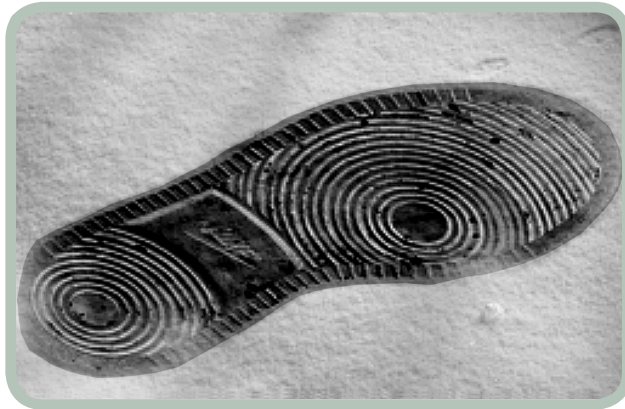


## CHAPTER 1

# Setting the Stage

Imagine walking into a middle school classroom where students are working on a statistics unit in which they are investigating patterns of association between two quantities. While students enter the classroom, the teacher gives each student a sheet of paper that contains the shoeprint shown in figure 1.1. The teacher explains that when investigators find shoeprints at the scene of a crime, forensic scientists can use the prints to identify suspects. She asks students to consider how a footprint could help someone solve a crime. After a brief discussion, students conclude that a shoeprint can indicate the type of shoe that a suspect wore, as well as the size of the suspect.



**Fig. 1.1.** Shoeprint distributed to students (From Pixabay)

The teacher explains that the students are going to investigate the relationship between shoe size and height so that they can determine the height of the suspect. While students work in pairs, measuring each other's height and shoe length, the

teacher monitors the activity and asks and answers questions as needed to support students' efforts. When pairs finish measuring, they add their data (red dots for girls and green dots for boys) to a large graph—with the  $x$ -axis labeled as shoe length and the  $y$ -axis labeled as height—posted in the front of the room. When all the students have added their data points to the graph, the teacher asks students to talk with their partners about the patterns that they notice. After a few minutes, the students share their observations, which the teacher records: for example, no two people have the same shoe size and height, most girls have smaller feet and are shorter than the boys, tall people have bigger feet than short people, the data go up from left to right, and the data are kind of linear.

The teacher tells students that their next step is to find a line that models these data—a line of best fit. She directs students to a Web-based applet, where they plot the class data in two-pair teams, guess at a line of best fit, and check their guesses. (An applet that supports this investigation is at <http://illuminations.nctm.org/Activity.aspx?id=4186>.)

The class concludes with a lively whole-group discussion, during which teams share their findings regarding the line of best fit, discuss the meaning of the slope and  $y$ -intercept in context, and consider how confident they are that the equation will be a good predictor of a person's height based on a shoeprint. In the final five minutes of class, students complete an exit ticket in which they indicate how tall they think the suspect is and present their reasons.

The authors have adapted the preceding lesson from NCTM *Illuminations*. <http://illuminations.nctm.org/Lesson.aspx?id=2838>

## A Vision for Students as Mathematics Learners and Doers

The lesson in the opening scenario exemplifies the vision of school mathematics for which the National Council of Teachers of Mathematics (NCTM) has been advocating in a series of policy documents for more than 25 years (1989, 2000, 2006, 2009). In this vision, as in the scenario, students are active learners, constructing their knowledge of mathematics through exploration, discussion, and reflection. The tasks in which students engage are both challenging and interesting, and students cannot quickly complete them by applying a known rule or procedure. Students must reason about and make sense of a situation and persevere when a pathway is not immediately evident. Students use a range of tools to support their thinking and collaborate with their peers to test and refine their ideas. A whole-class discussion provides a forum for students to share ideas and clarify understandings, develop convincing arguments, and learn to see things from the perspectives of other students.

In the shoeprint scenario, students faced a real-world problem, and they needed to collect and analyze data to solve it. All students could enter the problem by measuring, recording data on the graph, and making observations. Students' observations—including that “tall people have bigger feet than short people,” that data were “kind of linear” and “went up from left to right”—furnish evidence that students were attending to salient features of the graph and the relationship between the quantities. These observations then positioned the students to focus more narrowly on finding a line that modeled the data. Through the use of the applet, students were able to guess at the line of best fit and then check their guesses. During the discussion, students reported on their work, but the teacher also pressed them to consider what the equation meant in context. When the issue of how confident they should be about their equation came up, the teacher could then introduce and discuss the meaning of the correlation coefficient (which the applet generated). During the closing minutes of the lesson, the teacher asked students to determine how tall the suspect must be. This information could give the teacher insight into the extent to which students recognized the utility of the model to make predictions beyond the data set.

The vision for student learning that NCTM advocates and that our opening scenario represents has gained growing support over the previous decade while states and provinces have put into place world-class standards (e.g., National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010). These standards focus on developing conceptual understanding of key mathematical ideas, flexible use of procedures, and the ability to engage in a set of mathematical practices that include reasoning, problem solving, and communicating mathematically.

## A Vision for Teachers as Facilitators of Student Learning

Meeting the demands of world-class standards for student learning requires that teachers engage in *ambitious teaching*. Ambitious teaching stands in sharp contrast to the well-documented routine found in many classrooms. That routine consists of homework review, teacher lecture, and demonstration, followed by individual practice (e.g., Hiebert et al. 2003). This routine has been translated into the gradual release model: *I do* (the teacher tells students what to do); *we do* (the teacher practices with students); and *you do* (the students practice on their own) (Santos 2011). In instruction that uses this approach, the focus is on learning and practicing procedures with limited connection with meaning. Students have few opportunities to reason and solve problems. Although they may learn the procedure as intended, they often do not understand why it works and apply the procedure in situations in which it is not appropriate. According to Martin (2009, p. 165), “mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.” That is, at times students

get answers that make no sense; however, they have no idea how to judge correctness because they are mindlessly applying a procedure that they really do not understand.

In ambitious teaching, the teacher engages students in challenging tasks and then observes and listens while they work so that he or she can offer an appropriate level of support to diverse learners. The goal is to ensure that each and every student succeeds in doing high-quality academic work rather than merely executing procedures with speed and accuracy. In the opening scenario, a teacher is engaging students in meaningful mathematics learning. She has selected an authentic task for students to work on, provided resources to support their work (e.g., a method for measuring and recording data, an applet for investigating line of best fit, partners with whom to exchange ideas), monitored students while they worked, gave support as needed, and orchestrated a discussion in which students' contributions were essential. However, what we do not see in this brief scenario is exactly *how* the teacher is eliciting thinking and responding to students so that she supports every student in his or her learning. According to Lampert and her colleagues (Lampert et al. 2010, p. 130):

deliberately responsive and discipline-connected instruction greatly complicates the intellectual and social load of the interactions in which teachers need to engage, making ambitious teaching particularly challenging.

This book intends to support teachers in meeting the challenge of ambitious teaching by describing and illustrating a set of teaching practices that facilitate the type of “responsive and discipline-connected instruction” that is at the heart of ambitious teaching (Lampert et al. 2010, p. 130).

## Support for Ambitious Teaching

*Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014) gives guidance on what will make ambitious teaching, as well as rigorous content standards that it targets, a reality in classrooms, schools, and districts to support mathematical success for each and every student. At the heart of this book is a set of eight teaching practices that provide a framework for strengthening the teaching and learning of mathematics (see fig. 1.2). These teaching practices describe intentional and purposeful actions that teachers take to support the engagement and learning of each and every student. These practices, based on knowledge of mathematics teaching and learning accumulated over more than two decades, represent “a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics” (NCTM 2014, p. 9). Subsequent chapters of this book examine each of these teaching practices in more depth through illustrations and discussions.

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

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**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

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**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

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**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

---

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

---

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

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**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

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**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

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**Fig. 1.2.** Eight Effective Mathematics Teaching Practices (NCTM 2014, p. 10)

Ambitious mathematics teaching must be equitable. Driscoll and his colleagues (Driscoll, Nikula, and DePiper 2016, pp. ix–x) acknowledge that defining equity can be elusive but argue that equity is really about fairness in terms of access “providing each learner with alternative ways to achieve, no matter the obstacles they face” and potential, “as in potential shown by students to do challenging mathematical reasoning and problem solving.” Hence, teachers need to pay attention to the instructional opportunities that students receive, particularly

to historically underserved or marginalized youth (i.e., students who are black, Latina/Latino, American Indian, low income) (Gutierrez 2013, p. 7). Every student must participate substantially in all phases of a mathematics lesson (e.g., individual work, small-group work, whole-class discussion), although not necessarily in the same ways (Jackson and Cobb 2010).

Toward this end, throughout this book, we relate the eight effective teaching practices to specific equity-based practices that strengthen mathematical learning and cultivate positive student mathematical identities (Aguirre, Mayfield-Ingram, and Martin 2013). Figure 1.3 lists five equity-based instructional practices along with brief descriptions of them.

**Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.

**Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.

**Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.

**Challenge spaces of marginality.** Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.

**Draw on multiple resources of knowledge** (mathematics, language, culture, family). Tap students' knowledge and experiences as resources for mathematics learning.

**Fig. 1.3.** The Five Equity-Based Teaching Mathematics Practices  
(Adapted from Aguirre et al. 2013, p. 43)

Central to ambitious teaching—and at the core of the five equity-based practices—is helping each student develop an identity as a doer of mathematics. Aguirre and her colleagues (Aguirre et al. 2013, p. 14) define mathematical identities as follows:

the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives.

Many middle school students believe that they are not good at mathematics and approach mathematics with fear and lack of confidence. Their identity, developed through their previous years of schooling, can affect their school and career choices. Allen and Schnell (2016, p. 398) argue that “middle school mathematics teachers have a unique opportunity to steer their

students' mathematical development in a more positive direction." The effective teaching practices that this book discusses and illustrates attempt to help in this regard.

## Contents of This Book

This book is written primarily for teachers and teacher educators who are committed to ambitious teaching practice that provides their students with increased opportunities to experience mathematics as meaningful, challenging, and worthwhile. It is likely, however, that any education professionals working with teachers would benefit from the illustrations and discussions of the effective teaching practices.

Educators can use this book in several different ways. Teachers can read through the book on their own, stop to engage in the activities as suggested, or try activities in their own classrooms. Alternatively, and perhaps more powerfully, teachers can work their way through the book with colleagues in professional learning communities, department meetings, or when time permits. We believe that exchanging ideas with one's peers adds considerable value. Teacher educators or professional developers can use this book in college or university education courses for practicing or preservice teachers or in professional development workshops during the summer or school year. The book might be a good choice for a book study for any group of mathematics teachers interesting in improving their instructional practices.

This book supplies a rationale for and discussion of each of the eight effective teaching practices, and we connect them with the equity-based teaching practices when appropriate. We give examples and activities intended to help teachers of students in the middle grades develop their understanding of each practice, how they can enact it in the classroom, and how it can promote equity. Toward this end, we invite the reader to actively engage in two types of activities that the book presents throughout: Analyzing Teaching and Learning (ATL) and Taking Action in Your Classroom. ATL activities invite the reader to actively engage with specific artifacts of classroom practice (e.g., mathematics tasks, narrative cases of classroom instruction, video clips in More4U, student work samples). Taking Action in Your Classroom activities offer specific suggestions that indicate how a teacher can begin to explore specific teaching practices in her or his classroom. The Analyzing Teaching and Learning activities come, in part, from activities found in the *Principles to Actions Professional Learning Toolkit* (<http://www.nctm.org/PtAToolkit/>). We have added additional activities, beyond what the Toolkit includes, to facilitate a more extensive investigation of each of the eight effective mathematics teaching practices.

The video clips featured in the Analyzing Teaching and Learning activities show teachers who were endeavoring to engage in ambitious instruction in their urban classrooms and students who are persevering in solving mathematical tasks that require reasoning and problem solving. The videos, made available by the Institute for Learning at the University of Pittsburgh,



give images of aspects of effective teaching. As such, they are examples to analyze rather than models to copy. To access the video clips, visit NCTM's More4U website at [nctm.org/more4u](http://nctm.org/more4u) and enter the access code on the title page of this book.

While you read this book and engage with both types of activities, we encourage you to keep a journal or notebook in which you record your responses to questions that the book poses, in addition to noting issues and new ideas that emerge. These written records can serve as a basis for your own personal reflections, informal conversations with other teachers, and for planned discussions with colleagues.

Each of the next eight chapters focuses explicitly on one of the eight effective teaching practices. We have arranged the chapters in an order that makes it possible to highlight the interrelationships among the effective teaching practices. (Note that this order differs from the one shown in figure 1.2 and in *Principles to Actions* [NCTM 2014].)

- Chapter 2: Establish Mathematics Goals to Focus Learning
- Chapter 3: Implement Tasks That Promote Reasoning and Problem Solving
- Chapter 4: Build Procedural Fluency from Conceptual Understanding
- Chapter 5: Pose Purposeful Questions
- Chapter 6: Use and Connect Mathematical Representations
- Chapter 7: Facilitate Meaningful Mathematical Discourse
- Chapter 8: Elicit and Use Evidence of Student Thinking
- Chapter 9: Support Productive Struggle in Learning Mathematics

Each of these chapters follows a similar structure. We begin chapters by asking the reader to engage in an Analyzing Teaching and Learning activity that sets the stage for a discussion of the focal teaching practice. We then relate the opening activity to the focal teaching practice and highlight the key features of the teaching practice for teachers and students. Each chapter also highlights key research findings that relate to the focal teaching practice, describes how the focal teaching practice supports access and equity for all students, and includes additional ATL activities and related analysis as needed to provide sufficient grounding in the focal teaching practice. Each chapter concludes with a summary of the key points and a Taking Action in Your Classroom activity that encourages the reader to purposefully relate the teaching practice that the chapter examines to her or his own classroom instruction.

Although we present each of the effective teaching practices in a separate chapter, within each chapter we highlight other effective teaching practices that support the focal practice. In the final chapter of the book (Chapter 10—Pulling It All Together), we consider how the set of eight effective teaching practices relate to one another and how they work in concert to support students' learning. In chapter 10, we also consider the importance of thoughtful and thorough planning in advance of a lesson and evidence-based reflection following a lesson as



critical components of the teaching cycle that are necessary for successful use of the effective teaching practices.

## An Exploration of Teaching and Learning

We close the chapter with the first Analyzing Teaching and Learning activity, which takes readers into the classroom of Patrick Donnelly, where seventh-grade students are exploring proportional relationships. The case presents an excerpt from his classroom in which he and his students are discussing and analyzing the various strategies that students used to solve the Candy Jar task.

When chapters 2 through 9 introduce new teaching practices, we relate the new practice to some aspect of “The Case of Patrick Donnelly.” In so doing, we are using the case as a touchstone to which we can relate the new learning in each chapter. Hence, the case provides a unifying thread that brings coherence to the book and makes salient the synergy of the effective teaching practices (i.e., the combined effect of the practices is greater than the impact of any individual practice).

### Analyzing Teaching and Learning 1.1

#### Investigating Teaching and Learning in a Seventh-Grade Classroom

As you read “The Case of Patrick Donnelly,” consider the following questions and record your observations in your journal or notebook so that you can revisit them when we refer to this case in subsequent chapters:

- What does Patrick Donnelly do during the lesson to support his students’ engagement in and learning of mathematics?
- What aspects of Patrick Donnelly’s teaching are similar to or different from what you do?
- Which practices would you want to incorporate into your own teaching practices?

1 **Exploring Proportional Relationships:**  
2 **The Case of Patrick Donnelly**

3 Patrick Donnelly wanted his students to understand that quantities that are in a  
4 proportional (multiplicative) relationship grow at a constant rate and that students can  
5 use three key strategies to solve problems of this type: scaling up, a scale factor, and a unit  
6 rate. He selected the Candy Jar task for the lesson because it aligned with his goals, was  
7 cognitively challenging, and had multiple entry points.

8 **The Candy Jar Task**

9 A candy jar contains 5 Jolly Ranchers (JRs) and 13 jawbreakers (JBs). Suppose  
10 you had a new candy jar with the same ratio of Jolly Ranchers to jawbreakers,  
11 but it contained 100 Jolly Ranchers. How many jawbreakers would you have?  
12 Explain how you know.

13 While students began working with their partners on the task, Mr. Donnelly walked  
14 around the room, stopping at different groups to listen in on their conversations and to  
15 ask questions as needed (e.g., How did you get that? How do you know that the new  
16 ratio is equivalent to the initial ratio?). When students struggled to figure out what to  
17 do, he encouraged them to look at the work that they had done the previous day, which  
18 included producing a table of ratios equivalent to 5 JRs : 13 JBs and a unit rate of 1 JR  
19 to 2.6 JBs. He also encouraged students to consider how much larger the new candy jar  
20 must be when compared to the original jar.

21 As he made his way around the room, Mr. Donnelly also noted the strategies that  
22 students were using (see fig. 1.4) so that he could decide which groups he wanted to ask  
23 to present their work. After visiting each group, he decided that he would ask groups 4,  
24 5, and 2 to share their approaches (in that order), because each of those groups used one  
25 of the strategies that he was targeting and the sequencing reflected the sophistication and  
26 frequency of strategies.

27 During the discussion, he asked the presenters (one student from each of the  
28 targeted groups) to explain what their group did and why, and he invited other students  
29 to consider whether the approach made sense and to ask questions. He made a point of  
30 labeling each of the three strategies, asking students which strategy was most efficient  
31 in solving this particular task and asking them questions that helped them make  
32 connections between the different strategies and with the key ideas that he was targeting.  
33 Specifically, he wanted students to see that that the scale factor that group 5 identified  
34 was the same as the number of entries in the table that group 4 created (or the number of  
35 small candy jars that would make the new candy jar) and that the unit rate that group 2  
36 identified was the factor that connected the JRs and JBs in each row of the table.

37 The following is an excerpt from the discussion that took place around the unit-rate  
38 solution that Jerry from group 2 presented.

39 **Jerry:** We figured that there was 1 JR for 2.6 JBs, so that a jar with 100 JRs would  
40 have 260 JBs. So we got the same thing as the other groups.

41 **Mr. D.:** Can you tell us how you figured out that there was 1 JR for 2.6 JBs?

42 **Jerry:** We divided 13 by 5.

43 **Mr. D.:** Does anyone have any questions for Jerry? *[Pause]* Danielle?

44 **Danielle:** How did you know to do 13 divided by 5?

45 **Jerry:** See, we wanted to find out the number of JBs for 1 JR; so if we wanted JRs  
46 to be 1, we needed to divide it by 5. So now we needed to do the same thing  
47 to the JBs.

48 **Danielle:** How did you then get 260 JBs?

49 **Jerry:** Well, once we had 1 JR to 2.6 JBs, it was easy to see that we needed to  
50 multiply each type of candy by 100 so we could get 100 JRs.

51 **Mr. D.:** So Jerry's group multiplied by 100, but Danielle and her group (group 5)  
52 multiplied by 20. Can they both be right? Amanda?

53 **Amanda:** Yes. Jerry's group multiplied 1 and 2.6 by 100, and Danielle and her group  
54 multiplied 5 and 13 by 20. Jerry's group multiplied by a number 5 times  
55 bigger than Danielle's group because their ratio was one-fifth the size of the  
56 ratio Danielle's group used, so it is the same thing.

57 **Mr. D.:** Do others agree with what Danielle is saying? *[Students nod their heads and*  
 58 *give Danielle a thumbs-up.]* What is important here is that both groups kept  
 59 the ratio constant by multiplying both the JRs and JBs by the same amount.  
 60 We call what Jerry and his group found the *unit rate*. A unit rate describes  
 61 how many units of one quantity (in this case, JBs) correspond to one unit of  
 62 another quantity (in this case, JRs). *[Mr. Donnelly writes this definition on the*  
 63 *board.]*

64 **Mr. D.:** I am wondering if we can relate the unit rate to the table that group 4 shared.  
 65 Take two minutes, and talk to your partner about this. *[Two minutes pass.]*

66 **Mr. D.:** Kamiko and Jerilyn *[from group 4]*, can you tell us what you were talking  
 67 about?

68 **Kamiko:** We noticed that if we looked at any row in our table that the number of JBs  
 69 in the row was always 2.6 times the number of JRs in the same row.

70 **Mike:** Yeah, we saw that too, so it looks like any number of JRs times 2.6 will give  
 71 you the number of JBs.

72 **Mr. D.:** What if we were looking for the number of JBs in a jar that had 1000 JRs?

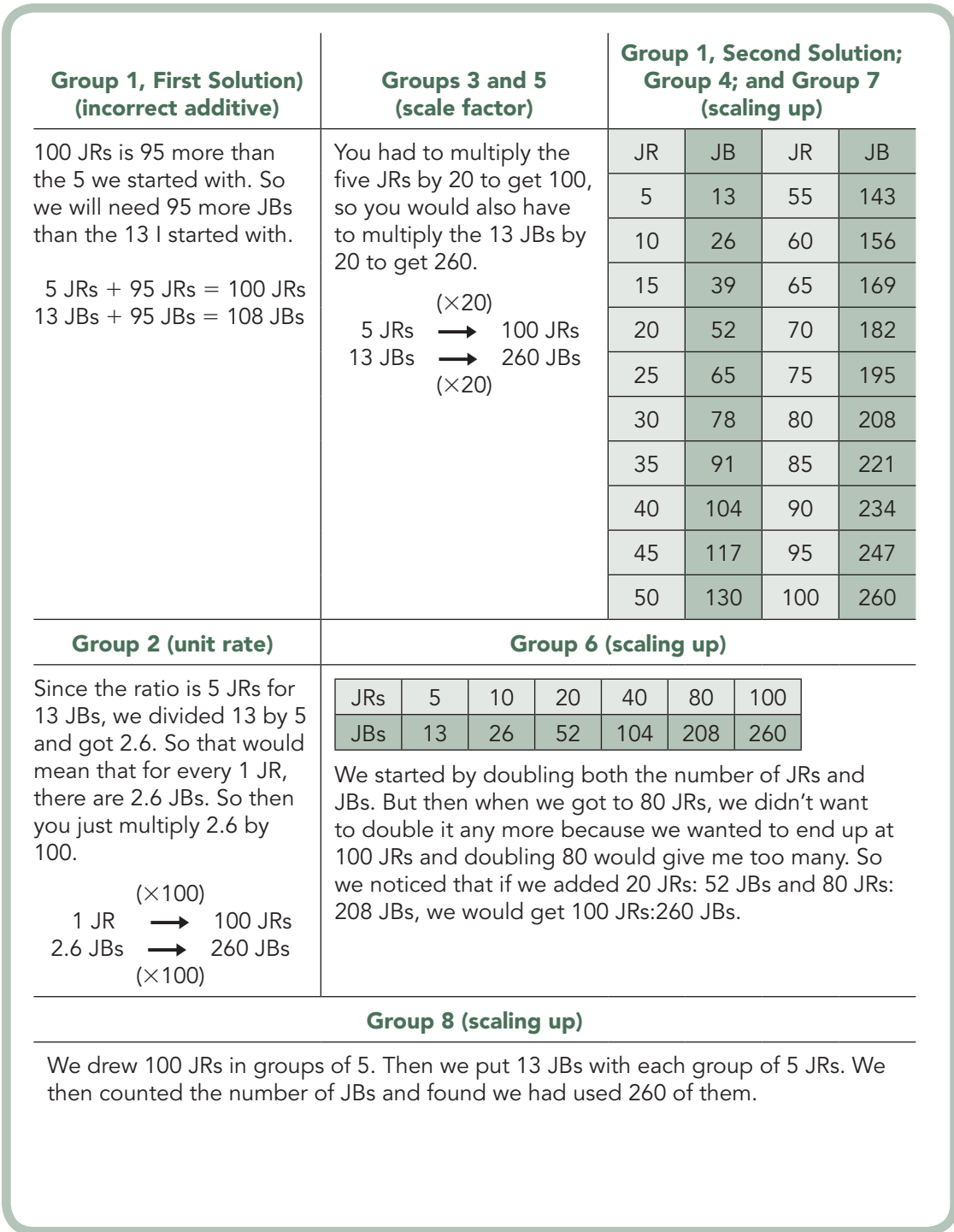
73 **Mike:** Well, the new jar would be 1000 times bigger, so you multiply by 1000.

74 **Mr. D.:** Take two minutes, and see if you and your partner can write a rule that we  
 75 could use to find the number of JBs in a candy jar no matter how many JRs  
 76 are in it.

77 *[After two minutes, the discussion continues.]*

78 Toward the end of the lesson, Mr. Donnelly placed the solution that group 1  
 79 produced on the document camera and asked students to decide whether this approach  
 80 was a viable one for solving the task and to justify their answer. He told them that they  
 81 would have five minutes to write a response that he would collect while they exited  
 82 the room. He thought that this exercise would give him some insight as to whether  
 83 individual students were coming to understand that ratios needed to grow at a constant  
 84 rate that was multiplicative rather than additive.

Margaret Smith of the University of Pittsburgh drew on two sources in writing this vignette: NCTM (2014) and Smith et al. (2005). The task is adapted and reprinted by permission of the Publisher. From Margaret Schwan Smith, Edward A. Silver, and Mary Kay Stein. *Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathematics Teaching and Learning* (volume 1) New York: Teachers College Press. Copyright 2005 by Teachers College, Columbia University. All rights reserved.



**Fig. 1.4.** The work of Patrick Donnelly's students

## Moving Forward

Mr. Donnelly's instruction has many noteworthy aspects, and the vignette gives examples of his use of the effective teaching practices. However, we are not going to analyze this case here. Rather, as you work your way through Chapters 2 through 9, you will revisit the case of Mr. Donnelly and consider the extent to which he engaged in the focal practice and the impact that it appeared to have on student learning and engagement. While you progress through the chapters, you may want to return to the observations that you made during your initial reading of the case and consider the extent to which you are viewing aspects of the case differently.

As you read the chapters that follow, we encourage you to continue to reflect on your own instruction and how the effective mathematics teaching practices can help you improve your teaching practice. The Taking Action in Your Classroom activity at the end of each chapter is intended to support you in this process. Cultivating a habit of systematic and deliberate reflection may hold the key to improving one's teaching as well as sustaining lifelong professional development.