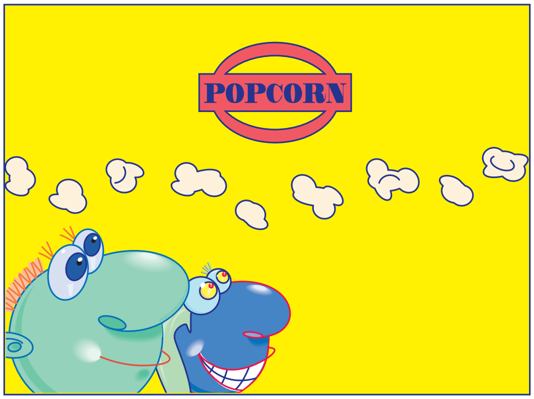
Math Challenge #3 POPCORN

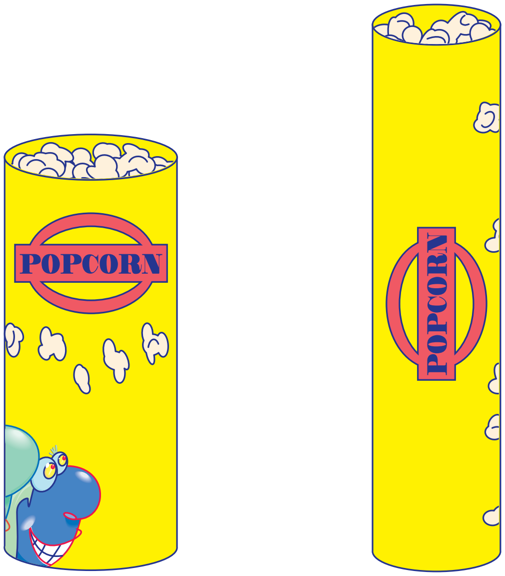
FigureThis!

Math Challenges for Families



If you like popcorn,
which one would you buy?

Figure This! Take two identical sheets of paper. [An ordinary sheet of paper measures 8 1/2 inches by 11 inches.] Roll one sheet into a short cylinder and the other into a tall cylinder. Set them both on a flat surface. Does one hold more than the other?



Hint: Place the taller cylinder inside the shorter one. Fill the taller one with dry cereal, rice, or popcorn; then remove it from the shorter cylinder. Which holds more?

Making visual estimates and finding volumes are useful skills. Designers and engineers use these skills to find economical ways to package and protect items.

Answer:
The shorter cylinder holds more.

Figure This!

Get Started:

Make a guess and then use the hint.

Complete Solution:

The process described in the "Hint" shows that the shorter cylinder holds more.

- To determine an answer mathematically, find the volume of each cylinder. The volume is the area of the base times the height. In this case, the bases are circular. The area of a circle is π • r • r or approximately 3.14 • r • r. To find the radius, r, use a ruler to estimate the width (or diameter) of the circle. Divide the diameter by 2 to get the radius. Another way to find the radius of a circle is to use the formula:

  Circumference of circle = 2 • π • radius = $2\pi r$
  Radius = Circumference divided by (2π)

  Once you have the radius, the table below shows how to determine the volume of each cylinder. The sheet of paper is 8 1/2 inches by 11 inches.

Cylinder	Base (inches) Circumference	Radius (inches) r	Height (inches) h	Volume (cubic inches) π • r • r • h
Short	11	11 ÷ (2π) or about 1.75	8.5	π • 1.75 • 1.75 • 8.5 About 81.8
Tall	8 1/2= 8.5	8.5 ÷ (2π) or about 1.35	11	π • 1.35 • 1.35 • 11 About 63.0

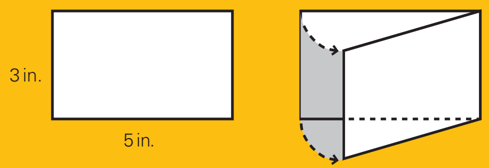
The volume of the shorter cylinder is about 82 cubic inches, and the volume of the taller cylinder is about 63 cubic inches.

Try This:

- Go to the grocery store and see what items come in different shaped or different sized cylinders.
- Look in your cupboard or go to the grocery. Find two different shaped containers that hold the same amount. What are the contents of each?

Additional Challenges:

1. For what size paper would the two cylinders hold the same amount?
2. Using any non-square rectangular sheet of paper, does the shorter cylinder always hold more?
3. Another way to describe a cylinder is to rotate an index card about one of its sides. Think about the cylinder traced by a 3 by 5 card as it turns. Which is larger: the volume of the cylinder formed when the card is turned about a short side or a long side?



4. Suppose you had two equal lengths of wire. Fold the wires to make two rectangles. Do you think the two rectangles will always have the same area?

Things to Think About:

- What items come packaged in cylindrical containers?
- What types of goods are packaged in boxes instead of cylinders? Why do you think companies use boxes?
- If a number is greater than 1, squaring it makes the result greater.

Did You Know That?

- Designing cans and labels is just one aspect of packaging technology. You see the results of this work every time you unwrap a CD, twist open a lipstick container, or open a soft drink.
- Several universities offer degrees in packaging technology. Many of them can be found at **packaging.hp.com/pkgschools.htm**
- Isopermetric figures are figures with the same perimeter. Fencing problems typically fall under this category.

Resources:
Books:

- Lawrence Hall of Science. *Equals Investigations: Flea-Sized Surgeons*. Alsip, IL: Creative Publications, 1994.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and E. Phillips. *Connected Mathematics: Filling and Wrapping*. Palo Alto, CA: Dale Seymour Publications, 1996.

Answers to Additional Challenges:

(1.)
The two cylinders would hold the same amount only for square sheets of paper.

(2.)
Yes. (This can be proven mathematically.)

(3.)
The cylinder with the larger volume is described when the card is turned about its shorter side. This problem compares π • 5 • 5 • 3 and π • 3 • 3 • 5.

(4.)
The rectangles may not always have the same area. Consider a piece of wire 16 inches long. You can make a square 4 inches by 4 inches with an area of 16 square inches, or a 1 inch by 7 inches rectangle that has an area of 7 square inches.