Chapter 1

The Physical and Virtual Environment

> Imagine a [learning environment] . . . where all students have access to high quality, engaging mathematics instruction. . . . The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. . . . Alone or in groups and with access to [tools and] technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.
>
> —National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics*

Open the door to any classroom and you enter a unique space, the home away from home for teachers and their students. Teachers spend quite a lot of time on their classrooms—debating over the desk arrangements, deciding what should go on the walls, and designing places to store learning resources—and rightfully so because a classroom is not just a room. It is an environment. Just as the conditions of a natural environment affect whether species flourish or decline, the classroom's physical environment influences the learning that takes place there (Arndt 2012; Cheryan et al. 2014; S. Martin 2002; Marx, Fuhrer, and Hartig 1999; McGregor 2004). It all begins with how we design and make use of the physical space.

The physical environment sets the tone for students' first experiences with what it means to be a thinker and learner in our classrooms. Looking at space is about more than the visible structures; it is about the social relationships that are possible and what is communicated about expectations and participation structures for learning (McGregor 2004). What counts as learning and doing mathematics? And who can learn mathematics and be recognized as successful? If teachers want all students to have access to learning powerful mathematical ideas and to see themselves as mathematical thinkers and as scholars who collaboratively support one another's success, classroom spaces must be organized and utilized in ways that reflect and promote this vision.

A Peek Inside a Classroom

So, how do these principles come together in the design of the physical setting? Let's consider an example by looking into Marie Sykes's second-grade classroom where students have been exploring the relationship between addition and subtraction (a copy of the student assignment is available at More4U). During the last math session, two of the students, Luke and Jacob, made a conjecture that the class is working on proving today. Mrs. Sykes and the students are gathered in the whole-class meeting area (fig. 1.1) as she poses the following task: "Luke and Jacob made a conjecture: You can use addition to solve subtraction, and you can use subtraction to solve addition problems. Do you agree or disagree with this conjecture? Prove your position using equations, words, and visual models."

Fig. 1.1. Create a central meeting place

STOP AND REFLECT

Before reading further, consider your stance. Do you agree or disagree with this conjecture? What tool or equation would you use to prove your response? Then, check to see how your response is similar to or different from the student responses that follow.

After Mrs. Sykes poses the task, students are given individual think time to consider their stance and to explain their thinking on paper. Students spread out, some working at tables, others at nooks and crannies around the room. Mrs. Sykes walks around documenting students' thinking in preparation for the later group discussion. After 10 minutes, Mrs. Sykes and the students gather around their meeting area to debrief (fig. 1.2). Mrs. Sykes asks students to give a thumbs up or down in agreement or disagreement with the conjecture. She calls on Esther, one of the quieter students in class, who had a claim similar to many of the students in the class in which she agreed with parts of the conjecture and not with others. Esther says, "Addition can be used to solve subtraction, but subtraction can only be used to solve addition some of the time." As Esther shares, quite a few of the students nod their heads. John raises his hand and builds from Esther's comment, explaining that there are different addition problem types and that subtraction can be used only for certain ones.

Fig. 1.2. The class in discussion

Mrs. Sykes writes this claim on the whiteboard and instructs students to work in quads to investigate this, using physical and visual tools as well as equations. Groups spread out around the room with students frequently getting up to get resources—paper, pencil, manipulatives—or walking to their math wall (a wall space documenting the big ideas and invented strategies discussed in class) to revisit strategies on addition and subtraction generated in the past month (see figs. 1.3 and 1.4).

The following day, the children meet with their teams to organize their findings and determine how they would present. Each team shares their work using the document camera. The majority of teams come to the same conclusion: "Yes, you can definitely add to subtract."

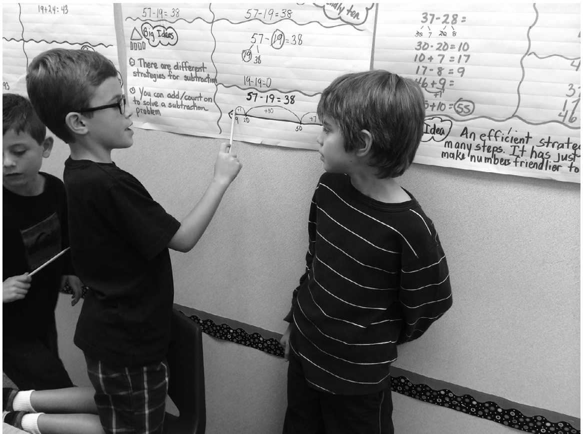


Fig. 1.3. Students solve problems on the math wall

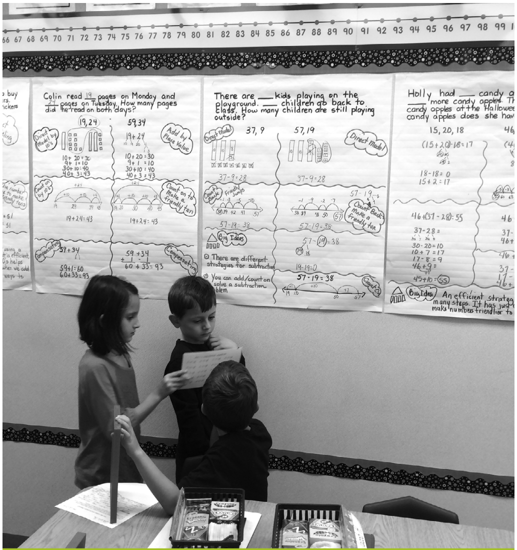


Fig. 1.4. The math wall in use

Many prove this by using a number line modeling *Think Addition* (see below) to find the difference. They also conclude that you can subtract to add. In problem situations such as $43 + n = 67$, you could solve with $67 - 43 = n$. You could also use subtraction with $n + 24 = 67$ by solving $67 - 24 = n$. One team claims that you could also solve $43 + 24 = n$ with subtraction. The class has quite a discussion about that one because the majority of students feel that it did not make sense to them and wonder why you would do that when it is easier to just add the numbers. This conjecture is left on the "Burning Question" poster on the math wall for later discussion.

MAKING SENSE OF STUDENT THINKING: THINK ADDITION

Think Addition, a strategy students often use early on to solve subtraction problems, reflects the inverse relationship between addition and subtraction. For example, in the equation $13 - 3$, students would think, "What can I add to 3 to make 13?" to get a difference of 10. Learning about fact families is an important aspect of this strategy. When students realize that addition and subtraction equations can be formed with the same three numbers, they develop greater fluency with using these operations.

This vignette illustrates a problem-solving activity that builds from a conjecture that emerged from the previous lesson. Students were asked to develop their own stance, provide evidence for their response using models and equations, and receive and provide peer feedback. Throughout the lesson, students engaged in mathematics across group settings (individual, small group, whole class) and with different representations (context, words, physical tools, and symbols). The classroom layout and resources were organized to support how students and the teacher engaged in their investigation.

In the remainder of this chapter, we discuss some of the critical elements in designing an environment where the physical space can serve as a teaching tool. We provide illustrations of layouts and classroom photos as well as teacher commentaries and rationales for their design. The following elements of classroom design are discussed in detail: classroom layout, math walls, and learning resources.

The Physical Configuration

Fig. 1.5. Heidi Kwalk's fifth-grade classroom

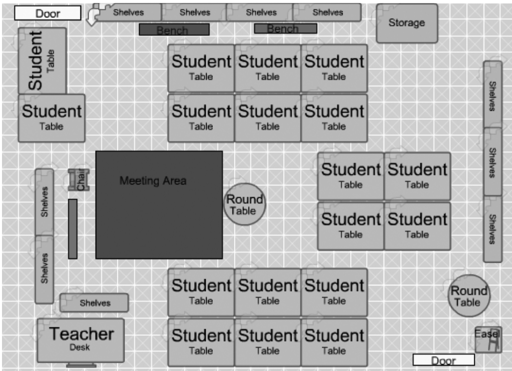


Fig. 1.6. Floor plan of Mrs. Kwalk's classroom

Fig. 1.7. Lauren Guite's first-grade classroom

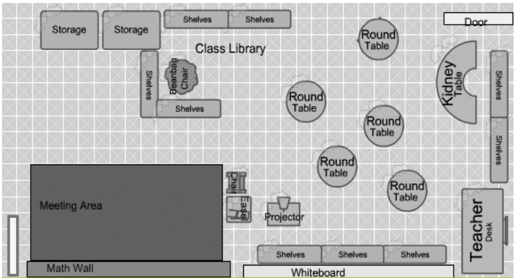


Fig. 1.8. Floor plan of Ms. Guite's classroom

STOP AND REFLECT

- Describe the physical arrangement of each of the two classrooms. How are the tables arranged? What types of space are available for varying types of group work (e.g., individual, partner, small group, whole class)? Where are learning resources and supplies stored?
- What is similar and what is different about the two rooms?
- What culture of learning is supported by each classroom layout?

The classrooms of Lauren Guite and Heidi Kwalk look different; however, both rooms are designed to support a common culture of learning that promotes students' individual and collective growth as mathematical thinkers. What are some features of these classrooms?

- Groups of desks to foster interaction
- Well-defined areas for working on the floor, at computers, alone, and as a group or whole class
- Easy access to learning resources and supplies—easy mobility from one workplace to another, to materials and supplies, to instructional resources on the walls

TEACHER COMMENTARY

I used to have individual desks but changed to tables. The students have their home spots where they come in the morning, but when they are working on reading or math workshop, they can work anywhere. It becomes a space for everyone. They are free to work at a table or get a clipboard and work on the rug. It's where they feel comfortable working. They collaborate and use and share the space more than when they first had their own designated spots.

—Lauren Guite, Grade 1

Although the room size and its shape, the number of students, and the available furniture influence the physical arrangements, attention to specific elements in the design of your physical space can help you promote interactions across learning resources. Organizing the classroom's physical space for active, interactive learning means creating well-defined spaces for individual, small-group, and whole-class discussions and providing easy access to learning resources. In addition, students must move easily from one work space to another and to mathematics tools and resources. Classroom transitions become smoother when students can efficiently navigate from one space to another during activities.

Designing space for mathematical community: Individual learning

Students need individual think time. The opportunity to think and wrestle with a problem independently encourages students' own sense making and perseverance as well as their self-esteem as mathematical thinkers. In most classrooms, students have an individual desk at which to work. Having easy access to clipboards allows students to move beyond their individual desks. An open space on the carpet, a pillow against the wall, or a small nook provides a quiet spot to work independently.

Designing space for mathematical community: Small-group learning

Figs. 1.9 and 1.10. Spaces for small groups

Collaborative learning requires that there be space for collaboration. Small, well-defined spaces encourage group interaction. Depending on the group size, such spaces can include rugs, tables, or benches (figs. 1.9 and 1.10). When working in small groups, students need to be able to face each other to make communication easier. Having tools and materials (e.g., whiteboards, pencils, erasers, manipulatives, norms for student-to-student talk) nearby also promotes and supports productive interaction (fig. 1.11). Much time can be saved in the long run by taking time at the start of the school year to establish and practice routines for transitioning into and out of group work.

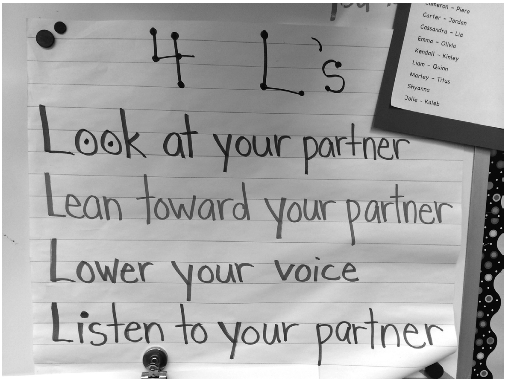


Fig. 1.11. Norms for communication

Designing space for mathematical community: Whole-class learning

The meeting area is a separate defined space for the class to gather. Students are more focused on the discussion if they are physically gathered close together. This is where the whole class has the opportunity to raise questions; share and defend their own models, ideas, and strategies; and create and argue solutions with one another. Students deepen their understanding of mathematics and their role as members of a learning community as they engage in collective activity, discourse, and reflection. The meeting area is often bordered by a wall and a whiteboard or chart board easel.

Figure 1.12 provides an example of a meeting area organized for whole-group discussion. Lauren Guite, a first-grade teacher in a Spanish Immersion classroom, uses an area rug to create a defined space for her whole-class discussions. The rug is placed next to a math wall where anchor charts and students' thinking are displayed. A chart paper easel is also placed next to the chair where she usually sits. The chart paper easel is where the students demonstrate their thinking and make comparisons between strategies and solution methods. The visual representation on the chart paper serves as a tool during their discussion. The chart paper is later moved to the math wall for future reference. Figure 1.13 provides an example of a chart Ms. Guite has created to introduce students to their problem of the day.

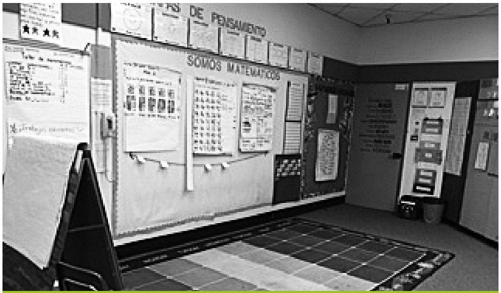


Fig. 1.12. Ms. Guite's meeting area

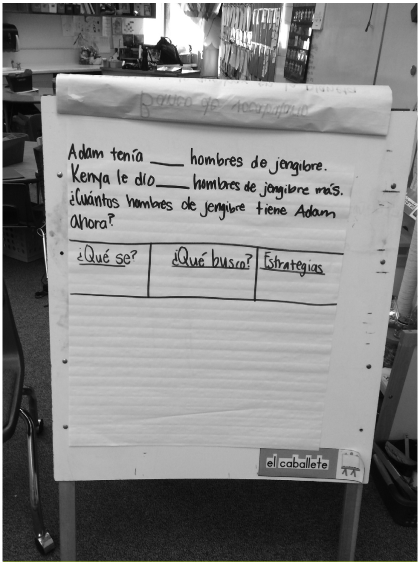


Fig. 1.13. An easel as focal point

STRATEGIES FOR YOUR CLASSROOM

Lauren Guite is using a story structure for problem solving (see fig. 1.13) that is based on Cognitively Guided Instruction (CGI). Spaces for quantities are left blank intentionally to encourage students to look closely at the context and its meaning for problem solving. Here, Ms. Guite asks students to consider "What they already know," "What they're trying to figure out," and "Possible strategies for problem solving" before the quantities are even presented. Additional strategies for launching mathematics problems will be shared in Chapter 4.

The Math Wall

The math wall is a wall space devoted to mathematics. In the vignette of Mrs. Sykes's class, students used resources on the class math wall to reflect, compare, and build on previously learned concepts and student-generated strategies.

The math wall serves as a visual record of students' methods of problem solving (fig. 1.14) and should be used to engage students in discussing connections and relationships between prior ideas and new content. Students too often see mathematics activities as discrete lessons instead of seeing the coherence between them. Having direct access to prior learning promotes an explicit focus on the interconnection of mathematical ideas and how they build on one another.

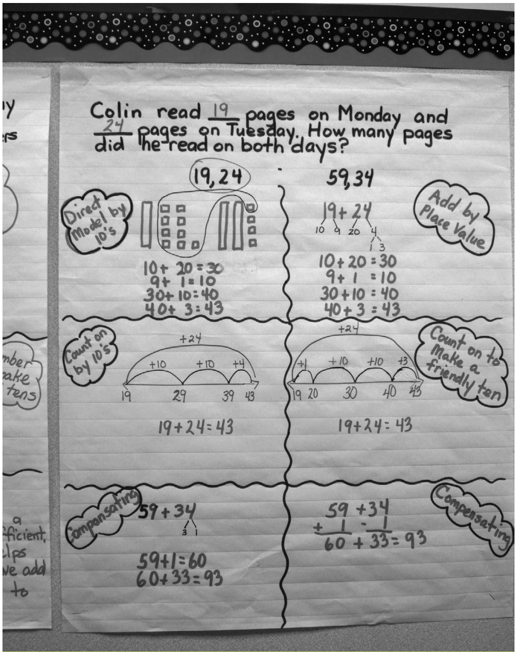
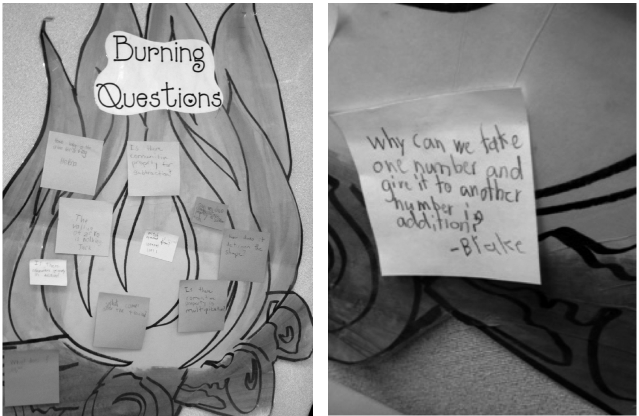


Fig. 1.14. Student strategy poster

Time constraints are another reason for saving student work. Time often seems to get the best of teachers when they are trying to orchestrate a rich discussion. Charting student work on poster paper makes it easy to continue the discussion the next day. Plus, a student may ask a rich question that should be pursued further, but there's limited time for investigation or the question is not directly related to the day's learning goal; putting the question on the math wall acknowledges its value and makes it easy to go back to it later (figs. 1.15 and 1.16).



Figs. 1.15 and 1.16. Burning question posters

Student thinking on display: What does it look like? How do I start?

Here we illustrate the use of the math wall as instructional space to reflect students' growing understanding and to share tools for problem solving and communication. The decision of what goes on the wall should be based on knowledge of the mathematical learning goals, students' needs, and their cognitive development.

MAKING SENSE OF STUDENT THINKING: EARLY DEVELOPMENT OF NUMBER CONCEPTS

Children's early development of number concepts moves from a concrete representation to the abstract. For example, students generally move from counting all (e.g., physically making four counters and then making twelve and counting all the counters to get sixteen), to counting on (e.g., starting at twelve and counting four to get to sixteen), to using part-whole (e.g., splitting apart the twelve to ten and two, adding the two to four, and then adding the ten to get sixteen), and finally to relational thinking (knowing that $6 + 10$ is 16 so $6 + 9$ would be just one less). Explicit discussions on student strategies and comparisons between strategies can support students in developing more efficient strategies.

In Ms. Guite's case, her understanding of children's progression from the concrete to the abstract guides decision making about what to include on the classroom math wall. The math wall is a reflection of the students' developing thinking and serves as a reference for students throughout their mathematical work. The charts and references created during class investigations and discussions and the student strategies on display represent varied approaches to problem solving. Ms. Guite intentionally selects strategies to include concrete representations (e.g., the use of base-ten blocks to solve $24 + 19$) as well as abstract representations (e.g., compensation strategy where $59 + 34$ becomes $60 + 33$). This is purposeful, and these displays are constantly referenced during whole-group interactions, as when students are encouraged to consider the relationship between strategies and the efficiency of the strategy used in relation to the problem posed.

Mrs. Sykes's math wall is bare at the start of the school year. From the first day, she lets students know that posters of their thinking will be added to reflect their growing knowledge, understandings, and conjectures (see fig. 1.17). The wall is kept at eye level so students can make reference to it during their own think time. Student strategies are first displayed with the student's name (e.g., "Saul's strategy") and later labeled with formal academic terms so the mathematical language is linked to meaning.

Fig. 1.17. Mrs. Sykes's math wall

TEACHING TIP: STUDENTS' STRATEGIES

When strategies are first shared, write the students' name next to the strategy (e.g., "Saul's strategy"). This provides easy reference back to the strategy and encourages student dialogue. For example, if a student wants to try out Saul's strategy, she or he can talk directly to Saul. Once students are familiar with the strategy, introduce the formal academic term for it. Developing academic language within context builds a stronger cognitive bond, making it more likely students will know and use it appropriately and with greater precision (Moschkovich 1999; Planas and Civil 2013).

An instructional wall: Supporting norms for doing mathematics

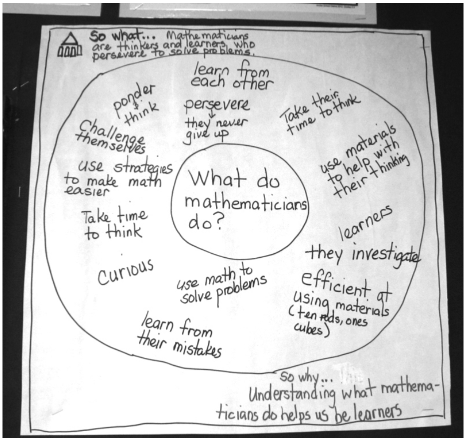


Fig. 1.18. Norms for engaging with mathematics as mathematicians

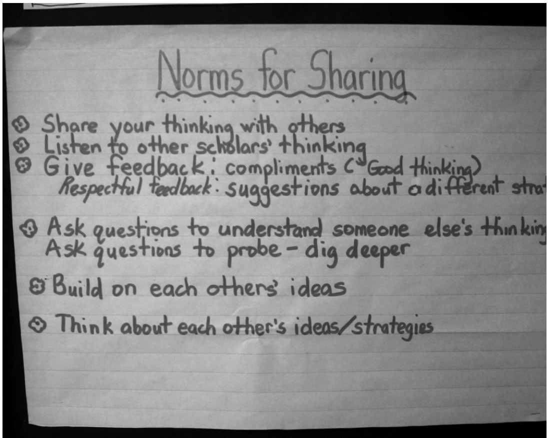


Fig. 1.19. Norms for student talk

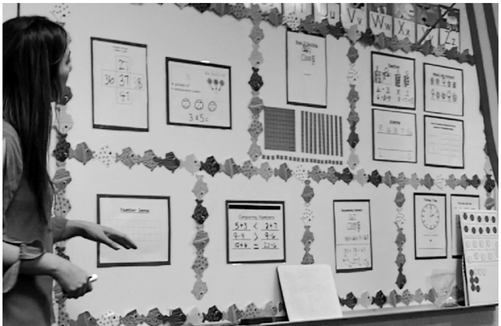
Before students can actively engage with the mathematics and with each other, norms for engagement need to be established. Teachers cultivate classroom norms in many different ways. It's important to pay attention to the norms you want to foster in your classroom and to make these norms explicit to your students. Set up norms for students to engage with the mathematics and their peers. Then, post a classroom poster of these norms near the whole-class meeting area to support students as they learn how to talk and engage in the mathematics and with each other's ideas (see figs. 1.18 and 1.19). (See Chapter 2 for more about strategies to develop classroom norms and to promote student math talks.)

A math focus wall

The math focus wall is an interactive mathematics bulletin board that reflects the daily learning in the classroom. It is often used in a ten- to fifteen-minute warm-up, and its purpose is to provide students with an opportunity to deepen their understanding of core concepts of mathematics at their grade level. In the mini-lesson, several tools, representations, and models are used to support students' thinking about connections between representations to operations.

Vanessa Frank, who teaches a kindergarten/first-grade combination class, reserves wall space near her whole-class meeting area for her math focus wall. Each day, Ms. Frank selects two or three problems that target specific content and process standards at her grade level. Problem solving and academic language use are integral in the activity.

Video clip 1.1, Math Wall Lesson, (available at More4U) shows Ms. Frank using the math focus wall to engage students in discussion around subtraction, place-value models, and decomposing a ten. Examine the nature of the conversation. How does the wall space serve as an instructional tool for Ms. Frank and her students? Why did Ms. Frank provide a story for the subtraction problem? What teaching strategies are used to reveal student thinking and encourage thoughtful responses?



Video: Math Wall Lesson (See More4U.)

Instructional walls: What else?

Heidi Kwalk develops fraction and decimal meaning from students' understanding of whole-number place value. To create a large place-value chart, she uses a row of cabinet doors with each door representing one of the positions in the place-value system (fig. 1.20).

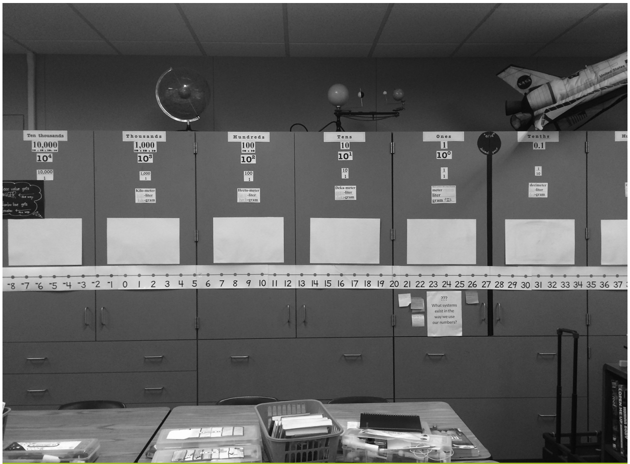


Fig. 1.20. Place-value chart posted on cabinet doors

She uses this to support students' understanding of the 10-to-1 relationship between the value of any two adjacent positions. This 10-to-1 relationship can extend infinitely in both directions. One hundred is the same as 10 tens, just as one-tenth is the same as 10 hundredths.

Darielle Tom, who teaches a kindergarten/first-grade combination class, gives each student her or his own space on the classroom math wall (see figs. 1.21 and 1.22). Dr. Tom conferences with each student to decide what should be added to this wall space. The wall space serves as a resource for students to reflect on strategies learned, and it provides a visual documentation of their mathematical growth throughout the year.

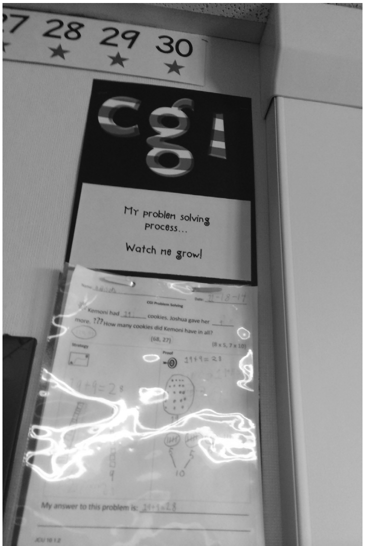


Fig. 1.21. Student work on math wall

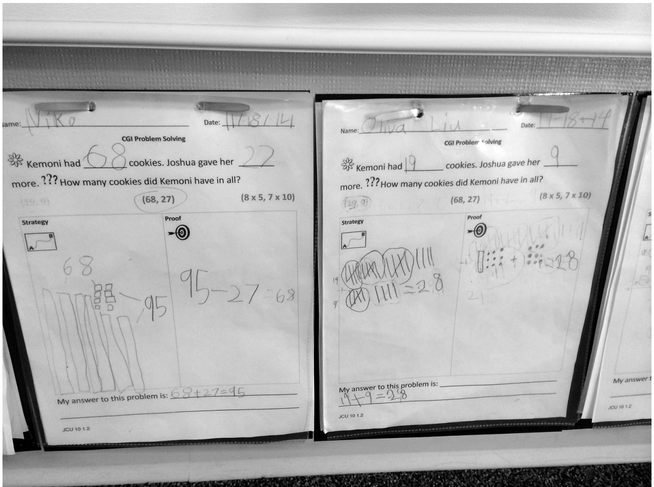


Fig. 1.22. Student work on math wall

Tools and Supplies

In the vignettes presented throughout this chapter, the students used mathematical tools and supplies during their independent think time and when proving their conjectures in their small groups. Students needed easy access to these materials to ensure that the flow of the mathematical inquiry is not interrupted while students or the teacher retrieves materials. Also, when tools are not easily accessible, students are less likely to use them.

> **TEACHING TIP: SELECTING THE RIGHT TOOL**
>
> Once a problem is assigned, instruct students to think about how they will approach the problem and the tool that would best support and represent this approach. Asking students to take the time to reflect about matching tools and resources to a specific strategy prevents the whole class from rushing to the supplies at the same time and encourages students to be more strategic in their selection.

Teaching and learning resources consist of two types: those used by us, the teachers, and those used by the students. Teacher resources should be stored inside a closet or on high shelves. Student resources should be kept low and be easily visible and accessible.

The shelves in figure 1.23 contain different resources that are easily accessible to the teacher and students. Labeled or clear containers enable students to quickly see what kinds of tools are available and know where to return them after use. Having clearly defined spaces for mathematical tools and resources (e.g., counting collections, counters, materials for math centers, writing materials, measuring tools) will ease access and prevent crowding.

Fig. 1.23. Storage for math tools

Thoughts about tool selection and use

> An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking. (NCTM 2014a, p. 78)

As you think about the range of tools to make available to students, consider how these tools (including technologies) may both support students' growth toward specific mathematical learning goals and increase access to demonstrate their mathematical thinking (King-Sears 2009; Pritchard, O'Hara, and Zwiers 2014; Sarama and Clements 2009; Suh and Moyer 2007). Although it is often best to allow students to choose the tools they want to use in a mathematical activity (Moyer and Jones 2004), there are times when teachers will find it useful to prescribe the tool choices. For example, when fifth-grade teacher Mayra Orozco found that many of her students had a fragile understanding of decimal numbers and were unable to translate between various representations, she designed a lesson that required students to rotate individually through the use of several tools, including base-ten blocks, coins, and 100-grids. The culminating activity challenged students to find connections among the various representations that helped them to better understand decimal numbers. See Chapter 4 on assessment for more about this lesson.

Figs. 1.24–1.27. Make materials accessible.

Digital learning spaces

Most schools now provide digital learning spaces to complement the physical space of the classroom (and some schools are entirely digital). Teachers must think about how to make use of digital tools and resources to support students' learning of mathematics in ways aligned with curricular goals (Pritchard, O'Hara, and Zwiers 2014; Sarama and Clements 2009). For instance, class websites offer a place to post resources for reference, review, and exploration. When posting resources to a class website, teachers should consider how well aligned these are to the way in which they want students to come to understand mathematical ideas and skills. Just as keeping desks in rows can work against the development of rich mathematical collaboration and discourse, posting videos showing how to perform an algorithm without any sense making or linking to games requiring minimal mathematical reasoning can disrupt efforts to create a community of mathematics learners engaged in practices of reasoning, sense making, explanation, and justification.

TECHNOLOGY INTEGRATION

Desiree Olivas uses Google Classroom to provide varied opportunities for students to share their thinking and respond to others' ideas. At times students are encouraged and prompted to respond to and expand on each other's thinking on an online forum. Other times, students use Sketchpad or watch or create their own videos explaining a mathematics concept or problem.

In addition, an ever-growing proportion of students have ready access to handheld computing devices, such as smartphones and tablets, and routinely use these for social purposes (Project Tomorrow 2011). These digital tools can be used to promote productive collaboration and communication through activities such as blog posts about recent mathematical tasks, screencasts of student problem solving, and shared images of mathematics in students' communities; such activities have the potential to increase engagement among a wide range of students (King-Sears 2009; Pritchard, O'Hara, and Zwiers 2014; Roschelle et al. 2010). And remember, just as norms help govern students' productive interactions in the physical classroom, it is important to establish and enforce norms for interactions in digital spaces (Ribble 2012). Throughout the remaining chapters, we include examples of the productive use of tools and technology within the context of establishing powerful mathematics learning environments.

Summary and Reflection

The vignettes in this chapter demonstrate ways that the strategic organization and use of a classroom's physical and virtual space can support rich mathematics learning. The classroom setting lays the foundation for all the other elements of the learning environment: discourse, task, assessments, and families and communities. A rich physical and virtual environment should have well-defined spaces for individual, small-group, and whole-class work and easy access to learning resources (i.e., other students and the teacher, tools and supplies, and spaces displaying student and class work).

STRATEGIES FOR YOUR CLASSROOM

Considerations for Designing Your Physical Environment

The physical configuration

- Well-defined spaces for individual, small-group, and whole-class learning where students can easily see each other
- Clear paths for mobility from one place to another (e.g., meeting area to individual seats)
- Easy access to learning resources (e.g., whiteboards, pencils, manipulatives, and norms for mathematics engagement)

The math wall

- Wall space to display student strategies, conjectures, and norms for engagement
- Clear path and easy access to the math wall
- Wall at eye level for students (not the teacher)

Tools/supplies

- Clearly defined spaces for mathematical tools, including manipulatives
- Easy access to student resources
- Labels or clear containers to facilitate return of materials and allow for easier accounting
- Separate spaces for resources to prevent overcrowding

Before you move on to the next chapter, take a moment to reflect on these questions:

STOP AND REFLECT

Consider pairing with another teacher to observe the structural setup in each other's classrooms:

- What type of space is available for students to work independently, in pairs, in small groups, and as a whole class?
- Do these spaces have easy access to—
  - manipulatives, tools, and general supplies?
  - norms to engage in the mathematics and with each other (e.g., norms for engagement, sentence starters for student talk)?
  - a variety of work spaces including digital ones?
- What wall spaces (and digital spaces) are available to share student thinking?

Make time to discuss the observations. Plan any changes accordingly.