A Family's Guide to High School Mathematics:

Making Reasoning and Sense Making the Focus

WHAT KINDS of experiences should high school mathematics offer students? Families that understand what their students should experience in mathematics throughout their high school years are in the best position to lend their support and encouragement.

The recommendations in this pamphlet are based on a new set of guidelines for high school mathematics, *Focus in High School Mathematics: Reasoning and Sense Making*, from the nation's leading advocate for more and better mathematics, the National Council of Teachers of Mathematics.

Why is mathematics important for your high school student?

A strong background in high school mathematics prepares students for success, regardless of what they aspire to do in the future. Mathematics is central to scientific and technical careers. Moreover, it is increasingly important in a wide range of careers in areas as diverse as finance and sports journalism. The Internet has resulted in an explosion of new careers in mathematics and statistics for those who have the skills to analyze and harness the power of the huge amount of data at people's fingertips.

Success in mathematics opens doors for students. By the same token, however, an inadequate preparation in mathematics often rules out certain careers or forces students who want to pursue them to complete years of make-up work in mathematics.

Furthermore, all citizens need a solid background in mathematics to make informed and reasoned decisions about their roles and responsibilities in their personal and public lives. Mathematical understanding helps individuals manage their personal finances, decide which public policies deserve their support, and select cost-effective insurance or health plans.

In today's highly technological world, having a solid preparation in mathematics is a tremendous asset. Those without a good mathematical grounding risk being taken advantage of or left behind.

Why is it important to have a broad preparation in mathematics?

You yourself may have experienced a high school mathematics curriculum that focused primarily on algebra and precalculus, along with some study of geometry. Today's students need mathematical experiences over a broader range to prepare them for their future success.

For example, your high school student needs more skill in working with statistics than students in previous generations did. Statistics now permeate many fields of endeavor, from business to medicine to the social sciences.

NCTM® NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

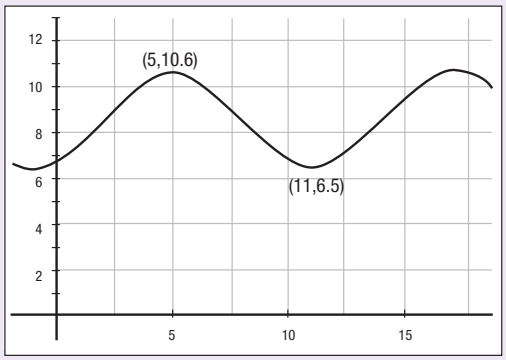
"TIDAL WAVES"

The captain of a shipping vessel must consider the tides when entering a seaport because the depth of the water can vary greatly from one time of day to another. Suppose that high tide in a certain port occurs at 5:00 AM, when the water is 10.6 meters deep, and the next low tide occurs at 11:00 AM, when the water is 6.5 meters deep. Develop a mathematical model that will predict the water's depth as a function of the time elapsed since midnight.

In the Classroom

The second-year algebra students working on this problem have had experience with transformations of linear and quadratic functions and are familiar with the graphs of the sine and cosine functions. The dialogue below shows several students' reasoning about this task:

Teacher: We have been given only two ordered pairs, so there are many types of graphs that could fit our data. What type of algebraic model would make sense in this situation?



Student 1: Two points determine a line, right? Couldn't we just connect the two points?

Student 2: No, the water level doesn't just keep going down forever—it goes back up again and then down again every day.

Student 3: That means it's probably going to be one of those wave-shaped graphs.

Student 1: Oh, yeah—I'll bet it's going to be sine or cosine. But how do we know which one?

Student 3: Well, let's try drawing part of the wave and see what we can figure out.

Student 2: If the pattern repeats like this every six hours, then there will be two high points and two low points every day. I suppose that makes the period 12 hours.

Student 3: Yeah, and if the highest and lowest the graph ever goes are 10.6 meters and 6.5, meters, then the amplitude is going to be 4.1 meters, right? Oh, wait a minute—the amplitude is only half of the height, so we need to change that to 2.05 meters.

Student 2: OK, now once we know that, we can find out that the vertical shift is halfway between the high and low points, which would make it 8.55 meters.

Teacher: Good job so far—now you just need to work on the period and horizontal shift. Do you think it would be easier to work with sine or cosine?

Student 1: I like cosine better for this graph because we can see that a high point happens 5 hours after midnight, so that will make it easy to find the horizontal shift.

The conversation might continue in this way in small groups, followed by a whole-class discussion of the observations made by various groups. Students could check the reasonableness of their solutions by using a dynamic graphing application on a calculator or computer. An interesting extension of this task would be to use the model to determine the times during which a ship with a certain depth requirement could navigate in and out of the port safely.

What are reasoning and sense making?

Reasoning and *sense making* are students' abilities to think about and use mathematics in meaningful ways. In any subject, it is not enough for teachers to expose students to topics. Nor is it enough for students to learn how to perform procedures or be able to recall facts.

Maybe in your own experiences with mathematics in high school you solved page after page of equations or factored page after page of polynomials. However, students today must understand more about algebra than they can learn by repetitively carrying out procedures. They need to develop critical thinking skills to succeed in mathematics—and in other areas of life and learning. For example, high school literature courses often call on students to analyze, interpret, or think critically about books that they are reading.

Reasoning is important in all fields—particularly mathematics. Mathematical reasoning involves drawing logical conclusions on the basis of assumptions and definitions. Sense making entails developing an understanding of a situation, context, or concept by connecting it with existing knowledge. Reasoning and sense making are interrelated.

Consider the example on page 2. A group of second-year algebra students use their knowledge of different types of functions to create a model for the depth of water in a seaport at a given time.

Several observations about this situation are useful:

- The students connect their knowledge of trigonometry with a context from everyday experience. Working with such a real-world context helps them see the meaning behind the mathematics while giving them motivation for learning it.
- The students draw logical conclusions about the situation, but they do not express their reasoning in a typical two-column proof. Instead, they express it in their own words—though they might be encouraged to write a more formal explanation at some other time.
- Although mathematical reasoning is often associated with writing proofs in geometry, students should reason and make sense of mathematics across the curriculum—in this case, in second-year algebra.

Why are reasoning and sense making important for your high school student?

Reasoning and sense making are simultaneously the purpose for learning mathematics and the most effective means of learning it. Unless students can reason with and make sense of the mathematics that they are learning, they are likely to ask the age-old question, "Why do we need to learn this?" They need to see a purpose in studying mathematics beyond the goal of preparing for the next mathematics course or a standardized test.

Moreover, research shows that students are more likely to retain mathematics that has its foundation in reasoning and sense making than mathematics that is presented as a list of isolated skills. For example, students who have explored the "Tidal Waves" scenario will have come to understand important facts about trigonometric functions and their usefulness by relating them to a context that makes sense to them.

Will reasoning and sense making provide the skills that your student needs for future success?

To be well prepared for success in their future endeavors, students need to have fluency in basic mathematical skills. This fluency must include knowing not only how to carry out basic procedures (such as solving quadratic equations), but also which procedure to choose, when to choose it, and for what purpose.

Without understanding the basis of procedures in reasoning and sense making, students may be able to perform the procedures correctly but may think of them only as a list of "tricks." As a result, they may have difficulty selecting an appropriate procedure to use in solving an unanticipated problem.

Or the students' seeming competence may evaporate when simple tasks give way to more complicated situations. Genuine procedural fluency requires both mastering technical skills and developing an understanding of how and when to use them appropriately.

Will reasoning and sense making prepare your student for success in college?

Organizations with an interest in college mathematics are increasingly calling for the same skills in reasoning, problem solving, and other areas of critical thinking that NCTM is advocating at the high school level. Moreover, students who develop a deep understanding of the mathematics that they study are more likely to remember it and be able to use it in the future. Retaining knowledge ensures that students will do well in college-level courses.

Do some students have a "mathematics gene?" In other words, can *all* students be successful in learning to reason mathematically?

Regardless of whether you or other members of your family did well in mathematics, you should expect and encourage your student to do well in it. Evidence shows that effort is a major factor in students' success in mathematics. Students who are held to high standards and expected to succeed are more likely to be successful. Family members who tell their students, "It's OK not to do well in mathematics—I didn't do well, either," give them an excuse not to make the effort to succeed.

However, just because a family member did well in mathematics does not mean it will come easily to a child. Nevertheless, with encouragement and support, all students can succeed.

What courses should your student take for an excellent preparation in mathematics?

In a 2006 Position Statement ("Math Takes Time"), NCTM made the case for a full four years of mathematics in high school: "Evidence supports the enrollment of high school students in a mathematics course every year, continuing beyond the equivalent of a second year of algebra and a year of geometry." Students intending to major in a technical or scientific field in college should take precalculus or calculus in high school, while other students may find senior-level courses in statistics or other areas of mathematics more productive.

Any college-level course that high school students take should adhere to high standards, such as those supported by the Advanced Placement Program. Less rigorous courses that focus primarily on learning procedures without developing reasoning and sense making will not provide students with the necessary foundation for further success in mathematics.

Encourage your student to take as much mathematics as possible, at the highest possible level. Although taking an honors class may seem like more work than taking a lower-level class, students who take more challenging classes learn more and are thus better prepared for college or other future options.

Furthermore, students who do not enroll in a mathematics course in their senior year may be at a disadvantage when they enter college. Those who take this year off from mathematics may find that their skills become rusty.

Some schools offer an integrated mathematics curriculum, which interweaves content from different areas of mathematic. Students at these schools do not take separate courses in algebra, geometry, and so on. Instead, each course includes material from geometry, algebra, statistics, and other areas of mathematics. Such curricular arrangements can be very helpful since they enable students to see the connections among areas of mathematics.

No matter how your student's high school arranges the curriculum, mathematics courses should provide experiences that encourage your student to engage in reasoning and sense making and see the connections among topics.

It goes without saying that students need a strong preparation in middle school mathematics to excel in math in high school. However, encouraging middle school students to take courses at the high school level calls for some caution. Be sure that taking courses early will not deprive your student of opportunities to develop critical mathematical ideas at his or her current grade level. Also be sure that such courses focus on reasoning and sense making so that your student will be adequately prepared for subsequent courses in high school.

What should your student's high school classroom look like?

Although no single way of teaching guarantees students' success in high school mathematics, whatever approach a teacher uses should emphasize active student involvement in thinking about mathematics. The "Tidal Waves" example shows a classroom where the students generate mathematical ideas for themselves rather than merely repeat what the teacher has told them.

Classrooms that focus on reasoning and sense making often—but not always—involve students in working together in groups on problems that may take some time to solve. Effective teachers ask many questions and expect students to communicate their reasoning to one another. Students justify their work and ask questions of themselves and their teacher. They engage deeply in understanding the mathematics that they study, exploring not just the *how,* but also the *why.*

What kind of homework should your student receive?

Teachers assign homework for a variety of reasons, so it may take different forms. Sometimes, they may ask students to practice some idea or method that they developed in class, so the homework may look more like the usual worksheet or problem set. At other times, however, they may ask students to explore problems or situations where they are not yet sure of what to do. In such instances, the homework will call on the students to come up with their own ways of solving the problems and explaining what they are doing. In class the next day, teacher may have the students share insights or help them develop productive ideas and methods.

If your student's teacher never assigns homework that calls for reasoning about and making sense of mathematics in this fashion, you might legitimately ask whether those activities are happening at all in the classroom.

What should your student's tests look like?

Like homework, tests may include some "routine" problems to see if students have mastered the skills that they need to continue their progress. However, tests should also include items that require students to come up with their own ideas and explain their thinking.

If your student's tests never provide opportunities for reasoning and sense making,

then he or she might conclude that these activities are not really all that important, but what matters most is being able to carry out routine computations. Consider discussing such tests with your student's teacher or school.

What is the best way to help your student study mathematics?

As you help your student complete his or her homework or prepare for a test, ask the student to explain what he or she is doing and why it makes sense. Remember, simply finding the answer to a problem is not enough; your student also needs to be able to explain how he or she found the answer and why his or her method works.

However, the method that your student uses may not be exactly like the one that you learned in school. Be patient, and listen to your student's explanations. If you do not understand what he or she says, ask questions to help you understand.

This interaction may be a learning experience for you—regardless of whether you have completed college-level mathematics or have a far less extensive mathematical background. Either way, discovering what your student is learning is likely to be interesting.

What if your student shows mathematical talent?

Families of gifted or mathematically talented children should recognize that although acceleration is one avenue of supporting such students, it is not the only route. Simply moving mathematically talented students faster and faster through more and more mathematics is not always the best way to develop their talents. Such students need enrichment, advanced content, and more in-depth study of that content.

Furthermore, it is important to recognize that mathematical talent or giftedness is fluid, and a student who shows a real aptitude for algebra, for example, might struggle or need additional help with geometry, or vice versa! In addition, an aptitude for mathematics might emerge at some later stage, even if it is not currently evident in a student.

What are the most important things that families can do to support the mathematics learning of their high school student?

Families need to be involved in the mathematical preparation of their high school students. Each year, be sure that your student is enrolled in challenging mathematics classes—courses that will move the student toward his or her long-term goals. Show interest in what is happening in the mathematics classroom, and look at your student's mathematics work. Encourage your student to persevere, even when the material is difficult, and urge him or her to ask questions in class when the material is not clear.

If your student is having serious difficulties, encourage him or her to set up sessions with the teacher for extra help. Remind your student that effort is the major determinant of long-term success. By the same token, if your student is doing well, encourage him or her to explore what is going on in class more deeply, doing additional research or solving extended problems provided by the teacher.

Finally, stay in contact with your student's mathematics teacher. If the teacher raises any concerns or asks for assistance, try to help. Likewise, if at any point you have questions about any aspect of your student's mathematical preparation, do not hesitate contact the teacher. Your student's success is a shared goal, with many people working together to achieve it.