Chapter 1

Contexts for Promoting Problem Solving

Mathematics is a participant sport. Children must play it frequently to become good at it. (National Research Council 2009, p. 125)

Our primary goal in mathematics education is to empower young children as problem solvers using mathematical understandings and skills featured in the Common Core State Standards for Mathematics (CCSSM). In order for children to achieve fluency as problem solvers, we need to expose them to multiple problem-solving strategies in different contexts that capture and sustain their interest and help them to reflect on the problem-solving process (National Research Council 2009).

From the ages of five to eight, children move through a significant period of cognitive growth. They increasingly make connections between what they have learned through experiences with materials in the concrete world and what they are learning through problems that require mathematical understandings and skills. Their early mathematical knowledge is directly linked to physical knowledge—that is, knowledge about the properties of objects gained through the senses (Rowan and Bourne 2001). Throughout these years, the balance shifts from learning based on discovering relationships between objects and actions through "hands-on" activities to learning through experiences of applying knowledge based on prior understandings. Children have an increasingly larger well of stored experiences, and this serves as the reservoir for mentally constructing answers to problems involving number, geometry, and measurement. However, the validation of solutions continues to require the use of real-world props, which in turn stimulates interest in learning more.

The major contexts that activate problem solving using mathematics in this developmental period occur during experiences in which children are doing the following:

1. Discovering mathematical content and relationships propelled by their own interests;
2. Organizing ongoing activities and planning activity sequences for materials management and scheduling purposes;
3. Engaging in playful activities and games organized by peers and adults;
4. Constructing products using art, craft, and other concrete materials;

5. Satisfying curiosity to solve a mathematical problem or figure out a mathematical relationship; and
6. Pursuing integrated curriculum activities in the form of topical studies, projects, and those activities related to literature.

The first five contexts are ones that permeate children's lives before they enter school. They serve as guidelines for designing curriculum activities that will engage and sustain children's interests in expanding mathematical knowledge that extends their problem-solving abilities. The last context is one that typically occurs in school.

CONTEXT 1—*Discovering mathematical content and relationships propelled by their own interests*

During the preprimary years, children are continually manipulating the rich variety of materials in their environment. For example, they count the peas on their plate, or they line them up or arrange them in a circle. Either deliberately or unintentionally, adults frequently use mathematical strategies related to measurement and number to facilitate daily events (e.g. "Do you want the bigger apple or a smaller one?" or "Just five more minutes, and we'll go to the park."). As children take on more responsibility for organizing their actions, they increase their awareness of number, geometry, and measurement relationships. Similarly, as they play with more of the objects that surround them, they not only compare and contrast the properties of these things, but they group them for different purposes. While research has documented that children learn a great deal from play, as they enter the formal schooling years "it appears that they can learn much more with artful guidance and challenging activities provided by the teachers" (Seo and Ginsberg 2004, p. 103). By the time children enter kindergarten, they have accumulated a wealth of experience that involves quantitative and geometric thinking, although they may not be able to communicate the degree of their understandings (Sophian 1999).

The instructional challenge with children at this developmental stage is to "capture them thinking mathematics" so that, as they pursue their own interests, their intuitive thinking is brought to the conscious level. The teacher's role here is to help children make a connection between the informal, intuitive mathematics and formal, school mathematics (Copley 2010). This is achieved by initiating conversation based on the adult observations. For example, if the child is pasting shapes to create a clown's face, an observational comment that translates action into words might be, "I noticed that you picked up a triangle to paste on the clown's face as the second eye and then decided to use a circle instead. Is that because the first eye was a circle?" Then, as the child continues by starting to take a circle and then selecting a triangle for the nose, the next question can focus on the child's decision: "I noticed that you were thinking about the circle or the triangle for the nose. Is there a special reason you chose a triangle for the nose instead of a circle?" In this instance, the child posed the problem and the adult invited an explanation of the solution (Schwartz and Copeland 2010). The kindergarten activity in chapter 3, which involves working with children in constructing with blocks, illustrates this context.

CONTEXT 2—*Organizing ongoing activities and planning activity sequences for materials management and scheduling purposes*

Children spontaneously plan many of their daily activities. For example, when several children are playing with a new set of materials, they distribute the items (i.e., *partitioning sets*) or place them in a central location accessible to everybody (i.e., *positioning a set*). When playing ball, they decide who goes first and who goes next in taking turns (i.e., *ordinal position and sequence*). Observing children as they pursue activities of their own choosing reveals how often they use mathematics to organize their use of materials. They show us in many ways how they use mathematical understandings and skills to control and anticipate their experiences.

In school, the need to solve problems using mathematics arises in planning and posting daily, weekly, and monthly schedules (i.e., *time measurement*); assigning classroom responsibilities and rotation of these responsibilities (i.e., *recording data*); and logging critical information for obtaining resources for such activities as snack and lunch (i.e., *number and computation*). Each of these activities serves a management purpose that creates opportunities for children to participate in solving time, space, number, and measurement problems in order to facilitate program activities. The two tasks for grade 1 found in chapter 4—planning a class trip and creating a calendar—involve children in this context.

CONTEXT 3—*Engaging in playful activities and games organized by peers and adults*

Playful activities designed and guided by the adult—such as copying and extending patterns with materials, following a musical beat with actions, or matching sets of objects in a mixed collection—may narrow the child's choices of how to use the materials or respond. But they can still retain the "high interest factor" associated with self-directed activities. These kinds of activities provide opportunities to increase the child's ability to identify mathematical relationships, and they feature problem solving in a given situation (e.g., inviting the child to justify the extension he has made to a bead pattern).

Similarly, playing games in which the player has to make some decisions narrows the choice of actions while simultaneously posing problems to be solved within the game structure (Eston and Economopoulos 1997; Kamii and DeVries 1980). Typical examples are a treasure hunt in which the players must figure out the meaning of locational and directional clues, a number game in which players must decide which numbers on their board to cover in order to make an equivalent match to the cue number, or a track game with alternative routes players may choose from to move to the end of the path. "In playing games, students tend to develop feelings of effectiveness and control because the actions they take in the game produce results" (Gordon 1972). The results of these choices contribute to the players' understandings of the mathematical relationships.

The adult role in this context includes:

- Designing games with increasing complexity so that the actions embedded in the game clearly focus on problem solving related to selected mathematical standards (Schwartz 2005).

- Observing the play of the game in order to collect information on the strategies the players are using to solve problems occurring during the play of the game for later discussion.
- Debriefing with the players about the problems they solved, in order to contribute to the players' thinking about the strategies they used.

Very simple games that offer no choices to the player, such as matching identical shapes with lotto cards or moving a specified number of spaces on a game board track, may be valuable tools for practicing mathematical skills, but they offer little opportunity to engage in mathematical problem solving. The adult role rests in the designing of the game and the debriefing with the players about the choices they made (Schwartz 2005).

Examples of tasks that fit within this context of playful activities include the kindergarten card games in chapter 2, the geometry game for grade 1 in chapter 3, and the kindergarten activity of matching lengths in chapter 4.

CONTEXT 4—*Constructing products using art, craft, and other concrete materials*

Making a product provides one of the richest opportunities for using mathematics to solve problems. Children quantify as they create, and they get immediate feedback on the success of their solution (Schirrmacher 1993). However, in order to realize this potential, children need opportunities for making choices that lead to thinking about problem solutions.

If a task is assigned (e.g., to make a three-dimensional model of the classroom), and the choice of materials is open, the problems posed involve selecting or making appropriately sized and shaped materials that can represent the objects in the classroom. This exercise also requires decisions about positioning the objects in the model at the right location and distance (Mitchell 1971; Sobel 1998). If, in contrast, the materials are assigned (e.g., precut paper shapes of different sizes and a collection of buttons, or a set of line stampers), the problems posed deal with selecting the goal of a product that can be created with the materials. In both approaches—either restricting the choice of product or restricting the choice of materials—the problems to be solved in producing a product involve linear measurement, joining shapes, and number (Barnett and Halls 2008; Lowenfeld 1958; Richardson 1964; Topal 2005).

For the least experienced learners, producing products with art, craft, and construction materials center more on discovering mathematical relationships than on applying knowledge to solve problems. The teacher's role here features helping children become aware of the mathematical relationships they are uncovering by sharing observations and extending conversations (e.g., "How did you decide on the size of the ears on this rabbit?").

For the more experienced learners, the decisions they make about what they want to produce provokes a greater need to use strategies to achieve their goal. Under these circumstances, the teaching role serves to help children to reflect on the strategies they used to solve problems and to think about alternative strategies (e.g., "How come you changed from using the string to using the wire when you were making the wheel for your car?").

The grade 2 activity in chapter 3, constructing three-dimensional objects from two-dimensional shapes, illustrates this context.

CONTEXT 5—*Being driven by curiosity to solve a mathematical problem or figure out a mathematical relationship*

It is not uncommon for a child or small group of children to become fascinated with exploring some mathematical relationship, such as figuring out how many ways they can match the weight of a specific object on a balance scale given objects of identical weights or several weights in ratio (Curcio and Schwartz 1997). The search serves no purpose other than to satisfy their curiosity. The inquiry may emerge spontaneously or be provoked by an adult who "wonders out loud" about a specific mathematical relationship related to an ongoing or recent curriculum activity.

The teacher role in this case rests primarily in setting up an investigation involving mathematical problem solving that captures the children's interests, and in focusing children's thinking on the mathematical strategies they are using to satisfy their curiosity.

The estimating tasks in the grade 2 section of chapter 2 offer an example of an activity driven by curiosity with significant mathematical possibilities. The grade 2 activity in chapter 4, measuring the power of magnets, also fits this context.

CONTEXT 6—*Pursuing integrated curriculum activities involving topical studies, projects, and literature*

Integrated curriculum activities, with their broad scope, present the most challenging contexts for focusing on problem solving using mathematical understandings. The many options make it essential to carefully plan in order to focus on identifying and solving problems that serve a purpose in the task while also featuring mathematical knowledge and skills related to the Common Core standards. For example, when studying growth and change in plants beginning with sprouting seeds, the most important embedded mathematical understanding relates to measurement. To support this learning, the adult needs to provide choices of measuring tools so that the children can solve the problem of which tool to use to measure the changes they observe as the seed matures into a seedling and then into a plant. The task of setting up a way to keep track of the changes associated with the various events they are observing provokes another kind of problem solving (e.g., how to record time intervals, change in size, appearance of leaves and their location on the plant). Both students and teachers need to participate in making decisions about what to measure and how to measure. In this type of activity, the role of the teacher involves helping students make choices of when, where, and how to use mathematical content to support the study of the topic.

When integrated curriculum activities take shape in projects, topics, and literature experiences, standards from all curriculum areas are naturally addressed, although not necessarily in each event that occurs as the project develops. One example would be if the theme is farm animals and the project is to build a model of a barn. In that case, life-science standards are addressed when identifying which farm animals need to be housed in the barn and what their needs are for care and feeding. Mathematics standards are addressed in the areas of measurement and number, as children figure out how many animals will be housed in the barn and the space each one needs. When using project activities to address standards, the broad scope of possibilities means the adult must carefully plan the choices offered to the children in order to create a focus on the use of mathematical strategies to achieve the goals in different segments of the project (Fromberg 2012; Helm and Katz 2011; Schwartz and Copeland 2010).

Similarly, the possibilities for pursuing a set of activities related to stories in literature have been extensively described in publications over the past two decades, as exemplified in the work of Whitin and Wilde (1992, 1995).

The grade 1 activities in chapter 2 on making mini-gardens illustrate several ways to make mathematics a central part of an integrated curriculum project.

Summary

The chapters that follow this introductory one deal with CCSSM K–2 standards and practices in the fields of number and operations, geometry, and measurement and data. All the tasks and activities presented in those fields are designed for engaging children in problem-solving situations and are framed in the six contexts identified above.

Kindergarten tasks feature activities that particularly capitalize on Context 1, *discovering mathematical content and relationships in activities propelled by children's own interests* (as in task 3.2, constructing with blocks), and on Context 3, *engaging in playful activities and games* (as in task 2.4, matching cards with pictured sets, and in tasks 4.1 and 4.2, comparing the length of wooden dowels).

Grade 1 tasks illustrate Context 2, *planning activity sequences and scheduling* (as in task 4.3, scheduling a class trip); Context 3, *engaging in playful activities and games* (as in task 3.4, playing a shape game); and Context 6, *pursuing an integrated curriculum activity* (as in task 2.5, a planting activity).

Grade 2 tasks focus on Context 4, *constructing products* (as in task 3.6, constructing a 3-D object from 2-D shapes), and on Context 5, *satisfying curiosity* (as in task 2.7, estimating quantities, and in task 4.5, measuring the power of magnets).

For all tasks and for all grades, the emphasis is on engaging and sustaining children's interest in using mathematical understandings to achieve a goal and to reflect on the strategies used to achieve it.