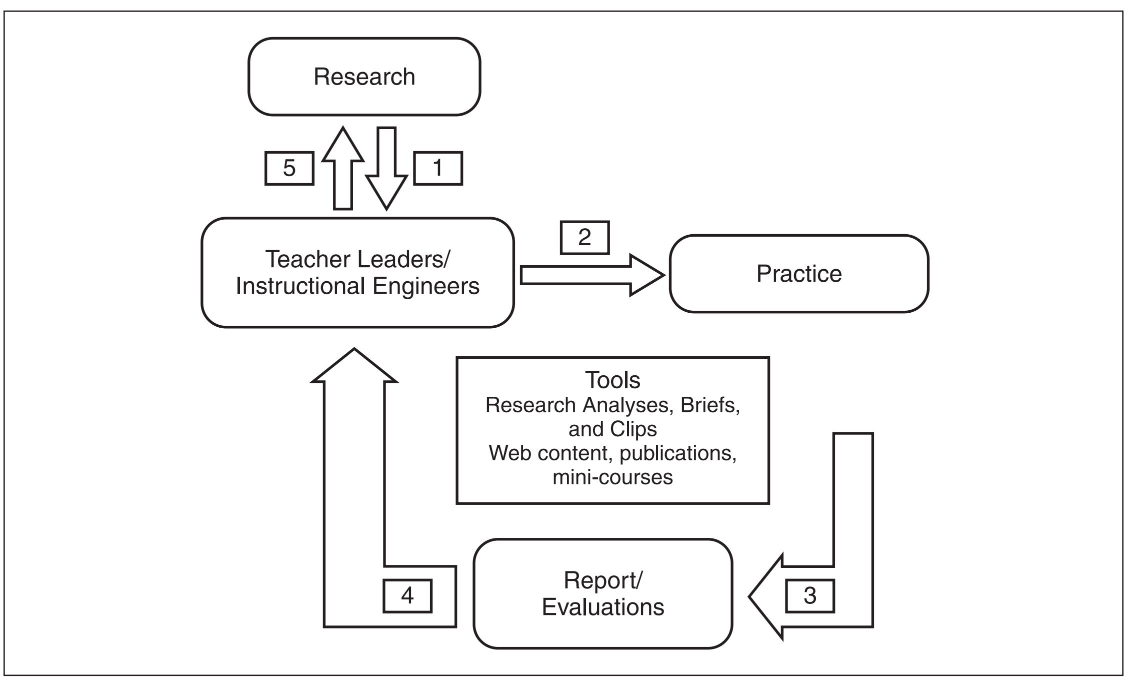


Figure 1. Diagram (adapted from the Linking Research and Practice Task Force report [Sowder et al., 2005]), depicting components in the flow of research information.

much as Silver (2003) described, the research and practice communities have much to contribute to each other's work; it is only through working in tandem that we can make a concerted effort to improve mathematics teaching and learning in the United States. As the NCTM Research Committee argued,

> At no time in the past have we had a better opportunity, nor has it been more important, to create a viable knowledge base for effective research-based practices to improve student learning. The community can best create this knowledge base by linking the work and purposes of mathematics education researchers and practitioners who, by working together, can advance, respect, and benefit from each other's work (Heid et al., 2006).

It was in this spirit that NCTM launched the Research Agenda Project and subsequently procured funding for a subproject titled Linking Research and Practice—Challenges and Opportunities (NSF DRL No. 0736558). This funding allowed NCTM to organize a small conference that would bring together mathematics education researchers and practitioners to identify a "research agenda that addresses significant problems of practice and is informed by experiences and expertise of practitioners" (NSF proposal, J. Rubillo, PI).

Subsequently, NCTM brought together approximately 60 mathematics education researchers and practitioners in August 2008 for a 4-day working conference focused on supporting the link between research and practice.[1] During this working conference, the participants analyzed over 350 mathematics education practitioner-generated questions in seven areas: assessment, curriculum, equity, student learning, technology, teaching, and teacher preparation/professional development. These inductive, theme-identifying analyses produced a set of 25 questions that communicate practitioners' needs for research that would affect their practices.

This document is a result of that working conference. Specifically, the purpose of this document is to (a) emphasize the need for communication and collaboration between practitioners and researchers around issues that are important to practitioners; (b) make practitioners' research needs, both as initiators and consumers of research, explicit to the mathematics education research community; (c) promote a set of research-guiding questions that focus researchers' attention on critical problems of practice; and (d) urge funding agencies, policymakers, and other mathematics education stakeholders to support research that is grounded in practitioners' problems of practice.

At the core of this document is a subset of the 25 questions developed at the conference. A subset of 10 questions (see chapter 2; a full listing of conference-generated questions is contained in Appendix D) were chosen, along with accompanying text, to present to the community in this document. It is important to note that this list of 10 questions is not exhaustive by any means—nor do we claim that these questions represent the corpus of important questions or the most important questions in mathematics education research. We present these questions as questions of importance to the practitioner community in hopes that they can serve as a resource for the mathematics education research community who choose to take more seriously the linking of research and practice.

It is also important to note that the questions presented in this document constitute what some might consider "a" research agenda for mathematics education. It is not our intention to imply that this set of questions constitutes "the" research agenda for mathematics education. The research-guiding questions presented in this document are representative of the kinds of questions that are based on the needs of practitioners. Thus, as readers engage with the content of this document, it is crucial to remember the origins of this work: mathematics education

[1] Details about the conference and participants are contained in Appendices A, B, and C.

practitioners' questions. It is not the intention of the NCTM Research Agenda Project to attempt to define the entire corpus of important mathematics education research domains and guiding questions; it is the intent of this project to communicate areas of research importance for practitioners.

This, we believe, is the strength and uniqueness of this document. For those mathematics education researchers committed to grounding their research in the needs of practitioners, this document contains invaluable information generated from approximately 200 mathematics education practitioners across the United States. For those researchers who have few opportunities to talk with practitioners about practice-based questions, this document is your window into their concerns. And for the mathematics education practitioners across the nation, this document gives voice to your questions in a manner that highlights your concerns and needs for research.

2
Research-Guiding Questions

> Teaching and research overlap in values, skills, and orientations, but the difference in emphasis between them is real and substantial because it is grounded in the positional constraints, incentives, and practices of these two forms of work.
>
> —(*Labaree, 2003, p. 21*)

IN THIS chapter we present a set of 10 research-guiding questions that focus attention on critical problems of practice. These are not research questions, per se, but questions that are broad enough to be used to guide a series of studies around a particular topic of importance to practitioners. Each research-guiding question in this chapter was generated by one of the conference working groups and is accompanied by text written by the members of the working group. Also included with each research-guiding question is a list of working-group-generated implications that could arise from research studies that are conducted in the quest of addressing the research-guiding question.

To start, however, we briefly discuss the process of generating these 10 research-guiding questions—a process that began with questions generated by practitioners and concluded with the transformation of these questions by researchers.

The Generation and Transformation of Practitioner Questions

Prior to the August 2008 Research Agenda Conference, each conference participant was charged with convening a focus group of practitioners to generate "important questions of practice most crucial to practitioners." The practitioner focus groups generated approximately 350 questions, which were then relayed to NCTM where they were sorted into the following eight domains: Assessment, Curriculum, Equity, Policy, Student Learning, Technology, Teacher Preparation/Professional Development, and Teaching. The Conference Planning Committee recognized that these domains are not mutually exclusive; that is, practitioner questions might fall in the intersection of two or more domains (e.g., How can teachers assess students' knowledge in order to make good instructional decisions?). Although these a priori domains served to structure (as well as potentially constrain) the nature of the research-guiding questions that were developed during the conference, the choice of these eight domains created manageable-sized sets of questions that helped to focus the work of the conference participants.

Conference participants were preassigned to one of eight working groups (one group for each of the aforementioned eight domains), and the practitioner focus group questions within a domain became the "data" for that domain's working group. The task assigned to each working group was to develop, from the practitioner focus group questions, a small set of representative research-guiding questions and to provide supporting narrative for each question.

The working group participants reported that in developing the set of research-guiding questions they worked hard to maintain the connection to the practitioners' questions—to honor the voices of the practitioners, whose questions were grounded in everyday work. Participants commented frequently during the conference, however, about the challenge of "translating" practitioner-generated questions into research-oriented questions while maintaining original intent and voice. This process of translating the practitioner-generated questions into research-oriented questions, or perhaps more accurately, the process of *transforming* the questions, underscores important differences in the worldviews of practitioners and researchers. As Larabee (2003) noted, the difference in worldviews reflects a shift from "normative to analytical, from personal to intellectual, from the particular to the universal, and from the experiential to the theoretical" (p. 16). Moreover, given the particularistic nature of teaching as a practice, "the reach for theory and generalization" (p. 20) is not necessarily reflected in the questions that practitioners generated as most crucial to K–12 education. As a consequence, the research-guiding questions that were developed at the conference were indeed a transformation of the practitioner-generated questions.

Research-Guiding Questions

The remainder of this chapter is dedicated to presenting 10 of the original 25 research-guiding questions that resulted from the August 2008 conference activities. Seven of the 10 questions were identified by conference participants (in their working groups) as being the most important question that was generated by the work of the group. The other three research-guiding questions were chosen for inclusion by consensus of the document writing group.

Research-Guiding Question 1: What are the characteristics of a comprehensive mathematics assessment system that provides instructional guidance, supports educational decision-making, measures continuous growth, and monitors system progress and accountability?

A comprehensive assessment program is defined as one that is coherent, balanced, equitable, and integrated (NCTM, 1995). There are two major conditions under which this question operates:

- Utilizing a range of assessments (e.g., formative, diagnostic, interim, summative)
- Maximizing the potential of technology (e.g., data storage and retrieval, representation and display, communication, tracking students' learning trajectories, analytics)

Although multiple audiences approach assessment in the education system for different purposes and outcomes, it is important that the assessment system provides data and results that inform instruction, support educational decision-making, measure continuous growth, and monitor system progress. It is also important to have a sense of how incentives impact the assessment process, including (a) how incentives affect development of high-quality assessments, (b) how incentives impact modification of curriculum and instruction based on assessment results, and (c) how incentives impact student effort in assessment situations.

Practitioners are very concerned about the types of assessment used and the purpose they serve related to student achievement. The continuum along which their questions are situated serves as evidence that they recognize "that multiple measures are needed to serve the assessment needs of an educational system. Currently, however, conflicts between classroom and large-scale assessments in terms of both goals and feedback cause confusion for educators, students, and parents" (NRC, 2003, p. xx).

Following are examples of the practitioner questions that guided the formulation of this question:

- What are the most effective ways of doing ongoing assessment?

- Which assessment improves student learning?
- What does research tell us about the relationship between higher scores on state assessments and increased student knowledge?
- How has the focus on high-stakes assessment affected the way teachers teach?
- To what extent do assessments align with and assess what we want students to know and be able to do?
- Should student assessments be directly keyed and benchmarked to high-stakes assessments or should they be independent and more skill oriented?
- What is the role and effective application of ongoing informal classroom assessment?

If the field had answers to

What are the characteristics of a comprehensive mathematics assessment system that provides instructional guidance, supports educational decision-making, measures continuous growth, and monitors system progress and accountability?

then teachers and administrators—

- would have a better sense of how the balance of different types of assessment affects student learning. This would include knowing the "optimal" mix of formative assessments in the classroom and the interaction between assessment and student and environmental characteristics (e.g., is the optimal mix of formative assessments different in urban as compared to rural schools?)
- would be better able to coordinate and interpret a variety of data types across the different types of assessment within a comprehensive data system.
- and policymakers could better understand how to use data appropriately to discuss issues related to equity.
- and local school officials could have a better sense of what data say about the success of their programs.
- and other school personnel could provide better rationales to parents and the public for classroom, school, and district-level assessment practices.

The field in general would—

- know which data analysis concepts, tools, and experiences that professionals in the field need to make sense of various data sets.
- know more about how large-scale assessment results should be formatted to be understandable by teachers and parents.

In addition,

- curriculum developers could be more specific in the suggestions for assessment that they provide in their materials.
- policymakers would have a better sense of how "assessment for accountability" can be a part of an assessment system that improves student learning.
- there would be better understanding of the interaction among assessment, curriculum, and grade level, and that understanding could be used to modify curriculum, instruction, and assessment practice.

Research-Guiding Question 2: In what ways do different curricular approaches and/or combinations of those approaches support or impede students' development of mathematical proficiency?

In this question, mathematical proficiency is defined as students having understanding of mathematics across five interwoven strands: computational fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001). Important goals of curriculum research, from practice, policy, and theoretical perspectives, include questions such as these: Under what conditions is the curriculum effective? What are the support requirements for various contexts? Why do certain sets of conditions decrease or increase the curriculum's effectiveness? (Clements, 2007). For example, different contexts (e.g., rural, urban, suburban; tracked math programs and nontracked mathematics programs), resources (e.g., access to technology, manipulatives, etc.), support structures (e.g., a district where there is intensive professional collaboration and educational experiences, a district where there is a single mathematics teacher in the high school, a school where there is a full-time technology support person), types of use of curriculum materials and tools (e.g., strictly using curriculum materials, supplementary materials, test preparation materials, manipulatives, or technology), and so on, all influence the enactment of the curriculum. The questions posed by the practitioners indicated an interest in understanding different conditions. For example:

- Which instructional/curricular approach, constructivist/discovery/investigative or traditional/lecture, leads to greater student achievement and later success in college?
- Do students in a specific curriculum take more mathematics?
- Do we have a curriculum that prepares people to be functional in today's society?
- Is developing relevance of mathematics to the real world important?
- What does research tell us about the role of fidelity of implementation in terms of effectiveness of standards-based math programs?
- Which approach is most effective: following a grade level curriculum or continually supplementing a curriculum (to meet the different needs in a classroom)?
- Textbooks and topics sequencing can sometimes stifle teaching. Should there be a required text in teaching of mathematics?

Moreover, practitioners were concerned about different groups of students including, for example, low- and high-achieving students who are typically underrepresented in higher-level mathematics, English language learners, and students who have been designated with an individual education plan (IEP). Some of the practitioner questions were phrased more generally (e.g., How do different curricula impact different subgroups of students?) and others were more specific to the populations of students with whom the teachers were working (e.g., Are there programs designed to motivate African American high school male students? Are there programs and materials that have been successful with low-achieving students?). As Clements (2007) pointed out, in the evaluation phase at least the following questions need to be answered: Is the curriculum usable by, and effective with, various student groups and teachers? How can it be improved in these areas or adapted to serve diverse situations and needs? (p. 43). In fact, Clements emphasizes the importance of considering issues of equity in the curriculum development process. He encourages researchers and developers to consider how curriculum impacts different groups of students in their work using both qualitative and quantitative research methods during curriculum development, enactment, and evaluation phases.

If the field had answers to

In what ways do different curricular approaches and/or combinations of those approaches support or impede students' development of mathematical proficiency?

then the field could—

- support teachers' understanding related to curriculum articulation across grade levels (e.g., transitions between elementary, middle, high schools, and colleges).
- broaden and clarify understanding of the goals and purposes for teaching specific aspects of the mathematics curriculum.
- better understand and reduce ineffective implementation of curriculum.

Teachers—

- could make informed decisions about which materials best meet the needs of their students and promote the development of the five mathematical proficiencies for all students.
- and curriculum coordinators could be empowered to provide informed responses to parental and administrative inquiries regarding curriculum choices and uses.

In addition,

- research results could inform the design of curriculum materials and lead to curriculum materials that facilitate better student learning and understanding of mathematics as well as meet the needs of our changing society and workforce.
- research results could inform decisions related to which concepts and procedures to include at each grade level (helping to eliminate the "mile wide, inch deep" curricular phenomenon).
- stakeholders could come to realize that curriculum selection is not an either/or issue (Standards-based versus traditional).

Research-Guiding Question 3: What is the impact of culturally responsive/relevant and critical pedagogies and programs on students' access and opportunity to learn mathematics and develop mathematics proficiency[2] in various school, cultural, and societal contexts?

This question is in response to practitioner calls for knowledge of what works in the way of approaches, tools, and methods in classrooms where issues of multilingualism, race/ethnicity, exceptionalities, gender, and socioeconomic factors overwhelm teachers. The field of mathematics education is at the point where although studies of this nature exist, they are limited in number, scope, and visibility.

This research-guiding question was developed from practitioner questions such as these:

- How can we close the achievement gap? Are there certain strategies, curricula, or cultural materials that are better than others that will help close the achievement gap?
- Have any outside-of-school programs been shown to improve performance among those who are more likely to be underserved in schools?
- How do we build more social capital for students through our math instruction?
- What role does culture play in student's mathematical learning?
- What connections are they (the children) making with their background? (For example, children who grew up on a ranch have a lot of knowledge about that life style. How do we build on that?)

Culturally relevant/responsive and critical pedagogies have been shown to be promising by seeking to promote academic success centered on students' cultural and community identities and their potential to engage in the critical pursuit of social justice. According to Enyedy and Mukhopadhyay (2007), the notion of relevance in teaching can be interpreted in at least three ways: (a) relevance of the content or context of the lesson, (b) relevance to students' community/social/cultural selves, and (c) relevance as the process of instruction.

Historically, connections between mathematics and what is being experienced in students' day-to-day lives have been overlooked (NCTM, 1989). The idea of drawing from the experiences of students to build mathematical knowledge has been steadily promoted in mathematics reform messages (NCTM, 1989, 1995, 2000). It must also be understood that these experiences are undoubtedly cultural in nature. As a result, a growing community of mathematics educators has explicated critical features necessary for implementing culturally relevant pedagogical practices in mathematics. Although there are several studies documenting examples of these practices in mathematics education, we know less about the impact of such practices (one line of work in this direction are Lipka and colleagues' curriculum and research projects; see Lipka, Webster, & Yanez, 2005). Because of this limited number of studies on "what works," it is necessary to cast a net that plows through a diverse set of studies including but not limited to disciplines beyond mathematics education such as cognitive studies/situational research in educational settings and international research in mathematics education and related fields of study. Implied in the focus-group questions from the practitioners is the notion of "what works" for whom and under what conditions.

---

[2] As defined in Research-Guiding Question 2.

If the field had answers to

What is the impact of culturally responsive/relevant and critical pedagogies and programs on students' access and opportunity to learn mathematics and develop mathematics proficiency in various school, cultural, and societal contexts?

then teachers—

- could have access to strategies that work well for all students with modifications based on the students' cultures and other special considerations.
- could have a variety of motivating contexts to use with students from different cultural backgrounds.
- could learn whether strategies that have been dubbed as good for all students are actually "good" for all students.
- and other stakeholders could have evidence that shows that by providing all students with opportunities to be exposed to high-quality mathematics, they can potentially learn a broad range of mathematical ideas and demonstrate proficiency in mathematical reasoning and problem solving.
- could learn strategies for helping all students to develop mathematics proficiency.

Teacher preservice and in-service programs would—

- be better equipped to address equitable teaching in mathematics.
- better understand existing challenges and enablers toward the development of mathematics teacher education programs that make equitable teaching a priority.

The field would—

- be better positioned to recommend culturally responsive/relevant programs that "work."
- be better informed about the links between these pedagogical approaches and the concept of mathematics proficiency (NRC, 2001).
- have a better understanding of what works for whom and how, thus enabling it to address the call for equity that is present in many current policy documents.

Stakeholders—

- would understand the features of highly effective mathematics programs for all students.
- would understand why the so-called achievement gap exists and flourishes in certain states and schools and is almost nonexistent in others.
- and teachers and policymakers would see why assessment, teaching, and curriculum should be seamless for all students.

In addition,

- textbook companies would publish books with a context that would interests and engages students from a variety of cultural groups.
- curriculum developers could use the findings from this question to inform the design of culturally relevant curriculum materials in mathematics.

Research-Guiding Question 4: What are coherent frameworks for characterizing the development of student thinking about specific mathematical concepts or processes?

As teachers design instruction and interact with their students, they become aware of the need to better understand their students' thinking in order to promote their students' mathematical development. In their efforts to understand students' thinking, they look for ways to help them understand their students, their curricular goals, and how to build a coherent sequence of instruction. Frameworks can provide one means of helping teachers understand their students' thinking and its development. A framework is a coherent and research-based "big picture" description of the development of student thinking. Frameworks can describe the development of student thinking about a particular mathematical concept or process—topic-specific frameworks (e.g., Battista's levels of sophistication about the concept of length; Harel and Sowder's proof schemes about the process of proving). Frameworks can also describe more generally the development of student thinking within an area of mathematics—general frameworks (e.g., van Hiele levels in geometry; Carpenter et al's CGI in addition and subtraction).[3]

The practitioners voiced their need for answers to questions related to students' mathematical development. The following practitioner questions illustrate their concerns:

- How do students at various ages develop mathematical thinking about specific mathematical topics?
- What skills/thought processes do we need to emphasize (and at what age/developmental level are students ready for them)?
- What are potential learning trajectories for the mathematical learning of middle school and high school students related to proportional reasoning, algebra, geometry, probability, and statistics?
- Are there situations where the best thing to do for a student having trouble is just wait and reteach the topic later?
- I want to know what is the average time line for students to develop mathematical understanding of various concepts. For example, research has shown that it takes people 9 years to develop true proficiency in a second language. How long does it take for students to develop mathematical understanding of a concept?
- Is there truly a progression of developmental stages of thinking that students go through as their mathematical understanding develops? If, and I emphasize if, there truly are developmental stages, how should the activities look for each stage, and can we expect these stages to happen at the same ages for all students or is it more experiential?
- How ready are young students to work meaningfully with large numbers?

Although some frameworks already exist, it is vital to develop additional topic-specific frameworks regarding the development of student thinking for other major K–12 topics as well as general frameworks regarding the development of student thinking for other K–12 areas (e.g., algebraic reasoning). Additional research that develops such frameworks is needed especially at the middle school and high school levels.

---

[3] General frameworks can also describe the development of student thinking more globally and can be applied for various topics and in various learning situations (e.g., Dubinsky's APOS theory as a means to characterize how mathematical ideas grow in sophistication; von Glasersfeld's psychological constructivism including the theory of abstraction).

If the field had answers to

What are coherent frameworks for characterizing the development of student thinking about specific mathematical concepts or processes?

then teachers, administrators, and school personnel would be able to—

- build learning on students' current ways of thinking.
- decide where to go next in instruction.
- ask questions that further promote the development of students' thinking.
- quickly understand the significance and validity of various types of student reasoning.
- diagnose and remediate student difficulties.

Teachers would—

- have strategies for helping struggling students learn mathematics.
- be able to identify effective strategies of remediation.
- be able to strike the appropriate balance between skill and concept development including having insight into "which comes first" (procedures or concepts) and whether it matters.

Curriculum/assessment developers would be able to—

- design curricular materials and assessments based on the guidance provided by the knowledge gathered.
- design curricular tasks and activities that support struggling students' learning of mathematics.

In addition, answers to these questions would—

- inform professional developers and teacher educators as they work with in-service and preservice teachers.

Research-Guiding Question 5: What are the mathematical concepts and reasoning processes that prepare and enable students to learn and use algebra?

The study of algebra is recognized outside the school setting as one of the hallmarks of a good education. It is positioned as the gateway to myriad courses, careers, and educational options. Furthermore, its importance has been elevated in recent national reports and initiatives, and the policies of many states now include an "algebra for all" thrust with successful algebra completion as a graduation requirement.

Historically algebra had been viewed as a course or two completed some time during the secondary school years. More recently, however, it has increasingly been viewed as a K-12 venture. NCTM's *Principles and Standards* (2000) advocates such a view based on the stance that a solid foundation of early algebra experiences better prepares students for later algebra. (See also the National Mathematics Advisory Panel Report.) Although mathematics educators have offered their best judgment, research does not yet provide a firm foundation to support the promise of algebra as a K–12 strand. Furthermore, regardless of when algebra is taught, little now exists that unifies our knowledge of how students build understandings of algebra. Moreover, little evidence exists of the extent to which algebra is better learned in any particular configuration (e.g., K–12, integrated secondary school courses, separate algebra courses). New curricula and new technological environments require evidence of the effects of curricular and pedagogical choices on the ways that students think about algebra. Technology places new demands on students' ability to *symbolize;* the capacity to create symbolic representations is one entry point to using technology. In light of significant curriculum efforts of the past quarter century that have developed algebra in the context of models of real-world phenomena or in technological environments, there is need to develop an understanding of how student understanding develops in these settings.

Practitioners also expressed a desire to learn ways to both better prepare their students for algebra and better enable their students for success while taking algebra. Practitioner-generated questions that illustrate this perspective included these:

- What are the key skills and concepts that students need to have mastered to be successful in 1st-year algebra?
- What is a typical eight- or ninth-grade student ready to grasp (conceptually) in algebra? Are there developmental trajectories for learning algebraic concepts?
- We have many students taking algebra in sixth and seventh grades. Are their brains actually ready for abstract thinking when they are this young?
- What are the critical ideas/concepts that algebra students should be developing to understand relationships among variables, especially linear and quadratic situations, and in systems of equations?
- Can students handle algebra concepts at an earlier age than previously thought (e.g., sixth grade or seventh grade)?

Given the importance of algebra and its capacity to enhance student thinking about quantifiable relationships, it is essential to understand how students develop this capacity. Progress in the proposed research arena would help practitioners in a wide array of areas.

If the field had answers to

What are the mathematical concepts and reasoning processes that prepare and enable students to learn and use algebra?

then teachers, administrators, and school personnel would be able to—

- build algebraic learning on students' understanding of arithmetic.
- ask questions that further promote the development of students' algebraic thinking.
- quickly understand the significance and validity of various types of students' algebraic reasoning.
- have strategies for helping struggling students learn algebra.

Curriculum/assessment developers would be able to—

- design, choose, and sequence algebra content in school mathematics.
- design curricular tasks and activities that support struggling students' learning of algebra.

In addition, answers to these questions would—

- inform professional developers and teacher educators as they work with in-service and preservice teachers.
- inform the revision of state standards and benchmarks so that they better reflect what students need to know in order to understand and use algebra.

Research-Guiding Question 6: What should be the goals of professional learning, and how will we measure attainment of these goals in terms of teacher growth?

This question deals with identifying the competencies that teachers need to have and prioritizing these competencies as desired outcomes of professional learning opportunities. Rather than specifying those competencies, this question notes that we need to make evidence-based decisions about what teacher competencies matter for student learning, for teacher retention, etc. (and thus should be priorities for professional learning), and we need ways to measure the development of these competencies so that we can evaluate the effectiveness of various professional learning approaches. Four constructs widely used as teacher competencies, or outcomes, are knowledge, practices, dispositions, and beliefs. Although these are important, we do not intend to limit the conception of outcomes for mathematics teacher development approaches. Rather, these might serve as placeholders for an array of such outcomes. Each of these competencies may be subdivided into specific important aspects of teacher qualities. For example, under dispositions, we might consider teacher professionalism; under knowledge and practice, we might consider teachers' knowledge about and use of technology; and under beliefs (and knowledge and disposition), we might consider teachers' orientation to equity.

Perhaps the most widely studied competency related to teacher professional education and teacher professional development is teachers' mathematical content knowledge. The practitioners posed questions that indicated they were concerned with this aspect of teacher competency. For example:

- How does a teacher's understanding of knowledge of content influence his/her effectiveness as a teacher?
- What content knowledge is necessary for an elementary teacher to be successful in his/her teaching of mathematics?
- How can confidence in mathematics be developed in teachers?

In a seminal paper in the teacher professional development literature, Ball and Cohen (1999) suggested that teachers need to become serious learners of *practice* rather than learners of strategies or collectors of activities. By placing practice at the core of teacher education and teacher professional development, Ball and Cohen called for a fundamental shift of focus, one that required rethinking the goals of teacher learning. In her synthesis of the literature, Sowder (2007) presented six overlapping goals for teacher learning. One of these goals is developing mathematical content knowledge. The other five are developing a shared vision, developing understanding of how students think about and learn mathematics, developing pedagogical content knowledge, understanding the role of equity, and developing a sense of self as a teacher of mathematics. Practitioner questions that indicated concerns related to these goals included these:

- If "excellence in mathematics education requires equity-high expectations and strong support for all students" (NCTM 2000), then what professional development opportunities will be provided for teachers to relearn mathematics, unlearn ineffective practices, and understand and confront their own beliefs and biases in order to accommodate differences among students' knowledge, experiences, and interests? The extent of equity is dependent on the expectations, decisions, interactions, and mathematics background of the classroom teacher.
- What is the minimum "tool set" required for teaching mathematics successfully?
- How does pedagogical content knowledge vary by teachers of mathematics?

- Is there a relationship between elementary teachers' beliefs concerning the nature of mathematics, their teaching of mathematics, and their students' success in mathematics?
- What is the effective way to break the cycle of teaching based on how the teacher (lecture mode) was taught?

If the field had answers to

What should be the goals of professional learning, and how will we measure attainment of these goals in terms of teacher growth?

then the field would—

- know where to put efforts in professional development (developing content knowledge, developing *mathematical knowledge for teaching*, focusing on beliefs, focusing on particular curriculum materials, etc.).
- have developed a more elaborated trajectory of teachers' evolution of their competencies (e.g., knowledge, beliefs, dispositions, and practices), beginning from one end of the continuum when teachers enter teacher preparation programs to the other end of the continuum when teachers establish themselves as effective teacher leaders.
- know how to prepare doctoral students, curriculum directors, teacher leaders, and existing university professors to be effective leaders of professional development.
- understand the similarities and differences between supporting prospective and practicing teachers in developing teacher competencies (e.g., content knowledge, beliefs, dispositions, or practices.)

In addition—

- funding agencies could tailor their RFP's to support professional development aimed at specified goals of professional learning.
- curriculum materials could be developed and aligned with the goals of professional development.
- publishing companies would know how to prepare their professional developers for effective work in a school district.
- resources could be applied more efficiently to effective forms of teacher learning.

Research-Guiding Question 7: How does students' engagement with mathematical, collaborative, and communication technologies influence what students learn and how they engage in mathematical processes?

At least two general forms of technology might be useful in mathematics teaching and learning: mathematical technologies and collaborative/communicative technologies. Mathematical technologies allow the user to operate on mathematical entities. These technologies serve to provide people with a range of mathematical activities and forms of mathematical representations. Collaborative and communicative technologies allow users to create, manipulate, edit, communicate, and share experiences, ideas, and products using words, numbers, symbols, images, audio, and video. Mathematical technology and collaborative/communicative technology are found in many, though not all, schools and are owned and used by many, though not all, students. Fundamental to arguing that all students have access to these tools or advocating that more technology be used in mathematics teaching and learning is an assumption that technology can enhance what students do and what they learn. Extant literature does provide insights to the general question of how technology influences what students learn and how they engage with technology. Research around this question would illuminate how emerging forms of technology and combinations of different types of technology might help or hinder student engagement and learning in mathematics. The body of research would address pressing practitioner questions, such as these:

- How does teaching mathematics using technology versus teaching by-hand mathematical skills first affect student learning?
- Can technology deter development of skills or does it enhance development of understanding?
- Does technology enhance the learning of basic facts?
- In what way can technology impact student understanding of the real-world connections of the mathematics?
- Can computer algebra systems be used to help students develop more efficient methods?

Related to the question of how technology affects how students engage in mathematics and what they learn is that of how technology affects how students perform on achievement tests. Technology use here might take many forms, such as the use of various mathematical technologies to develop concepts and procedures and the use of various collaborative/communicative technologies to enable students to work together and with teachers and others to develop and refine their understanding. The importance of connecting technology use with achievement is reflected in practitioner questions such as the following:

- What is the impact of (various forms of) technology on student mathematics achievement?
- What are the effective ways to use technology to promote student achievement?

Research around the general question of what students learn and how they engage in mathematics would also investigate more specifically how technology use relates to students developing mathematical ways of thinking (e.g., proof and reasoning) and mathematical understandings that apply across many mathematical topics (e.g., symbol sense and use of representations). Among the practitioner questions informed by investigations in this area are the following:

- Is there a correlation between graphing calculator use in mathematics and students' broader analytical abilities?

- Can computer algebra systems use help students develop symbol sense? Are certain paper-and-pencil algorithms necessary for learning mathematics?
- How do we help students to be facile in moving among representations, including those created by technology?

Aspects of this research-guiding question require extending research lines beyond existing arenas and investigating what happens when technology-enabled settings blend mathematics technology with collaborative/communicative technology. One example of where complex but intriguing questions and useful answers might be found is the use of technology to support inquiry. One practitioner articulated a set of questions that could be asked of mathematics technology and collaborative/communicative technology in the interest of furthering student inquiry:

- What are the effects of students engaging in technology-based inquiry?
- What outcomes can we expect (or not expect)?
- How does technology make inquiry different?
- What supports are needed for students to learn how to do inquiry?
- How do teachers teach the skills needed to do inquiry?
- What do you do with mathematically conflicting resources?

Knowledge gives one the ability to choose among tools, particularly within an inquiry setting. Engagement in mathematics includes involvement with and motivation to use available and appropriate technology to interact with meaningful mathematical tasks. Questions related to how students use technology to engage in the study of mathematics might include student preferences, attitudes, and beliefs. Research in the area of engagement might consider what choices students make when doing mathematics in technology-enabled settings and what students believe the technology should and does offer them.

If the field had answers to

How does students' engagement with mathematical, collaborative, and communication technologies influence what students learn and how they engage in mathematical processes?

then school-based educators could—

- move beyond the question of whether technology should be used to model processes and principles for how and when it can be used effectively and strategically.
- have a research-based set of principles and best practices appropriate in conjunction with technology use that would inform daily decisions and local policies.
- pursue available funding sources for technologies that prove effective with their student groups.

Textbook/technology developers could—

- provide ways to facilitate or extend teachers' experiences in useful ways.
- could align their ancillary materials with a focus on developing and drawing on the identified knowledge and experience.
- use findings to enhance products, and technology producers could adapt existing products to improve service to various groups.

Teacher preparation programs could—

- add or strengthen technology strands by focusing on the necessary knowledge and experiences.
- provide courses in pedagogy, mathematics, statistics, and related areas designed to provide relevant experiences for prospective and current teachers.

In addition—

- policymakers could make decisions that reflect a consistent picture, with a match among learning expectations, engagement expectations, and technology expectations.
- use of collaborative/communicative technologies might connect students' school mathe-matics experiences with the technological aspects of their daily life.
- issues of access could be dismissed or ways to address demonstrated access discrepancies could be developed through collaboration among funding agencies, schools, and communities.

Research-Guiding Question 8: What are the characteristics of a technology-enabled learning setting that contribute to students' learning of mathematics?

A "technology-enabled learning setting" is an environment in which teaching and learning occur through and with the aid of electronic forms of technology, with full recognition of the presence and impact of manipulatives and other nonelectronic technologies on student learning. A technology-enabled learning setting may have mathematical technologies, communicative and collaborative technologies, or a combination of both. (See Question 7 for an explanation of these terms.)

It is difficult to identify specific technologies or forms of technology that should be researched in this area, noting that innovation in this field constantly changes availability, access, and adoption of different technologies. Current technologies used in unexpected ways and future technologies may provide learning opportunities that are currently unimagined. For the purposes of this research-guiding question, technology includes any electronic tool that promotes a dynamic and interactive mathematics learning environment, which in turn allows students and teachers to engage in calculating, representing, creating, exploring, collaborating, and communicating mathematical ideas. Studies designed to address this research-guiding question might inform these types of practitioner questions:

1. What is the most effective balance between technology and traditional hands-on learning?
2. How can teachers make an informed decision about which technologies are best to use for their students?
3. When and how should technology be introduced?
4. What are some drawbacks to using technology in the classroom?

Within a technology-enabled learning setting, the learner and teacher might be geographically dispersed or in the same physical space, asynchronously or synchronously connected, or be engaged across various combinations of geography and time. A physical school setting or traditional classroom meeting time no longer bounds class discourse. Communicative and collaborative technologies help student-student and student-teacher discourses to continue beyond their origination in a traditional classroom or to be initiated outside of a physical classroom. The availability and use of such communicative and collaborative technologies raises a first research-guiding subquestion:

1. How does the *nature of mathematical activities* in which students and teachers engage together and apart vary as technology allows for classrooms that are not confined to space and time?

Related practitioner questions include these:

- What are the effects of student use of homework help lines or the effects of any dynamic environment that is asynchronous and ageographical? What makes any of these effective for mathematics learning?
- How can the use of online video/audio media be used to help mathematics instruction?
- What are productive blends of in-class and out-of-class technology-based work? What models can we use to think about new ways to work with students, given these technologies?
- What technologies can help home schooling in mathematics?
- What is the effectiveness of online learning for underserved populations?

Perhaps underscored by the potential of technology to allow classrooms to transcend time and geography, fundamental questions surround how student learning and understanding might be assessed in technology-enabled learning settings. The changing nature of classrooms and new forms and potential for discourse afforded by collaborative and communication technology, combined with previously unseen mathematical capabilities of mathematical technology, raise new questions about what to assess, how to assess, and when to assess. A second research-guiding subquestion attends to this situation:

2. What are *models of assessment* to inform student learning and teaching in technology-enabled settings?

Research here would address existing practitioner questions and new questions that practitioners are only now beginning to form around technology and assessment:

- Does teaching algebra with computer algebra systems (CAS) help students do better when tested without CAS?
- Can (students) use more technology on state-mandated tests, especially for open-ended questions?
- What are the effects of the technology, separating out such things as group work and open-ended tasks?

A need to identify general principles that characterize group dynamics and mathematical work also exists in effective technology-enabled learning settings. Inquiry in this area expands extant research into the development and importance of sociomathematical norms to what we might identify as technosocial and technomathematical norms—norms that might exist in the blending of technology with social and mathematical events. The goals of inquiry in this area are to understand what norms are supportive of learning in technology-enabled settings and how students and teachers establish such norms. This research-guiding subquestion arises:

3. What mathematical, social, technological, sociomathematical, technosocial, and technomathematical norms support mathematical teaching and learning in a technology-enabled learning setting?

Practitioner questions that might be informed by research on such norms include the following:

- What is the effect of technology (e.g., dynamic geometry or computer algebra systems) on encouraging an environment in which students ask mathematical questions?
- When should teachers let students use calculators and when should they not?

If the field had answers to

What are the characteristics of a technology-enabled learning setting that contribute to students' learning of mathematics?

then school-based educators could—

- have a research-based set of principles and best practices appropriate in conjunction with technology use that would inform daily decisions and local policies.
- along with students develop learning community norms that enhance learning within technology-enabled settings.
- use the characteristics of effective technology-enabled learning settings as targets and markers of their professional growth.

In addition—

- answers to these questions might change debates over technology use from opinion-based exchanges to evidence-based conversations. Answers might be helpful in convincing teachers who might not be inclined to embrace the use of technology in mathematics to see its value and consider its use. Alternatively, answers might be helpful in convincing technology enthusiasts of the limitations of technology use.
- schools could better match their choice and use of technology with the learning and engagement goals they embrace.
- research would have improved focus, moving away from uninformative comparisons of "technology using" classrooms with "nontechnology" classrooms to constructing a body of research that focuses on identifying and articulating characteristics of effective settings.
- policymakers, principals, administrators, researchers, parents, and others could have a common set of characteristics that could be observed as indicators of quality in technology-enabled classrooms.

Research-Guiding Question 9: What "interventions" help teachers reach students who they perceive have difficulty developing mathematical proficiency?[4]

Developing mathematics proficiency in every student requires that teachers build on students' prior knowledge and challenge them to learn more mathematics (Franke, Kazemi, & Battey, 2007; NRC 2001). In most mathematics classrooms, however, students vary in their mathematics knowledge and experiences. As a result, teachers constantly look for ways to help students who struggle to perform to their potential in mathematics.

Students struggle in many ways in mathematics classrooms. Some students struggle to understand a specific mathematics concept, whereas others may have substantial gaps in their levels of mathematics understanding and struggle to keep up with the mathematics presented in class. Research on the various types of interventions teachers use to support their struggling students develop mathematics proficiency is needed to support teachers as they teach.

The word *interventions* is used to broadly define instructional activities and programs that build mathematical proficiency in students by taking into account the students' current level of mathematics understanding, the mathematical learning goals, and the various context in which students learn and experience mathematics. These interventions are designed to accelerate the learning of struggling students and may occur within a particular classroom or take the form of more structured programs.

We purposefully framed the question as students who teachers *perceive* as having difficulty because research suggests that teacher perceptions of students frame how they engage with students in mathematics classrooms. For example, research suggests that low-income and minority students are often viewed as struggling or low-performing by teachers who do not understand students' cultural ways of communicating their mathematical knowledge (need a citation).

Specific questions about interventions raised by the practitioners include the following:

- How do we identify students in need of intervention prior to remediation?
- What are effective strategies for diagnosis and remediation? Currently, at least with "mainstream" material, all we can determine is what a student can do or cannot do, not how their minds are really dealing with mathematical information.
- What strategies work best with struggling students in urban settings?
- How does instruction need to change to reach the marginalized students?
- How can I help the students with disabilities in my inclusion classroom become more successful in understanding mathematics concepts?
- What instructional structures support learning for all students?
- What do we do with students who need intervention?
- What practices are being used around the country that are allowing at-risk children to excel?

[4] As defined in research-guiding question 2.

If the field had answers to

What "interventions" help teachers reach students who they perceive have difficulty developing mathematical proficiency?

then teachers could be better supported to—

- plan lessons that help students develop an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

The field would better understand how effective teachers—

- plan for and implement the kinds of classroom discourse patterns that help important mathematical ideas surface for discussion.
- anticipate the student thinking that might emerge during a lesson so that they can be alert for these ideas, or any unexpected ideas, as the lesson unfolds.
- reflect on lessons, considering the extent to which students were able to develop an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition and which features of their planning and their classroom instruction contributed to this students learning.

Additionally, the field would better understand—

- how attention to planning and reflecting on practice might be addressed in a range of professional development settings.
- when and how mathematical relationships should be made explicit, under what conditions different approaches work well, and what mathematically proficient outcomes can be expected from the various approaches.
- what teachers need to know in order to be able to support the mathematics learning of all students.

Research-Guiding Question 10: How can teachers engage students in productive struggle to support the development of mathematical proficiency?

Providing opportunities for students to struggle or wrestle with important mathematical ideas facilitates conceptual understanding (Hiebert & Grouws, 2007). The phrase *productive struggle* is intended to describe a process through which students engage in thinking, reasoning, and problem solving, expending effort to make sense of mathematics or figure out something they do not already know.

Mathematical tasks that provide the greatest opportunities for students to productively wrestle with mathematical ideas are the most difficult for teachers to implement well during instruction. Research suggests that the thinking associated with such tasks (often referred to as high-level or cognitively challenging thinking) often declines during implementation (Doyle, 1988; Stein, Grover, & Henningsen 1996). When this decline in thinking occurs, students are left to apply previously learned rules and procedures with no connection to meaning or understanding, and the opportunities for thinking and reasoning are lost.

Why are high-level tasks so difficult to implement in ways that maintain the rigor of the mathematical thinking? These tasks tend to be less structured, more difficult, and often take longer to complete than the kinds of tasks to which students are typically exposed. Students often perceive these types of tasks as ambiguous and/or risky because it is not apparent what they should do, how they should do it, and how their work will be evaluated (Doyle 1988; Romagnano, 1994). In order to deal with the discomfort that surrounds this uncertainty, students often urge teachers to make these types of tasks more explicit by breaking them down into smaller steps, specifying exact procedures to be followed, or actually doing parts of the task for them. Alternatively, the teacher may decide that students are unable to engage in the task and funnel the students' thinking along a particular pathway so that they can be successful. In either case, the challenging, sense-making aspects of the task are reduced or eliminated, thereby robbing students of the opportunity to develop thinking and reasoning skills and meaningful mathematical understandings.

Hence, a challenge faced and expressed by practitioners is how to support students' engagement in cognitively challenging mathematical tasks without taking over the thinking for them. Specific questions raised by practitioners include the following:

- What does "productive struggle" look like, and what steps do teachers need to take to not only understand the need to struggle but also develop lessons that provide students with opportunity?
- What are the levels of questions that should be used to challenge all students?
- What are the most effective ways to develop mathematical proficiency, especially conceptual understanding and procedural fluency?
- What is the best way to develop fact knowledge, computational fluency without compromising conceptual understanding?
- What instructional approaches support learning for all students?
- What are the best methods for enriching students' mathematics learning without creating accelerated classes?
- How do specific teaching strategies affect students' mathematics learning?

If the field had answers to

How can teachers engage students in productive struggle to support the development of mathematical proficiency?

then teachers could be better supported to—

- engage in ongoing reflection on their lessons with an eye to what students seemed to learn, the evidence of that learning, and what features of their planning and instruction contributed to that learning.

The field would better understand how effective teachers—

- plan and implement lessons that are clearly focused on key mathematical ideas and use identified activities and tasks as well as representational tools to address those mathematical ideas.
- reflect on lessons, considering the extent to which students were able to develop an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition and which features of their planning and their classroom instruction contributed to this students learning.

Additionally, the field would better understand—

- the tasks and sequence of tasks that help students develop an understanding of key mathematical relationships.
- the ways in which multiple representations can be used to make the key mathematical relationships visible and which representations are most fruitful in advancing student learning of mathematics.
- how to use students' thinking as the basis for explicating relationships that make sense to students.
- how to create a classroom environment where making sense of and understanding mathematical relationships is valued.

Conclusion

The 10 research-guiding questions presented in this chapter were developed from the synthesis of questions and concerns of approximately 200 practitioners from across the United States. A full list of the 25 research-guiding questions developed as a part of this process is contained in Appendix D. For those who are interested in the research-guiding question text (written by the working groups) for those questions that are not highlighted in this document, a downloadable PDF of all 25 research-guiding questions and accompanying text is available at **http://www.nctm.org/researchagenda**.

Although these questions resulted from working groups that focused on eight domains, it is obvious that many of the questions cross cut across several areas. For example, questions about student progressions in mathematics content areas fall under curriculum and student thinking. Similarly, questions about effective pedagogies and programs for students who have historically been marginalized in mathematics education have important implications for equity, teaching, and curriculum. Looking at this set of research-guiding questions from a curricular, pedagogical, and equity stance makes the most sense for truly getting to the root of practitioners' questions.

We ask readers to recognize that we could not capture every specific issue brought up by the practitioners who participated in the focus-group interviews in a single document, nor could we survey every practitioner nationwide. This set of research-guiding questions, however, does provide the field of mathematics education research with some insight into those critical issues of practice with which practitioners continue to struggle. In some cases, we already know something about the answers to some of the practitioner questions, and we urge researchers to make those answers accessible to practitioners. For other practitioner-based questions, more research clearly needs to be undertaken. In either case, work clearly needs to be done by the mathematics education research community.

3
Discussion and Implications

THE research-guiding questions presented in chapter 2 represent the results of an analysis of over 350 questions from mathematics education practitioners. These are certainly not the only questions of importance to mathematics education practitioners, but they are representative of some of their concerns. These questions serve as a starting point for those mathematics education researchers who seek to conduct their research across the boundary that connects the communities of mathematics education research and practice.

In this final chapter, we present a framework for delineating the relationship between researchers and practitioners: the Linking Research and Practice Continua. In an effort to illustrate projects that closely link research and practice, we present short descriptions of projects in which authors of this document have participated in the last decade. We close this document with a discussion of implications that arise from seeking to better link research and practice in mathematics education.

Linking Research and Practice Continua

In chapter 1 of this document, we presented a number of frameworks that scholars have proposed to address the problematic nature of linking research and practice. Those frameworks, however, more often than not present the relationship as unidirectional: They suggest that researchers seek ways to build research collaborations with, and disseminate research findings to, practitioners. We propose that this emphasis on practitioners as merely consumers of research be called into question. In doing so, we align ourselves with Wagner's (1997) perspective that all research involves some aspect of cooperation from practitioners. He proposed three different categories along the continuum of cooperation: (a) data-extraction agreements, in which the relationship between researcher and practitioner are formal and the practitioner mainly consents to the project; (b) clinical partnerships, in which the practitioners are more centrally involved and influence research decisions; and (c) co-learning agreements, in which both practitioner and researcher are not only involved in shaping the research but also involved in understanding what they have learned in the process. Although Wagner's continuum focuses more on the degree to which practitioners are *involved* in the research process, we see this continuum as related to the ways in which we conceive of the linking of research and practice.

As such, we present two frameworks that can support members of the mathematics education community as they seek to better understand *linking research and practice*. **Figures 2** and **3** contain depictions of continua describing the degree to which research and practice are linked from the perspective of researcher (**Figure 2**) and of practitioner (**Figure 3**). We focus each continuum on the extent to which research is engaged with, drawn upon, and valued by each audience—research done by people in academic institutions as well as by practitioners in schools and school districts. In this way, we see the end of each continuum labeled as "high" as a place where the research-practice connection is highly linked; the end of the continuum labeled as "low" includes activities and roles in which the relationship is quite limited.

It is important that those who are seeking to understand these continua do not misinterpret our message. We do not assume that engaging at a "high" linking level is "best." Rather, we argue that all points along each continuum have a place in the educational arena. We note, however, that the points at the mid to low ends of each continuum are more prominent in current practices in the field of mathematics education.

As indicated in **Figure 3**, we see low levels of linking research and practice as including aspects of research that may be described as "pure basic research." These studies are typically inquiries that researchers conduct without consideration of their practical implication and, indeed, many important studies that have contributed to the development of theories have been undertaken at this linking level (Silver & Herbst, 2007). Next on the continuum is research that might be described as "use-inspired basic research," in which researcher-generated problems of practice and utility influence the study. The researcher, in this case, may ground the research in a question of practice, but only communicates research findings to other researchers. At the next point on the continuum, the research questions also derive from a concern with practice, and the researcher not only communicates the findings to a research audience but also communicates the findings to a practitioner audience (see, for example, Sowder (2002) for a

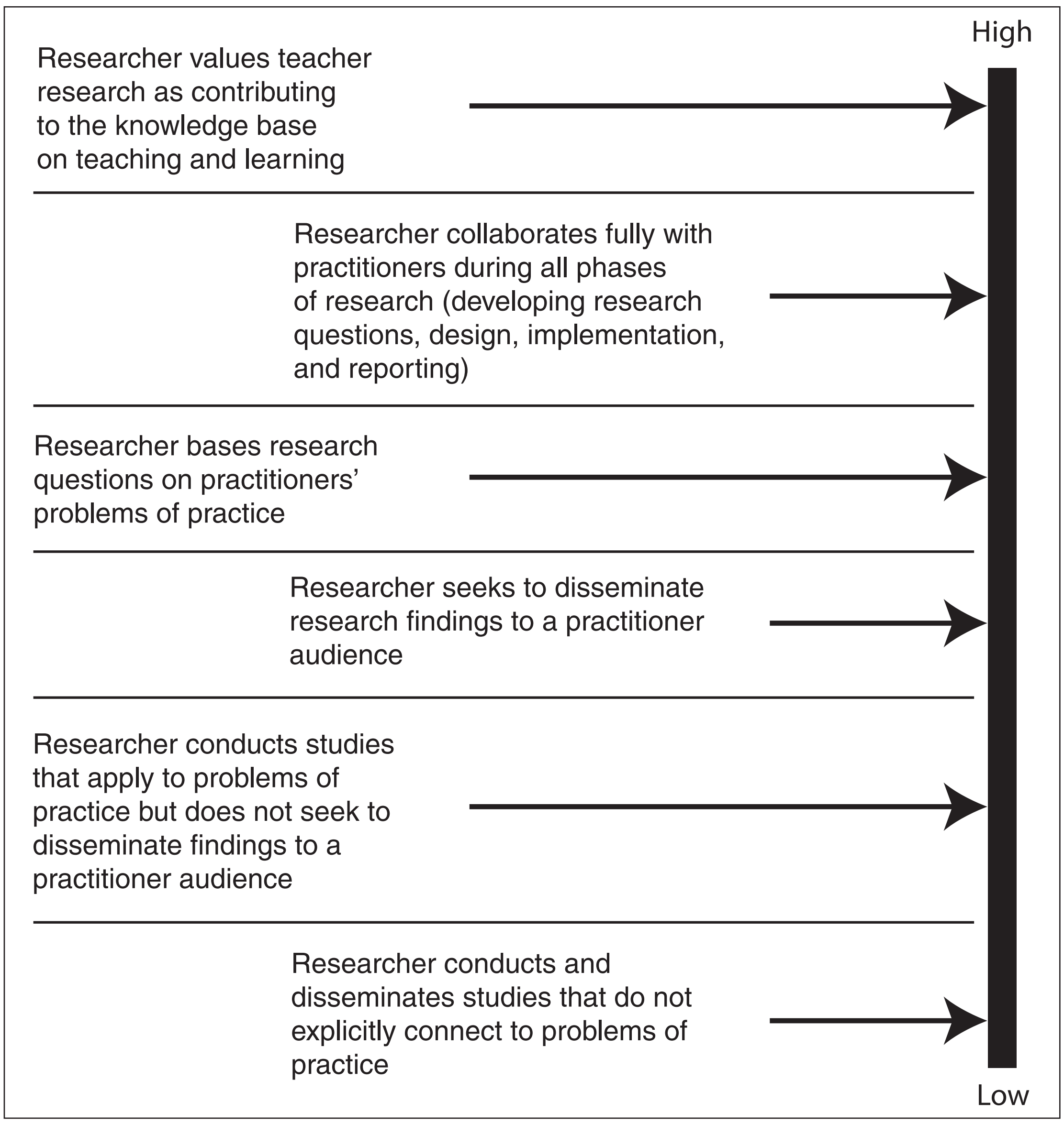


Figure 2. The degree to which research and practice are linked: The researcher's perspective.

compilation of such work). One commonality of these first three points on the continuum is that the research questions that guide such studies are often generated based on a review of the research literature or on suggestions made by other researchers about "directions for future research" (a common section in research articles). In these studies, research questions are typically developed from identifying a gap in the literature rather than being targeted to solve a practitioner-identified problem of practice (although they may fulfill this purpose, it is not the primary motivation for the work).

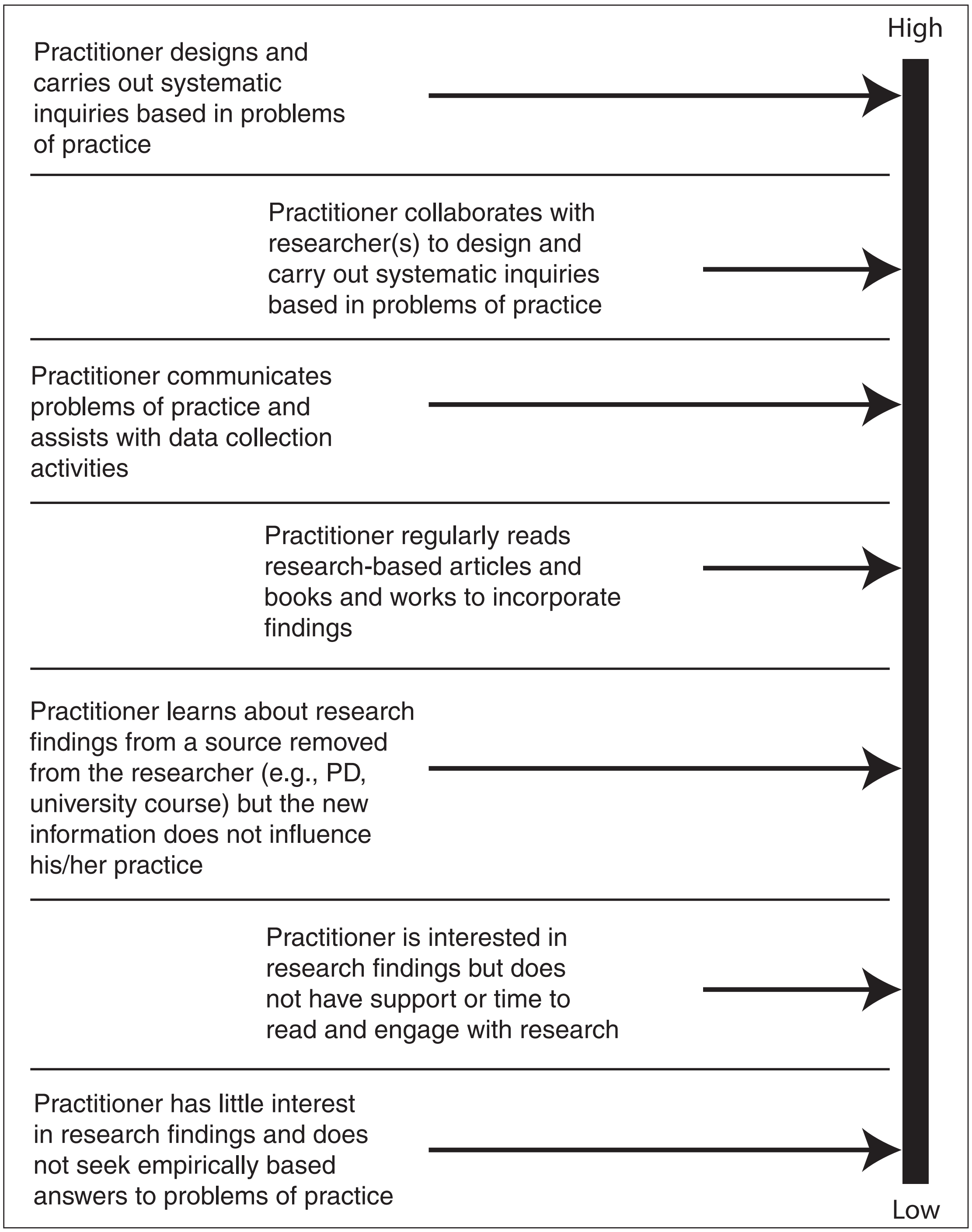


Figure 3. The degree to which research and practice are linked: The practitioner's perspective.

Thus far, the points on this continuum highlight researchers' questions and the audiences to whom the findings are communicated; at the next point on the continuum, researchers focus their inquiries on practitioners' questions and seek to try to solve problems of practice. The work of the conference reported in this document is a good example of this point on the continuum. A primary goal of this conference was to give voice to practitioners' questions in order to communicate to researchers areas in which practitioners would like to have answers.

At the next two points on the continuum, the role of the practitioner in the research process becomes more central. The difference between these two points is the degree to which the practitioners have ownership over the research endeavor. At the point that is second from the top of the continuum, the work is collaborative and shaped by input from researchers and teachers. Researchers and practitioners work together to design, implement, and report findings of research studies. At the highest point on this continuum, researchers recognize that teachers, through "systematic inquiry" (Cochran-Smith & Lytle, 1993), can generate research-based knowledge of their own.

It is important to highlight here that the goal of teacher research is not to mimic the kind of research that academics produce. Although not entirely different from university research, teacher research is its own genre with some features that make it distinct. Recently, mathematics educators have made an argument for more collaborative action research; it is an area that has not been valued in the same ways as other types of research in mathematics education. Atweh (2004), for example, argued that action research, as a form of collaborative work, is important for pragmatic, epistemological, and political reasons. In fact, these sentiments have been expressed by teacher-researchers themselves (Herbel-Eisenmann & Cirillo, 2009). As this kind of work is embraced as legitimate, it is necessary "to directly confront controversial issues of voice, power, ownership, status and role in the broad educational community" (Cochran-Smith & Lytle, 1993, p. 22). Glimpses of legitimating teacher research can be found in the recent publication of teacher research in the *Journal for Research in Mathematics Education* (Gutstein, 2003) as well as NCTM's publication of a set of books with chapters by teacher-researchers (e.g., Van Zoest, 2006).

We shift now to articulating **Figure 3** in order to say more about the continuum of linking research and practice in the context of the work of practitioners. The continuum of the research-practice link in the context of practitioners' lives echoes similar kinds of shifts between low-level links to high-level links. At the lowest level, we find a common observation: that teachers neither draw on nor are interested in findings from research. Slightly higher on this continuum is a point at which teachers would like to read and learn from research findings, but the ways in which the institution of schooling functions makes this desire difficult. For example, teaching five to six sections per day in secondary schools with little or no common planning time does not foster the desire to spend time reading, considering, and discussing research findings. We also recognize the critique that most research findings are not written for practitioner audiences and, hence, may make research difficult to access if a practitioner does have time to read.

The next point on the continuum includes communication from a source removed from the research. For example, the practitioner may attend a professional development workshop or take a university class and read or hear about findings from research. At this point, however, the practitioner sees this body of knowledge as separate from his or her classroom (whether it be a K–12 classroom or a university mathematics or methods course), and the new information does not necessarily influence or transform the person's practice.

The subsequent point on the continuum recognizes that practitioners can use findings from research to impact their practice. In some ways, this point in the continuum represents a shift in the linkage between research and practice because the practitioner sees research as relevant to what they are doing. The increased involvement of practitioners moves us to the next point on the continuum: The practitioner is no longer only reading and incorporating ideas from research, he or she is also communicating questions and has some input into the kind of data that one might collect to understand the phenomena of interest. This report, for example, is evidence of an attempt for practitioners' voices to be heard with respect to their interests and concerns but at a broader level and across multiple practitioners.

As the involvement of the practitioner increases, we reach the final two points on the continuum: collaborative work and teacher research. At each point, the amount of time, involvement, and agency of the practitioner increases until the practitioner sees him or herself as a researcher who can generate knowledge and engage in systematic inquiry. As we reach the point on the continuum in which practitioners are doing research on their own practice, we are reminded of differences between the context of academic institutions and what is valued and rewarded and the context of public schools and teaching institutions and what is valued and rewarded (see, for example, Goos (2008) for her discussion of some of the differences). In the context of public schools, Cochran-Smith and Lytle (1993) suggest the following, in order to encourage teacher research:

> We must first address incentives for teachers, the creation and maintenance of supportive networks, the reform of rigid organizational patterns in schools, and the hierarchical power relationships that characterize most schooling. (p. 22)

Such changes are considerable; without more encouragement and support from the broader mathematics education community (including researchers, administrators, policymakers, and funding agencies), real changes in the state of mathematics education in the United States are more likely to be next to impossible. It will only be through a concerted effort to move higher along both of these continua that improvement in mathematics education can be realized through a concerted effort by all stakeholders. It is only this kind of concerted effort that will allow us to work together toward improving the teaching and learning of mathematics for every student.

Linking Research and Practice: Two Examples of Funded Projects

In this section, we present short descriptions of two projects in which authors of this document have been involved over the last decade. Both projects fall at the higher ends of the continua described above, and these projects were supported through funding from different programs at the National Science Foundation. In example 1, Beth Herbel-Eisenmann describes a project in which she supported a small group of teachers in identifying problems of their practice and designing/implementing action research projects to seek solutions to those problems. In example 2, Henry Kranendonk describes a district-wide professional development project and the collaborative research efforts undertaken by the district and local university researchers. We provide these short descriptions as evidence of funded projects in which both the researchers and practitioners fall on the "high" end of the linkage continua; these are examples of projects that closely link research and practice in mathematics education.

Example 1
Discourse Analysis: A Catalyst for Reflective Inquiry in Mathematics Classrooms

Beth Herbel-Eisenmann

From 2004–2009, I was involved in a collaborative project in which a long-term goal of the work involved supporting teacher-researchers through cycles of action research as they worked to become more purposeful about mathematics classroom discourse. Because of the one-shot workshop professional development experiences I had as a teacher and, later, my apprenticeship into teacher education through working in professional development schools, I was compelled to propose an experience that would be collaborative and in which participants felt they would have a voice in the work. I decided that study groups and action research fit my beliefs and goals. These forms of professional development fall under the broader umbrella of "collaborative work" and are built on the assumption that learning is a social activity and that communication among professionals (e.g., teachers, university faculty, mathematics specialists) is key to developing common language to ask questions and reflect on teaching (Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). After writing a proposal to the National Science Foundation for the project and receiving a CAREER award (#0347906), I was fortunate to find eight middle-grades (Grades 6–10) mathematics teachers and a graduate assistant who were interested in working in such a collaborative relationship toward more purposeful mathematics classroom discourse.

In the first phase of the project work, we drew on an "apprenticeship model" (Doerr & Tinto, 2000) for beginning action research. The teacher-researchers allowed the university-researchers to collect baseline data by videotaping one class period each day for 4 weeks, 1 week each in September, November, January, and March of the 2005–2006 school year. During the months that we did not videotape, the group met and shared information about their teaching, the types of curriculum materials and tasks they used, and engaged in some activities in which we analyzed mathematical tasks or other artifacts of teaching and learning. After the videotaping was completed, the university-researchers did some quantitative and qualitative discourse analysis using the transcripts and videos (see Herbel-Eisenmann (in press) for specific information about these analyses). At a full-day retreat the following year, each teacher-researcher was provided a binder with a set of analytic memos describing some of the discourse patterns we noticed, along with our interpretations and illustrations of each pattern. The teacher-researchers spent a few hours individually reading and taking notes on his or her analytic memos. We then discussed the findings in small groups and as a whole group, encouraging the teacher-researchers to talk about their reactions, understandings, and alternative interpretations of the information in the analytic memos. In fact, throughout the duration of the project, almost all analyses that the university-researchers completed of the teacher-researchers' classroom discourse were discussed with the teacher-researchers to allow them to share their insights, interpretations, and disagreements (see, for example, Herbel-Eisenmann, Wagner, & Cortes, 2008; Wagner & Herbel-Eisenmann, 2008).

In the second phase of the project work, we spent about 1 year as a study group in which the teacher-researchers selected books, book chapters, and articles related to classroom discourse (from a library of readings compiled by myself) to read, and we met to discuss the readings. All our discussions began with the group sharing ideas that were interesting, important, unclear, troublesome, and so on, from the set of readings we had done. When the discussion seemed to slow down, I would then shift to going article by article to make sure that each teacher-researcher was able to talk about aspects of any article that resonated with him or her in some way. The discussions were often intense and went back and forth from trying to make sense of the ideas to trying to better understand the ideas in relationship to their classroom practices (for a detailed illustration of the movement between research and practice, see Herbel-Eisenmann, Drake, & Cirillo, 2009).

In the subsequent phase of the work, the teachers selected aspects of their classroom discourse to change and then carried out cycles of action research in which they studied the impact of the changes on students' social and mathematical experiences (see Herbel-Eisenmann & Cirillo (2009) for teacher-researchers' accounts of their action research projects). During this phase, the teacher-researchers collected their own videotapes and other artifacts of practice and used these to engage in systematic inquiry related to their goal(s). Prior to the action research cycles, each teacher-researcher created a visual mapping of what she or he felt was most important to his or her instructional decision-making. For example, they wrote, "Math is about thinking." These mappings became the standards by which the teacher-researchers evaluated their own teaching. When they read any analyses the university-researchers did, when they watched themselves on video, and when they asked for input from others in the group, the mappings were typically used as their underlying beliefs and values for what they wanted to happen in their classrooms. In this way, the criteria for success and progress were determined by the teacher-researchers and within the collective questioning and sharing with the group, rather than by an external source.

In summer 2008, the teacher-researchers joined me and three graduate assistants for a writing retreat in which I structured writing time based on a modified version of a writer's workshop. Drawing on information they shared in meetings, I provided each teacher with a potential idea related to his or her classroom discourse, along with three different framings of that idea. After 5 days, each teacher-researcher had a draft of a book chapter. The book (Herbel-Eisenmann & Cirillo, 2009) was released at the 2009 NCTM Annual Meeting and Exposition. It is through this process that the teachers feel their voices have been heard as they share their reflections, struggles, and future plans. It also allowed us to disseminate the collaborative model that we developed in our work together.

Example 2
The Milwaukee Mathematics Partnership

Henry Kranendonk

To understand how K–12 teachers, mathematicians, and mathematics education professors, as well as district administrators, worked together on the research undertaken in this project, a brief summary of the Milwaukee Mathematics Partnership (or MMP) provides insights into how significant research has the capacity to bring together the researcher and practitioner communities.

The Milwaukee Mathematics Partnership was awarded $20 million in 2003 for a 5-year comprehensive professional development program for mathematics teachers through the National Science Foundation's Mathematics and Science Partnership program. Although the grant concluded in 2008, the Partnership continues with support from the state of Wisconsin through educational reform funds and district funds. The primary partners of the MMP are the University of Wisconsin–Milwaukee (UWM), the Milwaukee Area Technical College (MATC), and the Milwaukee Public Schools (MPS). Mathematics achievement, as measured by the state's annual achievement test of public school students (Wisconsin Knowledge and Concepts Examination, or WKCE), has improved annually since the formation of the partnership. Research is clearly directed at analyzing how the partnership is involved in the changes in student achievement.

The research questions that evolved around the work of the MMP provide an example of how practitioners and researchers were partners in developing the scope of the project. As a result of this close link, thousands of teachers contributed data telling the story of their progress in understanding mathematical content as well as their connections to the leadership training. The evolution of the MMP is well documented in the type of research questions that emerged throughout the Partnership. The questions investigated in the early years of the project included questions investigating the central theme of the project, namely, the impact of the MMP within and across schools. The focus of the MMP evaluation was to analyze if student achievement changed over time in relation to constructs developed from responses on a project-developed survey. While informative, more specific and detailed analyses were needed.

A central strategy of the MMP was the establishment of a mathematics teacher leader position in every school and the inclusion of this individual on the school's learning team. Each team consisted of administrators and key teachers addressing instructional issues related to student achievement at their school. The specific impact of the new mathematics teacher leader (MTL) position in every school on the school professional community and possible links to mathematics achievement were initially not guided by clear researchable questions. However, this early work evolved into a refined data collection process that helped define the MTL position and the impact on the school community.

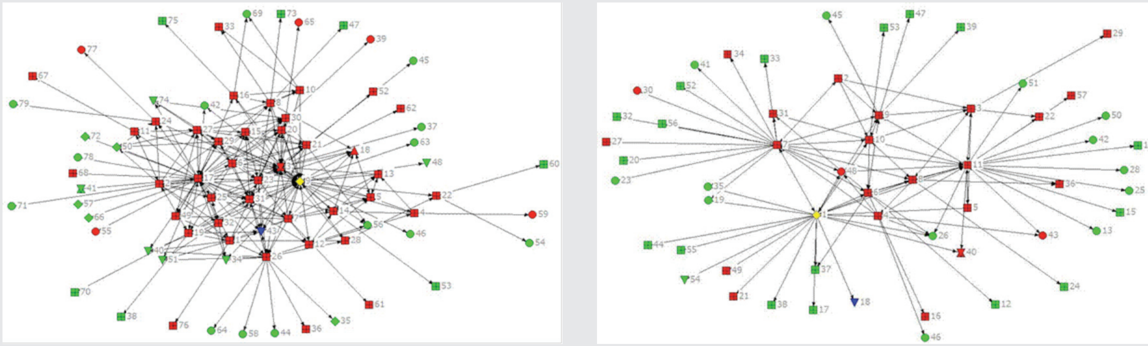
During the first 4 years of the project, the MTL was a full-time teacher of mathematics who willingly accepted a limited range of leadership responsibilities as defined by the MMP. An initial group of approximately 150 MTLs were involved in monthly professional development meetings that centered on the "three pillars" of the MMP, namely, formative assessment practices, mathematics content knowledge of teachers, and teacher leadership.

In the 2nd year, the MTLs were provided a more focused list of responsibilities and were clearly growing in their leadership skills and work within their schools. The MTL feedback indicated that each school was in varying stages of implementing improved mathematics instruction. To address a specific school's challenges, the MTLs were provided a self-assessment tool for gathering information about individual teacher needs, as well as the collective focus of the school. This tool was considered a roadmap for the MTL that provided direction in the primary contribution of this project, namely, providing teacher leadership for mathematics. The Learning Team Continuum outlined five stages of how teachers individually and collectively would evolve their school's culture into one that supports their students learning of mathematics. For the researchers, the Continuum provided a new way to measure the leadership provided by the MTL.

As the researchers developed metrics and descriptors to measure the stages of a school's involvement in the Continuum, the teachers were also given direction in providing leadership. Each year, the MTLs were asked to reflect upon and identify the placement of their schools on the Continuum and to consider plans for moving their schools forward. It was conjectured that the further along a school was on the Continuum, the more pronounced would be the achievement performance and achievement gain of a school. However, the specific approach to linking the Continuum to improved student achievement was still not clear, but the questions that evolved over this time reflect the interest that both researchers and practitioners had in analyzing the role of teacher leadership in mathematics and the links to student achievement.

The research completed after the 3rd year of the project indicated that the primary obstacle in effectively providing leadership as define by the Learning Team Continuum was the availability of the MTL to work with other teachers. As the MTL became a key person in changing the school culture for mathematics, the need to provide time for the MTL to complete the expectations of this position grew. Various constituencies, including the Wisconsin governor's office, reviewed the research and evaluation results. A review of this project-based research resulted in a $10 million special state grant to fund 113 "released-time" mathematics teacher leaders. The additional funding essentially created a new position in which a school's MTL was released from direct classroom instruction for approximately 80% of the day. The transition to this released-time model began 2nd semester during the 5th year of the MMP with full implementation the following fall. During this released time, the MTL was expected to implement an Action Plan that addressed how its school would advance at least one stage on the Continuum during an academic year. In addition, this new position provided the evaluators the opportunity to compare implementation of different MTL models, namely, released versus nonreleased, and how the specific model might contribute to improved student achievement.

A network that specifically measured the position an MTL held in the interactions and discussions involving mathematics was used to analyze the research questions related to leadership. The Social Network Analysis (SNA) started with teachers and administrators identifying whom they sought out in their mathematics discussions. The resulting network analysis was used to identify schools in which the connections to the MTL were more frequent, or "more dense." The visual representation of these data (see **Figure 4**) had a profound effect on teachers' understanding the project's goals involving leadership. The varying ranges of density provided both the researchers and the practitioners (the MTLs) opportunities to reflect on the leadership challenges. Over time, other factors of leader-



This is an example of a high-density network. The MTL is located in the center of this network, shown in yellow.

This network is more loosely defined. Although it is important to the network, the MTL is not as central. The density measure of this school is lower than the one represented to the left.

Figure 4. Examples of SNA analyses.

ship were analyzed and reflected in the SNA analysis, such as the inclusion of other key leaders in a school's implementation of their mathematics Action Plans and the effectiveness of a school's learning team.

The MMP was conceptualized to address the Milwaukee Public Schools fragmented mathematics instruction. At that time, teachers independently developed their own priorities of the mathematics skills and topics students were presented in their classes. The resulting partnership with the University of Wisconsin–Milwaukee provided the district teachers a vision for mathematics instruction and a more consistent delivery of instruction. The partnership also provided the university leaders the opportunity to understand the classroom challenges and to address the needs of teachers based on the feedback provided to them through regular meetings with the MTLs.

Initially, teachers were skeptical that the university leaders would provide meaningful connections to their needs. However, as the MTL evolved into a recognized point person in mathematics instruction, the interactions with the university improved. The need for all parties (the MTL, the principals, mathematics teachers, the university leadership) to understand the significance of this work resulted in respect for the evaluation process and in the role of the researchers to provide direction and clarity for teachers, MTLs, and school leaders. The research focused on an analysis of the varying models of leadership in the district schools and how these leadership models changed the way mathematics was taught. Teachers and administrators realized that they also gained insights from this analysis to improve their schools. An appreciation of researchers and their analysis of the data provided by practitioners and for practitioners resulted in a partnership that continues to evolve.

This project that links research and practice has resulted in several publications and presentations, including, but not limited to, Bedford, Hollinger, and Huinker (2006); Doyle, Hanssen, and Huinker (2007); Hanssen and Durland (2007); Hedges, Huinker, and Steinmeyer (2004); and Post, Pugach, Harris, and Hedges (2006).

Implications for Linking Research and Practice

In an effort to advance the purposes of the NCTM Research Agenda Project, we address a collection of implications of this work for the broader mathematics education community. It is our goal that all stakeholders accept the role they have in this endeavor and make a concerted effort to impact positively not only the culture that currently exists among mathematics education researchers and practitioners but also the extent to which research and practice are linked. Stakeholders from across our communities must accept responsibility for meeting this vision.

Forging a new alliance among practitioners and researchers and other professional stakeholders involved in mathematics education requires expanding communication and collaboration. Mathematics education researchers, practitioners, and those involved at the policy levels (including NCTM) have a responsibility to demonstrate dispositions that allow an open dialogue among the stakeholders. In particular, we propose the following responsibilities, by stakeholder group, that are necessary for building a collaborative research community:

Researchers have the responsibility to develop working relationships with practitioners at all levels to better understand specific district and classroom contexts and limitations. The development of these relationships enhances the collaborations needed to design research projects that meet the needs and interests of both practitioners and researchers while utilizing the expertise, experiences, insights, and knowledge of both. It is important that researchers both publish their studies in journals and present research at conferences that are closely connected to practitioners. In addition, established researchers need to encourage and support graduate students' contributions to practitioner journals and conferences. Attention to educating colleagues at research institutions about the importance of the research-practice link is also warranted.

Practitioners have a responsibility to develop alliances with researchers to identify similar interests and participate in research design and/or implementation. The articulation of classroom, district, and state-level challenges to researchers is a necessary component of a collaborative research community. Practitioners need to be actively involved in reading and reviewing research that is published or presented at conferences; further, they should consistently challenge the research community to provide meaningful explanations of what has been learned from research. Practitioners (and researchers) should participate in learning communities focused on research and the ramifications of the research. In addition, practitioners should identify and support the development of teacher leaders/instructional engineers (LPRTF, 2005) "who would, in turn, serve as the primary conduit for communicating these research results in a practical and useable fashion to the classroom teacher" (Heid, et al., 2006, p. 82). Supported by researchers, practitioners can design action research cycles related to the research-guiding questions presented in this document. Researchers and practitioners can analyze the data and contribute to the interpretations of the analyses.

Mathematics professional organizations and leaders, including NCTM, have the responsibility to continue to provide opportunities for researchers and practitioners to collaborate and learn from and with each other. This may require support or enhancement of current initiatives and/or development of new initiatives that support practitioners' understanding and use of research. NCTM, as well as other organizations, should encourage dialogue about mathematics education research through traditional workshops, e-workshops, and printed and electronic publications. NCTM should continue to creatively support the dissemination of research through its practitioner journals and Web site. Mathematics leaders and organizations should lead the establishment of a "research think tank," consisting of practitioners, policy-level stakeholders, and researchers, with a purpose of identifying a further focus for research that will move the mathematics education community forward. All organizations must encourage their members to form study groups to discuss, debate, and evaluate research findings and disseminate the information contained in this document to stakeholder groups.

Funding agencies also have some responsibility in the effort to better link mathematics education research and practice. Research that studies what mathematics students learn and when and how they learn it, as well as who teaches mathematics, how it is taught, and what resources are essential for learning and teaching can shape the direction, quality, and influence of mathematics education in the United States. These studies can help define policy on who has access to quality mathematics and how success in mathematics is defined. Resources required for the studies aimed at addressing the concerns of practitioners are limited. What is researched, when, by whom, and with whom is dictated by policies related to funding opportunities.

More funding opportunities are needed for researchers and practitioners to collaborate in *empirical studies* to inform policy deliberations in a timely fashion so as to be ready when policy decisions are on the table. An opportunity for this type of research presents itself today as many state departments of education entertain the establishment of end-of-course assessments, many of which will have high-stakes consequences for students. During the pilot phase, researchers and practitioners could conduct studies of the influence of such assessments on classroom instruction, student motivation, teacher satisfaction, and so on. There is a particular need for mathematics education researchers to undertake such research as they relate to the adoption of end-of-course high school assessments, as these assessments are becoming increasingly popular across the nation. Further, funding agencies could include a requirement that proposed projects include specific plans for disseminating research results to practitioner audiences, perhaps through targeted publications or presentations, workshops conducted for schools/districts in the area, short white papers sent to policymakers, and/or Web sites targeted to practitioners.

Funding also needs to be available for practitioners and researchers to work together to determine the impact of policies on mathematics education. This type of research can provide evidence of whether or not policies achieve their intended goals. It can determine the impact of policies including what groups are most affected. The results of these studies can be fed back into policy redesign and refinement. Coupled with studies of policy implementation, these impact studies can provide significant insight into understanding the influence of policies and can contribute to a broader understanding of the theories and concepts underlying those policies. Resources are needed for mathematics educators to focus on studying the implementation and impact of policies.

Strong collaborations among researchers and practitioners (including policymakers) are essential. Regular and ongoing structures and processes must be established and funded to connect these communities around developing, advocating, and evaluating the teaching and learning of mathematics as well as the implementation and impact of these policies on mathematics education. Researchers must be able and willing to respond to critical, and sometimes urgent, questions from the field if practices and policies are to improve mathematics teaching and learning at all levels. Practitioners must be willing to open their doors, share their data, and participate in research efforts. Funders must be willing to incorporate programs into their portfolios to support such efforts. The NCTM Research Agenda Project, as originally conceived, envisioned the formation of a new type of professional community that can sustain focused attention on use-inspired inquiry into key issues. The August 2008 Research Agenda Conference brought together mathematics education practitioners and researchers with the purpose of strengthening the link between mathematics education research and practice by creating this document. Underlying this work is the belief that to improve mathematics teaching and learning in the United States in a serious manner, communication among classroom teachers, district-level and state-level stakeholders and policymakers, and mathematics education researchers must be nurtured, supported, and sustained. With leadership from the researchers and practitioners in NCTM, the link between research and practice can be a powerful engine for progress in our field.

References

Atweh, B. (2004). Understanding for changing and changing for understanding: Praxis between practice and theory through action research in mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 14–15). New York: Kluwer.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). San Francisco: Jossey-Bass.

Battista, M. T., et al. (2007). Connecting research and practice at NCTM. *Journal for Research in Mathematics Education, 38*, 108–114.

Bedford, P., Hollinger, R., & Huinker, D. (2006). *Using a structured protocol for analyzing and learning from student work. Wisconsin Teacher of Mathematics*, *57*(1), 15–21.

Burkhardt, H., & Schoenfeld, A. H. (2003). Improving educational research: Toward a more useful, more influential, and better-funded enterprise. *Educational Researcher, 32*(3), 3–14.

Carnine, D. (1997). Bridging the research-to-practice gap. *Exceptional Children, 63*, 513–521.

Clements, D. (2007). Curriculum research: Toward a framework for "research-based curricula." *Journal for Research in Mathematics Education, 38*, 35–70

Cochran-Smith, M., & Lytle, S. L. (1993). *Inside/outside: Teacher research and knowledge*. New York: Teachers College Press.

Doerr, H. M., & Tinto, P. P. (2000). Paradigms for teacher-centered classroom-based research. In A. E. Kelly and R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 403–427). Mahwah, NJ: Erlbaum.

Donovan, M. D., Bransford, J. D., & Pellegrino, J. W. (Eds.). (1999). *How people learn: Bridging research and practice.* Washington, DC: The National Academies Press.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist, 23*(2), 167–180.

Doyle, L. H., Hanssen, C., & Huinker, D. (2007). *A study of school teams: Democratically functioning or malfunctioning.* Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Enyedy, N., & Mukhopadhyay, S. (2007). They don't show anything I didn't know: Emergent tensions between culturally relevant pedagogy and mathematics pedagogy. *The Journal of the Learning Sciences*, *16*(2), 139–174.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, NC: Information Age.

Goos, M. (2008). Sociocultural perspectives on learning to teach mathematics. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education, Vol. 4: The mathematics teacher educator as a developing professional* (pp. 75–91). Rotterdam: Sense Publishers.

Grouws, D. A. (1992). *Handbook of research in mathematics teaching and learning.* Reston, VA: National Council of Teachers of Mathematics.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*, 37–73.

Hanssen, C., & Durland, M. (2007). *Maximizing evaluation impact by maximizing methods: Social network analysis combined with traditional methods for measuring collaboration*. Paper presented at the annual meeting of the American Evaluation Association, Baltimore.

Hedges, M., Huinker, D., & Steinmeyer, M. (2004). Unpacking division to build teachers' mathematical knowledge. *Teaching Children Mathematics*, *11*, 478–483.

Heid, M. K., et al. (2006). The challenge of linking research and practice. *Journal for Research in Mathematics Education, 37*, 76–86.

Herbel-Eisenmann, B. (in press). Beyond tacit language choice to purposeful discourse practices. In L. Knott (Ed.), *The role of mathematics in producing leaders of discourse*. Charlotte, NC: Information Age.

Herbel-Eisenmann, B., & Cirillo, M. (Eds.). (2009). *Promoting purposeful discourse: Teacher research in mathematics classrooms*. Reston, VA: National Council of Teachers of Mathematics.

Herbel-Eisenmann, B., Drake, C., & Cirillo, M. (2009). Muddying the clear waters: Teachers' take-up of the linguistic idea of revoicing. *Teaching and Teacher Education*, *25,* 268–277.

Herbel-Eisenmann, B., Wagner, D., & Cortes, V. (2008). Encoding authority: Pervasive lexical bundles in mathematics classrooms. Paper read at Proceedings of the 32nd Conference of the International Group for the Psychology of Mathematics Education, Morelia, Mexico.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte, NC: Information Age.

Labaree, D. (2003). The peculiar problems of preparing educational researchers. *Educational Researcher*, *32*(4), 13–22.

Lester, F. K. (2007). *Second handbook of research on mathematics teaching and learning.* Charlotte, NC: Information Age.

Linking Research and Practice Task Force. (2005). Harnessing the power of research for practice. Report presented to the National Council of Teachers of Mathematics Board of Directors, Reston, VA.

Lipka, J., Webster, J. P., & Yanez, E. (Eds.). (2005). *Journal of American Indian Education* [Special Issue], *44*(3).

Lloyd, J. W., Weintraub, F. J., & Safer, N. D. (1997). A bridge between research and practice: Building consensus. *Exceptional Children, 63*, 535–538.

Loucks-Horsley, S., Love, N., Stiles, K. E., Mundry, S., & Hewson, P. W. (2003). *Designing professional development for teachers of science and mathematics* (2nd ed.). Thousand Oaks, CA: Corwin Press.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel*. Washington, DC: U.S. Department of Education.

National Research Council (NRC). (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Washington, DC: National Academy Press.

National Research Council. (2003). *Assessment in support of instruction and learning: Bridging the gap between large-scale and classroom assessment—Workshop report*. Washington, DC: National Academies Press.

Post, L., Pugach, M., Harris, S., & Hedges, M. (2006). The Teachers-in-Residence program: Veteran urban teachers as teacher leaders in boundary spanner roles. In K. Howey & N. Zimpher (Eds.), *Boundary spanners* (pp. 213–236). Washington, DC: American Association of State Colleges and Universities and National Association of State Universities and Land-Grant Colleges.

Romagnano, L. (1994). *Wrestling with change: The dilemmas of teaching real mathematics.* Portsmouth, NH: Heinemann.

Silver, E. A. (2003). Border crossing: Relating research and practice in mathematics education. *Journal for Research in Mathematics Education, 34*, 182–184.

Silver, E. A., & Herbst, P. (2007). Theory in mathematics education scholarship. In F. K. Lester (Ed.), *Second handbook of research on teaching and learning mathematics* (pp. 39–67). Charlotte, NC: Information Age.

Sowder, J. T. (2000). Relating research and practice. Opening Session, National Council of Teachers of Mathematics (NCTM) Research Presession 2000, Chicago.

Sowder, J. T. (Ed.). (2002). *Lessons learned from research*. Reston, VA: National Council of Teachers of Mathematics.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–223). Charlotte, NC: Information Age.

Stein, M. S., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal,* 33, 455–488.

Wagner, Jon. (1997). The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher 26*(7), 13–22.

Wagner, D., & Herbel-Eisenmann, B. (2008). Just don't: The suppression and invitation of dialogue in mathematics classrooms. *Educational Studies in Mathematics, 67*(2), 143–157.

Van Zoest, L. (Ed.). (2006). *Teachers engaged in research: Inquiry into mathematics classrooms*. Greenwich, CT: Information Age.

Appendix A

The Phases of the NCTM Research Agenda Project

The NCTM Research Agenda Project, led by NCTM Presidents Francis (Skip) Fennell and Henry Kepner, involved a multitude of volunteers. In the following paragraphs, we describe different aspects of the project, providing specific information about the evolution of this document and the people involved in its creation.

Evolution of the NCTM Research Agenda Project. The NCTM Research Agenda Project, having its roots in the Linking Research and Practice Task Force report described above, was further conceptualized by a number of groups from 2006–2007. First, a Research Agenda Planning Committee met in June 2006 and was led by Jim Fey (see the Acknowledgments section for a list of members of this committee). This group discussed the need and feasibility of hosting a conference that would bring together practitioners and researchers and would result in a set of research priorities based on practitioner needs. As part of this initial work, the members of this committee created a proposal that could be used for soliciting external funding. The last act of this committee was to put forth a motion to the NCTM Board of Directors, supporting such a conference and asking the Board to approve the solicitation of external funding to monetarily support the conference. The NCTM Board of Directors voted "yes" to this motion in July 2006.

Subsequently, a new group was established to begin conceptualizing such a conference (see Appendix B for a list of the members of the first iteration of the NCTM Research Agenda Conference Planning Committee). This planning committee, led by Nora Ramirez, conducted a series of conference calls in Fall 2006 and ultimately finalized a grant proposal that was sent to the National Science Foundation (NSF), seeking funding for the Research Agenda Conference.

In Summer 2007, once funding had been secured from the NSF, the NCTM Research Agenda Conference Planning Committee was reconfigured to include both practitioners and researchers (see Appendix B for a list of members). This committee, as described below, worked from Fall 2007 through August 2008 and was disbanded at the completion of the Research Agenda Conference.

National Science Foundation Funding. The National Science Foundation, under the Division of Research on Learning in Formal and Informal Settings (DRL), provided funding to NCTM for a project titled "Linking Research and Practice—Challenges and Opportunities" (0736558). In the proposal for this project, NCTM requested funding to support a working conference with the following goals:

1. Building a Community

1.1 Begin building a community of researchers, practitioners, and policymakers working collaboratively to address important problems of practice in mathematics education.

1.2 Begin building a network of practitioners who are supportive of mathematics education research that investigates significant problems of practice.

2. Identifying Research Questions

2.1 Develop an initial set of use-inspired basic research territories—major domains of work in mathematics education that impact achievement of mathematical proficiency of all students.
2.2 Reach consensus on the major researchable questions in each of the important research territories—a *research agenda* that addresses significant problems of practice and is informed by experiences and expertise of practitioners.
2.3 Develop an overview of what is currently known and what remains to be studied about each critical research question.

3. Initiating a Sustainable Process

3.1 Develop strategies for facilitating the professional community's accumulation of knowledge about critical aspects of practice in mathematics education.
3.2 Develop mechanisms for ongoing review and reporting on the state of our knowledge about the practice of mathematics education and for the identification of new research questions. (NSF Proposal, J. Rubillo, PI)

The NCTM Research Agenda Conference and accompanying activities resulting from this funding are described below. All activities, including the conference, were conceptualized, planned, and organized by the committees described above.

August 2008 NCTM Research Agenda Conference. The NCTM Research Agenda Conference was held at the Maritime Institute outside of Baltimore, Maryland, August 4–8, 2008. For the majority of the conference, participants (see Appendix C) worked in their groups as described below, with whole-group feedback sessions occurring at the end of each working day. The conference opening session included a panel presentation by Bill Bush, Linda Davenport, and Randy Philipp. Joan Ferrini-Mundy provided a keynote on the last day of the conference.

Prior to the August 2008 Conference, each participant conducted a focus group with practitioners, gathering "important questions of practice most crucial to practitioners" (preconference e-mail to conference participants). Approximately 350 questions resulted from these focus groups. The conference participants relayed the focus group questions to NCTM, where they were sorted into the eight domains that would constitute the working groups for the August 2008 Conference: Assessment, Curriculum, Equity, Policy, Student Learning, Technology, Teacher Preparation/Professional Development, and Teaching. The Conference Planning Committee recognized that these domains are not distinct and that a number of practitioner questions might fall in the intersection of two or more domains (e.g., How can teachers assess students' knowledge in order to make good instructional decisions?). The choice of these eight domains, in the end, created manageable-sized working groups for the conference and focused the work of the participants.

Conference participants were preassigned to a working group and received a number of readings, chosen by the working group coleaders and particular to the work of the group, prior to the conference. The focus group questions, gathered and organized prior to the conference, became the "data" that the working groups used to identify the research-guiding questions contained in chapter 2. The working groups were charged with analyzing the practitioner questions and identifying themes that represented the majority of the practitioner questions. Working group members were sent to their groups with this prompt: "Ten years from now, if we had answers to these questions, then we would have really moved the field of mathematics education forward and helped to better define the field." The Policy group followed a different path than the other groups, resulting not with a list of research-guiding questions but with a set of recommendations for policy-based research (see chapter 3, which includes portions of this working group's report). August 2008 conference working groups submitted a working group report at the end of the conference containing research-guiding questions and supporting text connecting those questions to the original practitioner questions as well as justifying the importance of the questions for the field.

The NCTM Research Agenda Writing Group. NCTM President Henry Kepner asked six of the August 2008 Conference participants (the authors of this document) to continue the work of this project as members of the writing group that would assemble the full document. This group met via conference call and e-mail in Fall 2008 and for a writing meeting in January 2009, where they wrote the preface, introduction, and chapters 1 and 4. In addition, the writing group edited the Conference Working Group documents (found in chapter 2) and the Policy Working Group document (portions of which are found in chapter 3).

Full document review. This document underwent review and revision. In February 2009, the full document was sent to all participants of the August 2008 Conference for review and comment. Then, in April 2009, the writing team held two different sessions at the NCTM annual meeting: one during the Research Presession and one during the main conference. Session attendees were introduced to the project and document and were provided the opportunity to comment on the contents of the document. All review comments were compiled at NCTM, considered by the writing team, and used to revise the document to its present form.

Appendix B

NCTM Research Agenda Planning Committees

NCTM Research Agenda Planning Committee

Nora Ramirez, NCTM Board Liaison
Arizona State University

Matt Larson
Lincoln Public Schools

Michael T. Battista
Ohio State University

Karen D. King
New York University

M. Kathleen Heid
Pennsylvania State University

Cathy Brown
Indiana University

Jim Barta
Utah State University

Jim Fey
National Science Foundation

Judith Reed Quander, Staff Liaison
National Council of Teachers of Mathematics

Ken Krehbiel, Staff Liaison
National Council of Teachers of Mathematics

NCTM Research Agenda Conference Planning Committee

Fran Arbaugh, Cochair
University of Missouri

Vanessa Cleaver
Little Rock School District

Brad Findell, Cochair
Ohio Department of Education

Jim Fey
National Science Foundation

Karen D. King
New York University

M. Kathleen Heid
Pennsylvania State University

Judith Reed, Staff Liaison
National Council of Teachers of Mathematics

Rick Scott
New Mexico Department of Education

Margaret (Peg) Smith, Board Liaison
University of Pittsburgh

Appendix C

August 2008 Research Agenda Conference Participants

Assessment Working Group

Jere Confrey
North Carolina State University

Linda Griffith, Working group leader
University of Central Arkansas

Peter Kloosterman, Working group leader
Indiana University

Henry Kranendonk
Milwaukee Public Schools

Marge Petit
Marge Petit Consulting

Jeff Shih
University of Nevada

John Sutton
RMC Research Corporation

Equity Working Group

Marta Civil
University of Arizona

Lou Matthews
Georgia State University

Jerry Lipka
University of Alaska

Nora Ramirez, Working group leader
Arizona State University

Rick Scott
New Mexico Department of Education

Marilyn Stutchens, Working group leader
Auburn University

Curriculum Working Group

David Bressoud
Macalester College

John Choate
Groton School

Kathryn Chval, Working group leader
University of Missouri

Beth Herbel-Eisenmann
Michigan State University

May Samuels, Working group leader
Newark Public Schools

Policy Working Group

Bill Bush, Working group leader
University of Louisville

Brad Findell
Ohio Department of Education

Karen D. King
New York University

Cathy Seeley
Dana Center, The University of Texas at Austin

Mary Kay Stein
University of Pittsburgh

Student Learning Working Group

Mike Battista
Ohio State University

Susan Brown
York High School

M. Kathleen Heid, Working group leader
Pennsylvania State University

Eric Knuth
University of Wisconsin–Madison

Olga Torres, Working group leader
Tucson Unified School District

Delinda van Garderen
University of Missouri

Teacher Preparation/Professional Development Working Group

Angela Allen
Brookline Public Schools

Pat Baltzley, Working group leader
Baltimore County Public Schools

Tim Boerst
University of Michigan/South Redford School District

Denise Mewborn
University of Georgia

Randy Philipp, Working group leader
San Diego State University

Teaching Working Group

Linda Davenport
Boston Public Schools

Megan Franke
University of California Los Angeles

Jim Hiebert
University of Delaware

Matt Larson, Working group leader
Lincoln Public Schools

Margaret (Peg) Smith, Working group leader
University of Pittsburg

Dorothy White
University of Georgia

Technology Working Group

Glen Blume
Pennsylvania State University

Karen Hollebrands
North Carolina State University

Darshan Jain
Adlai Stevenson High School

Jon Wray, Working group leader
Howard County Public Schools

Rose Zbiek, Working group leader
Pennsylvania State University

Appendix D

Set of 25 Research-Guiding Questions Generated at the August 2008 NCTM Research Agenda Conference

1. What are the characteristics of a comprehensive mathematics assessment system that provides instructional guidance, supports educational decision making, measures continuous growth, and monitors system progress and accountability?
2. In what ways do different curricular approaches and/or combinations of those approaches support or impede students' development of mathematical proficiency?
3. What is the impact of culturally responsive/relevant and critical pedagogies and programs on students' access and opportunity to learn mathematics and develop mathematics proficiency in various school, cultural, and societal contexts?
4. What are coherent frameworks for characterizing the development of student thinking about specific mathematical concepts or processes?
5. What are the mathematical concepts and reasoning processes that prepare and enable students to learn and use algebra?
6. What should be the goals of professional learning, and how will we measure attainment of these goals in terms of teacher growth?
7. How does students' engagement with mathematical, collaborative, and communication technologies influence what students learn and how they engage in mathematical processes?
8. What are the characteristics of a technology-enabled learning setting that contribute to students' learning of mathematics?
9. What "interventions" help teachers reach students who they perceive have difficulty developing an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition?
10. How can teachers engage students in a productive struggle to support the development of an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition?
11. What knowledge, skills, and dispositions does the education community need to validly utilize student assessment data in mathematics to effectively promote equity, improve and focus instruction, evaluate programs, identify students' needs for intervention, and positively impact student learning?
12. What are viable progressions of students' mathematical thinking and learning for specific mathematical concepts and procedures? How are progressions of students' mathematical thinking and learning for specific mathematical concepts and procedures influenced by different curricular approaches?

13. How do the different curricular approaches and combinations impact different groups of students under different conditions? How well suited are different curricular approaches for different purposes?
14. What are proactive approaches for addressing concerns and informing decisions of parents, administrators, and policymakers about mathematics curriculum?
15. What elements of mathematics programs and practices result in achieving equity related to multilingualism, race/ethnicity, exceptionalities, gender, and socioeconomics in various school, cultural, and societal contexts?
16. What is the long- and short-term impact of mandated assessments on students' access and opportunity to learn mathematics and develop mathematics proficiency in various school, cultural, and societal contexts?
17. What elements of the preservice-in-service mathematics education continuum promote and sustain equitable teaching related to multilingualism, race/ethnicity, exceptionalities, gender, and socioeconomics in various school, cultural, and societal contexts?
18. What relationships exist among procedural knowledge, conceptual knowledge, and mathematical thinking?
19. What are characteristics of student thinking that impede effective mathematics learning?
20. To what extent are various forms of teacher learning effective?
21. What should be the qualifications of those who deliver teacher learning, and how should these individuals be prepared?
22. What *knowledge and experiences* do prospective and practicing teachers need to help them incorporate technology in doing, learning, and teaching mathematics in technology-enabled settings?
23. What are the differential effects of specific types of technology-enabled settings on learning by students from various groups, including those receiving special services?
24. How can teachers facilitate explicit attention to key mathematical relationships within and across lessons to support the development of an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition?
25. How can teachers plan for, reflect on, and improve instruction to help students develop an integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition?