Introduction

In this book, we focus on students' learning of powerful mathematics. We use the term "powerful" in our description of mathematics for several reasons. First of all, mathematics carries particular power in our society; success in mathematics is critical for academic success. More than any other subject, mathematics affects students' college and career opportunities. The highest high school mathematics class for which a student earns credit has a major influence on college acceptance and college graduation (Lee 2012). However, while mathematics success is significant for college and career success, by the time students reach middle school, the majority think it is too boring or too hard, with over half expressing both a lack of interest and a dislike for the subject (Mullis et al. 2012; U.S. Department of Education 2014). In addition, U.S. students in the early elementary school years learn to associate mathematics success with speed and accuracy (Boaler 2015), rather than with reasoning and sense making. Being good at mathematics comes to be seen as a talent or skill one is born with: some may be born mathematically gifted, while others are not (Boaler 2015). But this is simply not true!

We have written this book with the goal of challenging narrow conceptions of mathematics learning and mathematics achievement. Decades of research, as well as the work of the teachers and students highlighted in this book, demonstrate that mathematical success can and should be available for all. This body of work makes clear that each student's success in mathematics is greatly influenced by the opportunities and support for learning provided by teachers. *Principles to Actions* (NCTM 2014a) calls for schools and classrooms to shift from "pockets of excellence" to "systemic excellence" where every aspect of our instruction is carefully considered to maximize student learning. Thinking about the learning environment as a system is important because all facets of instructional decision

making—the classroom layout, the opportunity for student talk, the tasks assigned, the assessments used, the availability and use of technology and tools, and the involvement of families and communities—affect student learning and convey to students what is valued and who is valued in the classroom.

Three principles guide our discussion of creating productive, inclusive mathematics learning environments:

1. Teach toward the understanding of powerful mathematics.
2. View students as sense makers with valuable, important ideas.
3. Nurture a mathematics community of learners.

These principles are at the heart of creating classrooms where children can participate meaningfully and equitably with mathematics and with each other. The challenge, of course, lies in putting these principles into practice, which is what we hope this book will support teachers in doing. The classroom vignettes in this book reflect insights we have gained about designing productive mathematics learning environments from working closely with and learning from teachers who are committed to equity in student learning opportunities and outcomes. These vignettes are inspired by the ways these learning environments have been designed to support teacher and student practices that demonstrate the transformative potential within student-centered instruction. Just as these teachers have inspired us, we hope their everyday practices will illuminate possibilities for other teachers seeking to create classrooms that disrupt long-standing assumptions about what it means to know and understand mathematics and who can and cannot excel in mathematics.

We'll now talk a bit more about each of these principles and then provide an outline of what lies ahead in the rest of the book.

Principle 1: Teach Toward the Understanding of Powerful Mathematics

The building of powerful mathematics starts from understanding. What is understanding of mathematics? Our conception of mathematical understanding is rooted in a notion of proficiency as knowing more than isolated facts and methods. It requires "an integrated and functional grasp of mathematical ideas . . . which enables students to learn new ideas by connecting those ideas to what they already know" (National Research Council [NRC] 2001, p. 118). Knowledge that is learned with understanding is rich in connections and is generative, helping to support further learning. To develop learning with understanding

requires that teachers know strategies to connect new knowledge with students' prior knowledge and know which connections are most productive in supporting students' problem solving and future mathematics learning.

This view of mathematical understanding requires approaching learning as building from students' existing knowledge, posing questions that provoke curiosity, and supporting students in developing habits of mathematical reasoning and sense making. Through habits, or practices, of reasoning, children begin to make connections between rules, procedures, and concepts. They come to see coherence among mathematical ideas, recognize a purpose for mathematics in their lives, and maybe even discover pleasure in exploring mathematics. This book will share strategies for planning and implementing tasks to support such learning.

Principle 2: View Students as Sense Makers with Valuable, Important Ideas

Mathematical knowledge and understanding cannot be transferred directly from teacher to student. Students must be provided with opportunities to construct their own knowledge and understanding in ways that reflect, refine, and extend their prior knowledge, skills, and experiences.

All children come to class with a great deal of practical mathematical knowledge and reasoning skills from their lived experiences. Students' existing knowledge and intuitive strategies for solving problems should be the basis for further learning. For example, kindergarten students without prior schooling have some intuitive knowledge of fractions from their experiences with fair sharing (e.g., sharing half a cookie with a sibling is about making two parts that are equal in size) but may not know that one-half can take on different shapes while remaining equivalent (e.g., cutting a square diagonally versus horizontally) or that whether one-half is more than one-third depends on whether they share a common whole. Students' rich experiences with mathematics, formal and informal, should serve as a starting point for building additional mathematical knowledge.

This leads us to realize that teachers must intentionally choose or design tasks that (a) reflect a deep understanding of mathematics as well as a deep understanding of those learning mathematics, and (b) promote and elicit students' thinking with and about mathematics. When these tasks are implemented, teachers must use instructional strategies that help students see mathematics as part of themselves and see themselves as owners and users of mathematics.

Principle 3: Nurture a Mathematics Community of Learners

> A community approach enhances learning: It helps to advance understanding, expand students' capabilities for investigation, enrich the questions that guide inquiry, and aid students in giving meaning to experiences. (NRC 1996, p. 46)

Learning for understanding is not an isolated or passive activity. It is above all a process of sense making and establishing meaning, both individually and collectively. Nurturing a learning community with the shared purpose of making sense of mathematics is vital to learning powerful mathematics.

We view a mathematics community of learners as an ecological system. In an ecosystem, such as a wetland or a desert, all aspects of life are interrelated and depend on each other's well-being. Teaching and learning are also built on relationships and interdependence. Students learn with and from each other in interaction with teachers and other adults in their lives. This book will highlight collaboration across varied activity structures—opportunities for students to work independently, in teams and small groups, and as a whole class—as well as within and beyond the classroom, a space that provides opportunities for every student to actively participate and to persevere in learning powerful mathematics. This sort of learning community must be intentionally created and nurtured, and it relies on clear and inclusive routines for participation so that societal inequities are not mirrored in the distribution of learning outcomes.

A Note about Tools and Technology

> An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking. (NCTM 2014a, p. 78)

Tools (e.g., manipulatives, rulers, physical models) and technology (e.g., interactive whiteboards, computing devices, tablets) are resources that can help teachers and students enact the three principles to support powerful mathematics learning. In our view, it is not a question of whether to use tools and technology but how to use them to connect students to mathematics in ways that enable them to develop their own sense of ownership of mathematics. For this reason, we have integrated examples of tools and technology throughout each chapter. Look for text boxes that highlight specific-use cases of a particular tool or technology. At the end of the book, we also provide an annotated list of the tools and technology that are mentioned in the text.

How to Use This Book

This book is intended to help you, the educator, design a learning environment that engages students individually and collectively in meaningful learning experiences that promote sense making of mathematical ideas and mathematical reasoning. Each chapter describes one aspect of the learning environment by synthesizing the research and providing practical strategies that teachers can apply directly to their work.

Chapter 1 describes the physical attributes of the learning environment—the classroom layout, the wall space, and storage of manipulatives—as well as the use of virtual spaces to encourage teachers' and students' collaborative engagement with mathematics. Chapter 2 highlights the centrality of discourse and provides specific strategies, norms, and routines to support student discourse and promote equitable participation. Chapter 3 unpacks the characteristics of a rich mathematical task and offers guidance on how to carefully plan and enact lessons to allow and support students' productive engagement and perseverance. Chapter 4 examines the importance of assessment in supporting powerful learning and provides ideas on formative assessments that measure students' mathematical knowledge (conceptual, procedural, factual/notational) and mathematical practices (e.g., reasoning, justification, problem solving). Chapter 5 describes best practices and concrete examples of strategies to promote and strengthen home-school-community partnerships that support students' success with mathematics. Chapter 6 illustrates how these five elements come together in practice with vignettes of classroom teachers who are working to create learning environments that promote powerful mathematics for every student.

Each chapter contains the following features, either in the book itself or online at NCTM's More4U website:

- A synthesis of research applied to specific topics of this dimension of the learning environment, with concrete examples of how teachers have applied the concepts in their classrooms
- Video and classroom examples that offer a close look at how real teachers use the strategies from the chapter to teach mathematical concepts. Interviews with both students and teachers provide interesting insights on the rationale and outcome of specific teaching strategies.
- Handouts and materials for next-day use

Materials that are available online at More4U are listed in Appendix B, and readers may gain access to them by going to www.nctm.org/more4u and entering the access code located on the title page of this book.

It is our intention that this book portray ideas and images that reimagine the mathematics classroom as a place of possibility and powerful learning for all students,

challenging what are often taken-for-granted beliefs and habits of mathematics teaching and learning that constrain opportunities to learn. While the way in which we have organized and described this content is ours, the research and practitioner knowledge and experiences that informed it come from a large community of mathematics educators—classroom teachers, school district administrators, engaged families, and university faculty—with a shared commitment to promoting powerful mathematics learning for all students. As you read through these pages and explore the digital resources at More4U, we hope you find support for both your efforts to make instructional shifts that support deeper learning in mathematics and your professional growth as a teacher of mathematics. And as you put some of these ideas into practice, we hope you find joy, as we have, in students' enthusiasm for making sense of mathematics, individually and collectively.

Finally, we want to acknowledge that the vision for mathematics teaching and learning we articulate requires collaboration among teachers, administrators, and families to have maximum impact, and that, in the process of working toward this goal, there will be lessons learned that inform us all about how to successfully engage in this work. We look forward to learning with and from these efforts and thank you in advance for sharing with us your successes and challenges along the way! Our understanding of putting research into practice is continually enriched and refined by what we learn from the passionate, committed teachers who on a daily basis create and nurture mathematics learning environments that support all students.