Introduction

This *Journal for Research in Mathematics Education* monograph had its origins in a conference titled *An Interdisciplinary Conference on Assessment in K–12 Mathematics: Collaborations Between Mathematics Education and Psychometrics*, which was held from September 25 to September 27, 2011, in Atlanta, Georgia. A grant from the National Science Foundation supported the conference, with additional funding from the University of Georgia's College of Education. The 64 attendees included faculty members, postdoctoral researchers, and doctoral students from the fields of mathematics education and psychometrics.

Two factors led us to propose the conference to the National Science Foundation. First, an increasing number of mathematics education research projects are using psychometric methods, including projects that study teacher knowledge and learning trajectories. Second, the field of psychometrics is experiencing a renaissance in which an increasing variety of psychometric models are becoming available through advances in computer hardware and software. The co-occurrence of these two factors is opening new avenues for studying the mathematical knowledge of teachers and students, but few opportunities have existed for researchers from both fields to discuss the opportunities for and challenges of harnessing psychometric models for mathematics education research.

Discussions between mathematics education and psychometrics are critical to interdisciplinary collaboration in light of different approaches and framing assumptions that guide researchers in the two fields. Mathematics education researchers and psychometricians can adopt priorities and approaches that are not readily compatible. Psychometricians are well positioned to contribute measurement expertise but may not be sensitive to nuances of mathematical knowledge that mathematics education research seeks to measure. Mathematics educators, however, often lack adequate knowledge of psychometric modeling techniques and requirements and are not always familiar with essential trade-offs when selecting such models to accomplish particular research goals. Furthermore, some mathematics educators may feel constrained by limits of the psychometric tools available or by the lack of fit to the purpose at hand.

Our aim for the conference was to promote conversation about aims, framing assumptions, knowledge, methods, and applications across these two fields—conversation that could lead to further collaborative research. We believe that interdisciplinary research projects can advance the fields of mathematics education and psychometrics: The application of psychometric models can help mathematics education researchers leverage accumulating results from studies with smaller samples to study larger samples of teachers and students, and practical

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problems measuring mathematical knowledge can lead to the further development of psychometric models.

The following overarching questions for the conference reflected the potential for productive give and take between the two fields:

- What opportunities do current and emerging psychometric models afford, and how can researchers use them for measuring mathematical knowledge?
- To what extent are various psychometric models consistent with conceptualizations of knowledge and learning used in mathematics education research? When are discrepancies consequential, and when are they not?
- What are the trade-offs of using different psychometric models for measuring mathematical knowledge?
- What new psychometric models might researchers need for measuring mathematical knowledge before it is possible to make significant additional advances in research at the intersection of mathematics education and psychometrics?

Conference activities included workshops on mathematical knowledge and psychometric models, as well as invited panels during which researchers presented opportunities and challenges that they were encountering when using psychometric models as tools for research in mathematics education. Several chapters in the monograph have their roots in presentations and conversations that took place during the conference, and the editors subsequently solicited other chapters to round out the collection.

PURPOSE AND BOUNDARIES

The overarching purpose of the monograph is to guide further interdisciplinary collaborations between mathematics education researchers and psychometricians by examining theoretical and conceptual issues that have arisen in recent efforts to apply contemporary psychometric models to mathematics education research. Specifically, we intend chapters in the monograph to (a) illustrate for mathematics education researchers the two main categories of psychometric models—models that locate individuals along continua and models that place individuals into discrete groups, as well as hybrids of these approaches; (b) provide examples that apply these different categories of psychometric models to mathematics education research; (c) illustrate how researchers have selected different psychometric models depending on the researchers' goals; and (d) demonstrate issues related to item development. With these goals in mind, we hope that the monograph will enhance awareness among mathematics education researchers that it is increasingly possible to select from a variety of model options when pursuing particular research goals and that choosing among models involves trade-offs. Such awareness may help mathematics education researchers have informed

discussions with psychometricians about the implications of different modeling options for particular projects.

Employing psychometric models in mathematics education is sufficiently complex that a single monograph cannot treat all aspects that might interest readers. For two reasons, we do not intend the monograph to be a primer in psychometric methods that would allow mathematics education researchers to apply those methods in the absence of substantive collaboration with experienced psychometricians. First, good introductory texts for more established psychometric models, such as item response theory (e.g., Bond & Fox, 2007; Hambleton, Swaminathan, & Rogers, 1991), already exist. Second, we believe that interdisciplinary teams that combine expertise in conceptualizing mathematical knowledge with expertise in psychometrics can best pursue innovative applications of psychometric modeling to mathematics education research.

Furthermore, a single monograph cannot engage the full range of discussions and debates about the strengths and weaknesses of different conceptualizations of mathematical knowledge or psychometric models. Mathematics education researchers have debated the merits of different conceptualizations of mathematical knowledge, some of which foreground knowledge as the property of individuals stored in their minds and others of which foreground knowledge distributed across individuals and surrounding physical or sociocultural settings (e.g., Sfard, 1998). It is fair to say that psychometric models, as well as the examples discussed in this monograph, emphasize knowledge as the property of individuals. Readers will see a multitude of ways that the following chapters describe knowledge of individuals—from computation skills and procedures to concepts and components for reasoning with quantities to broad categories such as pedagogical content knowledge. At the same time, psychometricians have debated the merits of various psychometric models. As one example, there have been vigorous debates about the relative merits of two families of psychometric models presented in this monograph—item response theory models and diagnostic classification models—and the reader may detect that the contributing authors take different positions in these debates. Research that combines psychometric models with mathematics education is likely to engage such debates in both fields.

OVERVIEW OF THE MONOGRAPH

The monograph contains 10 chapters. Chapters 1 through 4 introduce a range of psychometric models. These include item response theory models, which are used to order examinees along continua, and diagnostic classification models, which place examinees into distinct groups. These chapters communicate some of the diversity in psychometric models and provide examples of applications to mathematics education research. Chapters 5 through 8 address a range of issues to consider when designing assessments and selecting psychometric models for research. Chapter 5 discusses key decisions in designing research that uses psychometric models—decisions that include how to conceptualize the construct to

be measured, the trade-offs of different item types and scoring options, and different choices for the types and numbers of variables to use. Chapter 6 discusses dimensionality of test data, both as a choice that researchers and test developers make and as a matter of degree. Chapter 7 examines the implications of different purposes for assessment development, and Chapter 8 discusses ways that developing assessments can be a form of research in which the assessments and the constructs that they measure inform one another. None of the chapters require knowledge of advanced statistics. Chapters 9 and 10 provide commentaries, one written from the perspective of mathematics education and one written from the perspective of psychometrics.

The chapters of the monograph complement one another and, at the same time, stand on their own. Readers new to psychometrics may want to read the chapters in order from the beginning to gain initial familiarity with the most common examples of contemporary psychometric models. References in the early chapters point to more technical treatments of psychometric models. Readers already familiar with some psychometric models and their applications to mathematics education studies may be interested in only some of the initial chapters, as well as chapters in the second half. We imagine that educators could use various chapters in courses or seminars in mathematics education research or in interdisciplinary research design. Ultimately, we hope that this monograph will help researchers from both fields see the potential for innovative research that can be realized when collaborating across the two disciplines.

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