A Policymaker's Guide to High School Mathematics:
Making Reasoning and Sense Making the Focus

What kinds of experiences should high school mathematics offer students? *Focus in High School Mathematics: Reasoning and Sense Making* presents a new set of guidelines, prepared by the nation's leading advocate for more and better mathematics for all students, the National Council of Teachers of Mathematics. To help policymakers gain a better understanding of what students should be experiencing in mathematics throughout their high school years, this pamphlet presents recommendations based on these new guidelines.

Why is mathematics important for high school students?

A strong preparation in high school mathematics readies students for future success in their jobs, their continued education, their personal lives as citizens, and their social responsibilities in our democratic society.

Of course, mathematics is and will continue to be central to scientific and technical careers. However, mathematics is increasingly essential for a wide range of careers, including positions from inventory strategists and cost analysts to sports journalists. In addition, as technology continues to expand communication and software capabilities, more opportunities are emerging for new careers in mathematics and statistics based on harnessing the huge amount of data at people's fingertips. In short, knowledge of meaningful mathematics opens career doors for all students. However, all too often, an inadequate preparation in mathematics in high school means that doors will be closed to some students without years of make-up work in mathematics.

When our students become adult citizens, they will need a solid background in mathematics to make informed and reasoned decisions about their lives and society. They must be able to handle their personal finances, decide which public policies deserve their support, and evaluate which insurance or health plan option to select. In today's highly technological world, having a solid preparation in mathematics is a tremendous asset. Those without a solid mathematical background risk being taken advantage of or left behind. Good decisions by individuals benefit us all.

Mathematics will play a fundamental role in addressing many current global concerns, such as economic security and competitiveness, resource management (including sustainability and independence), and health-care costs. In addition, mathematics will most likely be a central component of our nation's success in meeting the challenges that will appear in our future. Successful resolutions of such problems require knowledgeable efforts on many fronts—from our citizenry, the workforce, and our policymakers.

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Mathematical knowledge is a vital asset to individuals in each of these roles.

Why is a broad preparation in mathematics important?

Many of today's adult citizens may have experienced a high school mathematics curriculum that focused primarily on algebra and precalculus, along with some study of geometry. However, students today need experiences in a broader range of mathematics to prepare them for their future success. For example, students need more preparation in statistics than they did in the past, as statistics permeates many fields of endeavor, from business to medicine to the social sciences.

What do *reasoning* and *sense making* mean?

Reasoning and *sense making* refer to students' abilities to think about and use mathematics in meaningful ways. Genuine mathematical fluency requires both mastering technical skills and developing an understanding of how to use mathematics appropriately. A focus on reasoning and sense making fosters the ability to use mathematical tools and methodology in new situations.

To this end, simply exposing students to topics is not enough. Nor is it enough for students simply to know how to perform procedures or be able to recall facts. Students today must be able to do more than repetitively carry out procedures. They need to develop the skills that allow them to think critically to determine why particular mathematical procedures or approaches work and when they might use a procedure or approach effectively. *A focus on reasoning and sense making raises the standard of mathematics education by providing students with the ability to use mathematics in flexible and unanticipated ways—in their jobs, their personal lives, and their responses to critical issues affecting our society.*

In this context, mathematical reasoning includes drawing logical conclusions based on assumptions and definitions. Reasoning begins and is supported by "reasoning habits" outlined in *Focus in High School Mathematics: Reasoning and Sense Making*. Sense making involves developing an understanding of a situation, context, or concept by connecting it with existing knowledge. Reasoning and sense making are thus closely interrelated.

As students engage in the intertwined processes of reasoning and sense making, they develop their understanding of the connections between mathematics and the world in which they live. Consider "Tidal Waves" (see page 3), a brief example in which students need to draw on their knowledge of different types of mathematical functions to suggest a model for a real-world situation.

Several observations about this situation are useful. First, it is the students who are doing the reasoning. At the beginning of the discussion, note that rather than blindly applying the dictum that "two points determine a straight line," the students use reasoning and sense making to reject a linear model and to suggest a better model based on trigonometric functions. Here they are connecting their knowledge of trigonometry with a problem drawn from a practical context. The context is new to them, but working on the task helps them to strengthen their understanding of the meaning of mathematical concepts as they reason about ways to "fit" a trigonometric function to the two data points. In addition, the task provides both motivation and reason to do the mathematics. Although the students are drawing logical conclusions about the

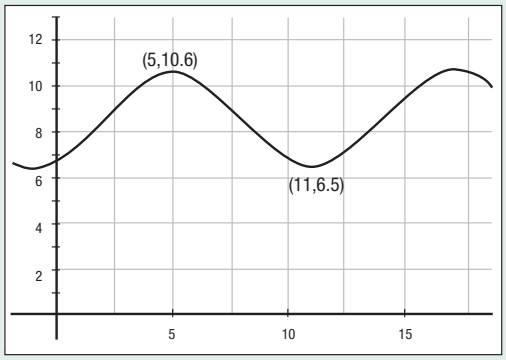
"TIDAL WAVES"

The captain of a shipping vessel must consider the tides when entering a seaport because the depth of the water can vary greatly from one time of day to another. Suppose that high tide in a certain port occurs at 5:00 AM, when the water is 10.6 meters deep, and the next low tide occurs at 11:00 AM, when the water is 6.5 meters deep. Develop a mathematical model that will predict the water's depth as a function of the time elapsed since midnight.

In the Classroom

The second-year algebra students working on this problem have had experience with transformations of linear and quadratic functions and are familiar with the graphs of the sine and cosine functions. The dialogue below shows several students' reasoning about this task:

Teacher: We have been given only two ordered pairs, so there are many types of graphs that could fit our data. What type of algebraic model would make sense in this situation?



Student 1: Two points determine a line, right? Couldn't we just connect the two points?

Student 2: No, the water level doesn't just keep going down forever—it goes back up again and then down again every day.

Student 3: That means it's probably going to be one of those wave-shaped graphs.

Student 1: Oh, yeah—I'll bet it's going to be sine or cosine. But how do we know which one?

Student 3: Well, let's try drawing part of the wave and see what we can figure out.

Student 2: If the pattern repeats like this every six hours, then there will be two high points and two low points every day. I suppose that makes the period 12 hours.

Student 3: Yeah, and if the highest and lowest the graph ever goes are 10.6 meters and 6.5, meters, then the amplitude is going to be 4.1 meters, right? Oh, wait a minute—the amplitude is only half of the height, so we need to change that to 2.05 meters.

Student 2: OK, now once we know that, we can find out that the vertical shift is halfway between the high and low points, which would make it 8.55 meters.

Teacher: Good job so far—now you just need to work on the period and horizontal shift. Do you think it would be easier to work with sine or cosine?

Student 1: I like cosine better for this graph because we can see that a high point happens 5 hours after midnight, so that will make it easy to find the horizontal shift.

The conversation might continue in this way in small groups, followed by a whole-class discussion of the observations made by various groups. Students could check the reasonableness of their solutions by using a dynamic graphing application on a calculator or computer. An interesting extension of this task would be to use the model to determine the times during which a ship with a certain depth requirement could navigate in and out of the port safely.

situation, they do not express their reasoning as a formal proof. Instead, they express it in their own words, although they might be encouraged to write a more formal explanation at a later time. Reasoning about and making sense of mathematics should occur continually, across the high school mathematics curriculum—not just in a particular course or a particular chapter of material.

Why are reasoning and sense making important for high school students?

Unless students can reason with and make sense of the mathematics that they are learning, they are likely to ask the age-old question, "When will I ever use this?" Minimizing the roles of reasoning and sense making devalues the learning of mathematics and prevents students from internalizing the fact that studying mathematics has purpose beyond preparing for the next mathematics course or standardized test. Reasoning and sense making are both the reason for learning mathematics and the means by which mathematics is learned best. Moreover, research shows that when students learn mathematics with a foundation in reasoning and sense making, they are more likely to remember it than when they encounter it as a list of isolated skills. For example, students who have explored the "Tidal Waves" scenario will understand important facts about how trigonometric functions behave by relating them to a context that makes sense to the students.

Will students who focus on reasoning and sense making learn the skills that they need for future success?

To be well prepared for future success, students need to have mathematical fluency, which includes knowing not only how to carry out basic mathematical procedures but also which procedures to choose, when to choose them, and for what purpose. Being successful in our fast-paced, economically competitive society will increasingly require innovation and creativity. Such success most often depends on hard work and builds on a firm foundation of usable knowledge. Mathematical reasoning and sense making are keys to such a foundation and consequently belong at the core of a high school mathematics education.

Will students who focus on reasoning and sense making be prepared for success in college?

Organizations that devote attention to college mathematics are increasingly calling for the same kind of reasoning, problem solving, and other critical thinking skills that *Focus in High School Mathematics: Reasoning and Sense Making* advocates. Moreover, students who develop a deep understanding of the mathematics that they study are more likely to remember it and to be able to use it in the future, thus ensuring that they will do well in college-level courses.

What can policymakers do?

Policymakers can help ensure that the answers to the following questions are *yes*.

Are reasoning and sense making inextricably integrated with content topics in frameworks or standards that guide curricular decisions in schools, districts, states, or the nation? Reasoning and sense making are not separate topics to be added to lists of content topics consisting of items such as "solving two-variable systems of equations." Instead, reasoning and sense making are essential mathematical processes that need to permeate every content goal. Standards that require reasoning and sense making raise the level of

expectations for our students, bringing these expectations in line with students' needs for the twenty-first century. Therefore, frameworks and standards that shape classroom curricula need to emphasize that reasoning and sense making are indispensable to the study of mathematical content.

In our current culture, standards that simply list topics briefly are insufficient. *Focus in High School Mathematics: Reasoning and Sense Making* offers a more complete discussion of reasoning and sense making and calls attention to role of the Process Standards in high school mathematics, as described in *Principles and Standards for School Mathematics,* published by the National Council of Teachers of Mathematics in 2000.

Do state and local assessment policies emphasize the need for and importance of items that examine students' abilities to reason and make sense of mathematical situations? It makes sense to test what we value. Regardless of our educational aims, students, teachers, administrators, and many others equate what we test with what we value. This means that assessment instruments need to include items that call for reasoning and sense making. Such items should, for instance, require students to explain their thinking and reasoning or show that they can use mathematics flexibly in nonroutine situations. Students need to have these kinds of assessments regularly in high school mathematics classrooms.

Moreover, items requiring reasoning and sense making should be incorporated in high-stakes and accountability measures. This means that we will need to modify, strengthen, and improve the assessment instruments that we use in high-stakes assessments and accountability measures. Evidence indicates that this can be done, and we cannot settle for less. Together, we can address the limitations of current practices, which may involve the use of single, short-answer end-of-year examinations.

Some important types of assessments that measure reasoning and sense making may call for more than a single response to each item, and so may require more time to take and more time to grade. To accommodate these needs, accountability assessments might be incorporated into actual classroom work throughout a semester in place of a separate examination at the end of the term. A collection (portfolio) of students' work during the semester, for example, might be used as justification for an accountability score.

Do all students have access to a rich mathematics curriculum based on reasoning and sense making? All students, in all demographic settings, need to experience a high school mathematics curriculum that is rich in reasoning and sense making. Such a curriculum sets higher expectations for students. Providing the necessary support to ensure that these expectations are met extends beyond the stated educational goals and specified assessment instruments of the education system. It includes support that can improve the capacity of the system, such as good instructional materials (e.g., textbooks, appropriate technology, manipulatives, and classroom activities); qualified teachers who can effectively initiate and facilitate student reasoning and sense making by using sound pedagogical practices; and knowledgeable administrators who support a learning environment conducive to such curricular goals.

Some examples of instructional materials (classroom activities) are available that demonstrate what reasoning and sense making look like in the classroom and how they can benefit all students. *Focus in High School Mathematics: Reasoning and Sense Making* and its related topic books provide useful examples and also discuss some of the many pedagogical strategies that help all students improve their reasoning and sense-making abilities.

Are adequate resources allocated to assist schools and districts in effective efforts to implement a curriculum based on reasoning and sense making? It is possible to provide all students with a curriculum that is rich in reasoning and sense making. The *will* to offer such a curriculum is critical. Policymakers can help forge that will. One initial step, for example, is to incorporate the goals of reasoning and sense making in mathematics standards at all levels.

At the same time, however, successfully raising the standards for mathematics programs in secondary schools depends on allocating many resources of time and money appropriately. Teachers who are currently serving students in high school mathematics classrooms need strong, ongoing professional development if they are to meet these new goals. Good, coherent instructional materials need to be available in all classrooms. This may mean adapting some materials that already exist or creating new ones. Such work is not easy and often requires cycles of research, writing, and testing. Assessment instruments must be aligned with new, higher expectations for learning.

All of these requirements underscore the continuing need to foster partnerships among school, higher education, and business constituencies.

Are we prepared to accept the challenge?

The goal of engaging students in significant reasoning and sense making every day in every high school mathematics classroom places these intertwined processes at the core of every high school mathematics curriculum. Achieving this goal is essential if we are to meet the educational needs of 21st-century students. However, accepting this goal represents a shift from current practice in most high school mathematics classrooms.

It is vital to put policies in place that foster a focus on mathematical reasoning and sense making. Anything less will not serve our students—or our nation—well.