

CHAPTER 1

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A Vision for School Mathematics

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

The vision for mathematics education described in *Principles and Standards for School Mathematics* is highly ambitious. Achieving it requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence. The challenge is enormous and

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meeting it is essential. Our students deserve and need the best mathematics education possible, one that enables them to fulfill personal ambitions and career goals in an ever-changing world.

Since the release in 1989 of the *Curriculum and Evaluation Standards for School Mathematics*—followed in 1991 by the *Professional Teaching Standards for School Mathematics* and in 1995 by the *Assessment Standards for School Mathematics*—the National Council of Teachers of Mathematics (NCTM) has remained committed to the view that standards can play a leading role in guiding the improvement of mathematics education. As an organization representing teachers of mathematics, NCTM shares with students, school leaders, and parents and other caregivers the responsibility to ensure that all students receive a high-quality mathematics education. All interested parties must work together to create mathematics classrooms where students of varied backgrounds and abilities work with expert teachers, learning important mathematical ideas with understanding, in environments that are equitable, challenging, supportive, and technologically equipped for the twenty-first century.

The Need for Mathematics in a Changing World

We live in a time of extraordinary and accelerating change. New knowledge, tools, and ways of doing and communicating mathematics continue to emerge and evolve. Calculators, too expensive for common use in the early eighties, now are not only commonplace and inexpensive but vastly more powerful. Quantitative information available to limited numbers of people a few years ago is now widely disseminated through popular media outlets.

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. For example:

- *Mathematics for life.* Knowing mathematics can be personally satisfying and empowering. The underpinnings of everyday life are increasingly mathematical and technological. For instance, making purchasing decisions, choosing insurance or health plans, and voting knowledgeably all call for quantitative sophistication.
- *Mathematics as a part of cultural heritage.* Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects.
- *Mathematics for the workplace.* Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem solving needed in the workplace, in professional areas ranging from health care to graphic design.
- *Mathematics for the scientific and technical community.* Although all careers require a foundation of mathematical knowledge, some are mathematics intensive. More students must pursue an educational path that will prepare them for lifelong work as mathematicians, statisticians, engineers, and scientists.

In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed. NCTM challenges the assumption that mathematics is only for the select few. On the contrary, everyone needs to understand mathematics. All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence.

Principles and Standards calls for a common foundation of mathematics to be learned by all students. This approach, however, does not imply that all students are alike. Students exhibit different talents, abilities, achievements, needs, and interests in mathematics. Nevertheless, all students must have access to the highest-quality mathematics instructional programs. Students with a deep interest in pursuing mathematical and scientific careers must have their talents and interests engaged. Likewise, students with special educational needs must have the opportunities and support they require to attain a substantial understanding of important mathematics. A society in which only a few have the mathematical knowledge needed to fill crucial economic, political, and scientific roles is not consistent with the values of a just democratic system or its economic needs.

The Need for Continued Improvement of Mathematics Education

The vision described at the beginning of this chapter is idealized. Despite the concerted efforts of many classroom teachers, administrators, teacher-leaders, curriculum developers, teacher educators, mathematicians, and policymakers, the portrayal of mathematics teaching and learning in *Principles and Standards* is not the reality in the vast majority of classrooms, schools, and districts. Evidence from a variety of sources makes it clear that many students are not learning the mathematics they need or are expected to learn (Kenney and Silver 1997; Mullis et al. 1997, 1998; Beaton et al. 1996). The reasons for this deficiency are many: In some instances, students have not had the opportunity to learn important mathematics. In other instances, the curriculum offered to students does not engage them. Sometimes students lack a commitment to learning. The quality of mathematics teaching is highly variable. There is no question that the effectiveness of mathematics education in the United States and Canada can be improved substantially.

Standards can play a central role in the process of improvement. The previously released NCTM *Standards* (NCTM 1989, 1991, 1995) have influenced state standards and curriculum frameworks (Council of Chief State School Officers 1995; Raimi and Braden 1998), instructional materials (U.S. Department of Education 1999), teacher education (Mathematical Association of America 1991), and classroom practice (Ferrini-Mundy and Schram 1997). As with any educational innovation, however, the ideas of the *Standards* have been interpreted in many different ways and have been implemented with varying degrees of fidelity. Sometimes the changes made in the name of standards have been superficial or incomplete. For example, some of the pedagogical ideas from the NCTM *Standards*—such as the emphases on discourse, worthwhile mathematical

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tasks, or learning through problem solving—have been enacted without sufficient attention to students’ understanding of mathematics content. Efforts to move in the directions of the original NCTM *Standards* are by no means fully developed or firmly in place.

The Role and Purpose of Standards

The introduction to the 1989 *Curriculum and Evaluation Standards* noted three reasons for a professional organization to formally adopt standards: to ensure quality, to indicate goals, and to promote change. One way in which standards documents help meet these goals is by shaping conversations about mathematics education. As with the previous NCTM *Standards*, *Principles and Standards* offers common language, examples, and recommendations to engage many groups of people in productive dialogue. Although there will never be complete consensus within the mathematics education profession or among the general public about the ideas advanced in any standards document, the Standards provide a guide for focused, sustained efforts to improve students’ school mathematics education. *Principles and Standards* supplies guidance and vision while leaving specific curriculum decisions to the local level. This document is intended to—

- set forth a comprehensive and coherent set of goals for mathematics for all students from prekindergarten through grade 12 that will orient curricular, teaching, and assessment efforts during the next decades;
- serve as a resource for teachers, education leaders, and policy-makers to use in examining and improving the quality of mathematics instructional programs;
- guide the development of curriculum frameworks, assessments, and instructional materials;
- stimulate ideas and ongoing conversations at the national, provincial or state, and local levels about how best to help students gain a deep understanding of important mathematics.

An Overview of *Principles and Standards*

Principles and Standards for School Mathematics builds on and consolidates messages from the previous *Standards* documents. The document is organized into four main parts:

- Principles for school mathematics (chapter 2)
- An overview of the Standards for mathematics education in prekindergarten through grade 12 (chapter 3)
- Standards for four separate grade bands: prekindergarten through grade 2 (chapter 4), grades 3–5 (chapter 5), grades 6–8 (chapter 6), and grades 9–12 (chapter 7)
- A discussion of the steps needed to move toward the vision embodied in *Principles and Standards* (chapter 8)

The principles are statements reflecting basic precepts that are fundamental to a high-quality mathematics education. The discussions in chapter 2 elaborate on the underlying assumptions, values, and evidence

on which these Principles are founded. The Principles should be useful as perspectives on which educators can base decisions that affect school mathematics. NCTM's commitment to mathematics for all is reaffirmed in the Equity Principle. In the Curriculum Principle, a focused curriculum is shown to be an important aspect of what is needed to improve school mathematics. The Teaching Principle makes the case that students must have opportunities to learn important mathematics under the guidance of competent and committed teachers. The view of learning that is the basis for the document is taken up in the Learning Principle. The important roles of assessment and technology in school mathematics programs are discussed in the Assessment and Technology Principles.

Chapters 3–7 outline an ambitious and comprehensive set of curriculum standards for all students. Standards are descriptions of what mathematics instruction should enable students to know and do—statements of what is valued for school mathematics education. Each of the ten curriculum standards proposed in this document spans the entire range from prekindergarten through grade 12. Chapter 3 discusses each Standard in turn to convey its main ideas. In addition, these discussions give a sense of how the ideas encompassed in a Standard develop over all four grade bands, highlighting points at which certain levels of mastery or closure are appropriate. Chapters 4–7 present the Standards in detail for each grade band.

The first five Standards describe mathematical content goals in the areas of number and operations, algebra, geometry, measurement, and data analysis and probability. The next five Standards address the processes of problem solving, reasoning and proof, connections, communication, and representation. In each grade-band chapter, a set of “expectations” is identified and discussed for each Content Standard. The appendix displays the Content Standards and expectations in a chart that highlights the increasing sophistication of ideas across the grades. Each grade-band chapter discusses what each Process Standard should “look like” in that grade band and what the teacher’s role is in supporting the development of that process.

The mathematical Content and Process Standards discussed in chapters 3–7 are inextricably linked. One cannot solve problems without understanding and using mathematical content. Establishing geometric knowledge calls for reasoning. The concepts of algebra can be examined and communicated through representations.

One purpose of this document is to offer teachers, curriculum developers, and those responsible for establishing curriculum frameworks a way to focus curricula. Focus is promoted through attention to the idea of “moving on.” School mathematics programs should not address every topic every year. Instead, students will reach certain levels of conceptual understanding and procedural fluency by certain points in the curriculum. Teachers should be able to assume that students possess these understandings and levels of fluency when they plan their mathematics instruction. Teachers and policymakers can then fashion instructional programs and curricular frameworks that develop progressively over the grades and that focus on important mathematical areas.

Chapter 8 discusses what it will take to move toward the vision described in the previous chapters. In particular, it discusses critical issues

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related to putting the Principles into action and outlines the key roles played by various groups and communities in realizing the vision of the *Principles and Standards*.

As We Move Forward

Attaining the vision described at the beginning of this chapter will require the talents, energy, and attention of many individuals, including students, teachers, school administrators, teacher-leaders, policymakers, parents and other caregivers, mathematicians, mathematics educators, and the local community. It will require that the vision of this document be shared and understood and that all concerned be committed to improving the futures of our children. The task is enormous and essential. All students need an education in mathematics that will prepare them for a future of great and continual change.