Grade 6:
Representing Ratios in Various Formats
NCTM Interactive Institute, 2015

Name
Title/Position
Affiliation
Email Address

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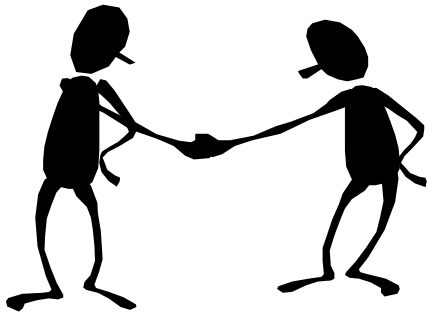
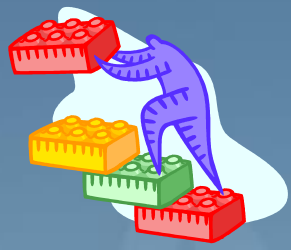
Introductions….

- Introduce yourself to others at your table.



- Discuss success and challenges you encounter when teaching the topic of **ratios** to students in your classroom.

Common Core Progressions



GRADE 6	GRADE 7	GRADE 8
Understand ratio concepts and use ratio reasoning to solve problems.	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**	**Understand the connections between proportional relationships, lines, and linear equations.**
• Concept of ratio • Use ratio language • Concept of unit rate • Use ratio and rate reasoning to solve real-world and mathematical problems (tables, diagrams, double number lines, equations)	• Compute unit rates • Represent proportional relationships between quantities • Use proportional relationships to solve multistep ratio and percent problems	• Graph proportional relationships, interpreting the unit rate as the slope of the graph • Use similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line

Big Ideas for Ratios

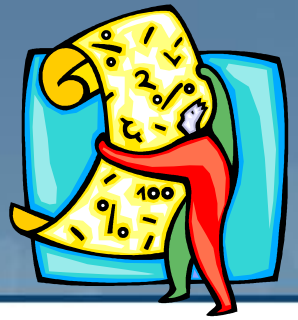
A ratio is an ordered pair of numbers or measurements that expresses a comparison between the numbers or measures.

- Reasoning with ratios involves attending to and coordinating two quantities.

  Forming a ratio involves isolating one attribute from other attributes.

- A ratio is a multiplicative comparison of two quantities or it is a joining of two quantities in a composed unit.

Ratios



- **Ratios are expressed in fraction notation.**
  - Ratios and fractions do not have identical meaning.
  - Ratios and fractions can be conceived as overlapping sets.

- **Fractions are ratios, but not all ratios are fractions.**
  - Some ratios make "part-part" comparisons or relate more than two parts.

- **Rates are ratios, but not all ratios are rates.**

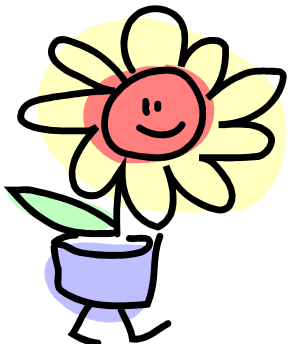
Ratios are building blocks for proportions and proportional reasoning.

Types of Ratios

- **Part-to-Whole Ratios**
  - Compare two measures of same type
  - Nonfiction books to all books in library, percentages, probabilities
- **Part-to-Part Ratios**
  - Compare two measures of same type
  - Fiction books to nonfiction books in library, odds of an event
- **Rates as Ratios**
  - Comparison of measures of two different things / quantities
  - Prices, time and distance, miles per gallon, inches per foot
- **Special Ratios**
  - Golden ratio, π, slope of line, geometric similarity, trigonometric functions from right triangles

Ratios:
Student Thinking

Two weeks ago, two flowering plants were measured at 8 inches and 12 inches. Today they are 11 inches and 15 inches tall, respectively.



Which flowering plant grew more – the 8-inch or 12-inch flower?

Defend two different "answers" to this problem.

Additive Versus Multiplicative Reasoning

Multiple Representations

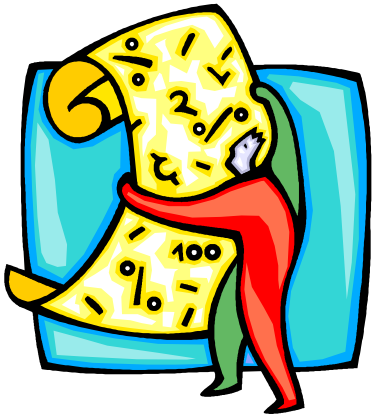
During this session, we are going to use various representations to help students develop conceptual understanding of ratios.

- Unit ratio
- Ratio table
- Double number line
- Tape diagram

Comparing Ratios

Unit rate

Distinguish equivalency between 2 or more ratios



Comparing Ratios

Two camps of Scouts are having pizza parties.

The Bear Camp ordered enough so that every 3 campers will have 2 pizzas.

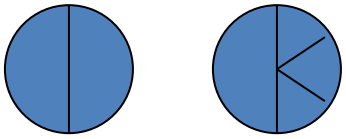
The leader of the Raccoons ordered enough so that there would be 3 pizzas for every 5 campers.

Which campers had more pizza to eat: the Bear campers or the Raccoon campers?

Unit Rate

"Pizzas per Camper" Approach

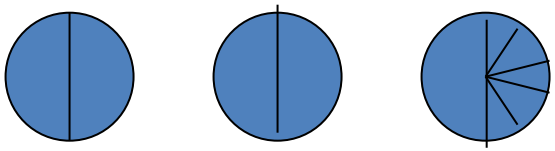
Bear Campers



Each of the 3 campers will get $\frac{1}{2}$ pizza and $\frac{1}{6}$ pizza.

$\frac{2}{3}$ pizza per camper

Raccoon Campers



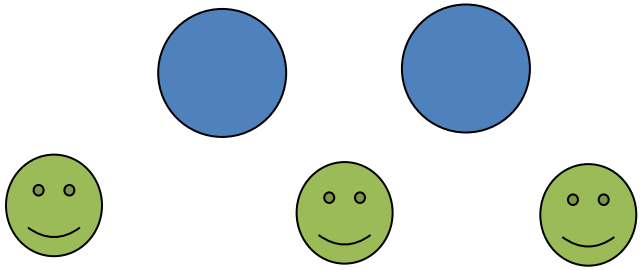
Each of the 5 campers will get $\frac{1}{2}$ pizza and $\frac{1}{10}$ pizza.

$\frac{3}{5}$ pizza per camper

Unit Rate

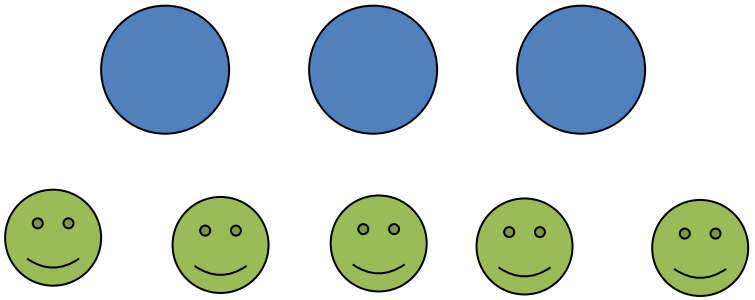
"Campers per Pizza" Approach

Bear Campers



$1\frac{1}{2}$ campers per pizza

Raccoon Campers

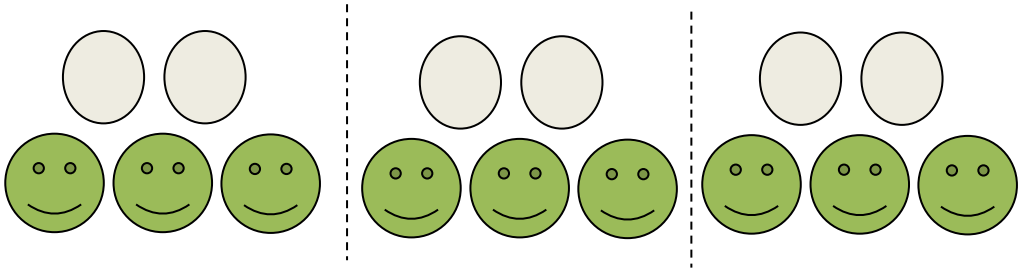


$1\frac{2}{3}$ campers per pizza

Unit Rate

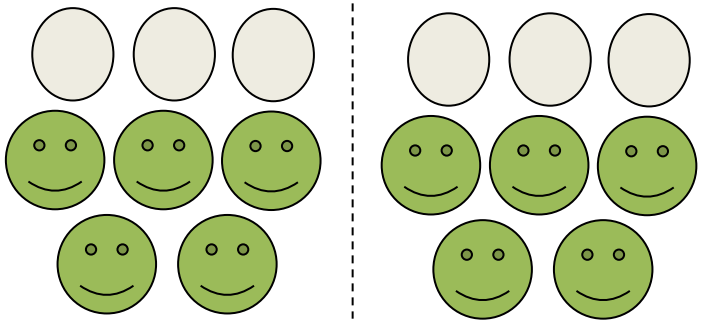
Compare equivalent number of pizzas to number of campers

Bears



6 pizzas for 9 campers

Raccoons

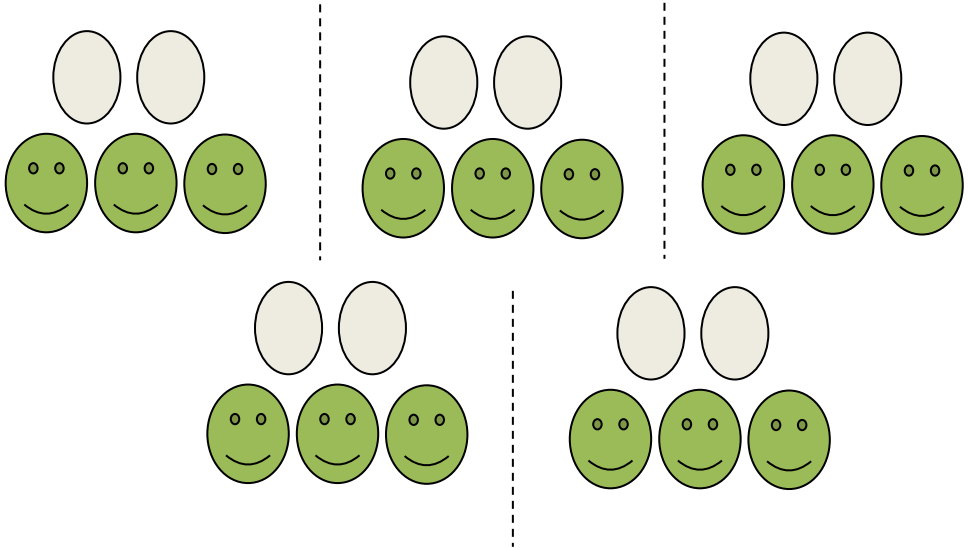


6 pizzas for 10 campers

Unit Rate

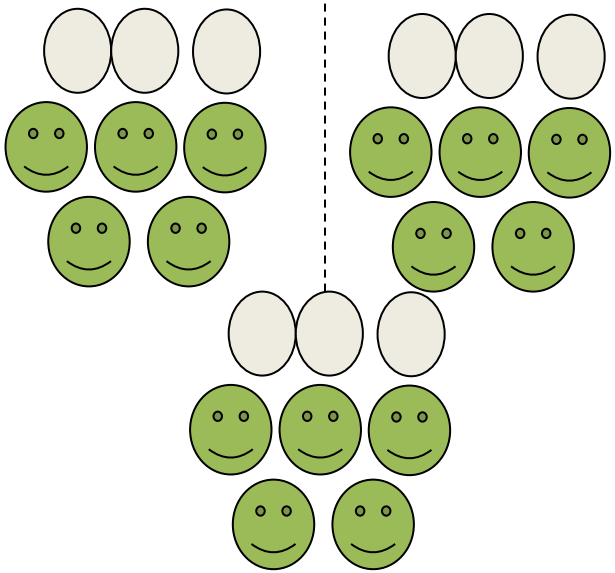
Compare equivalent number of campers to number of pizzas

Bears



15 campers for 10 pizzas

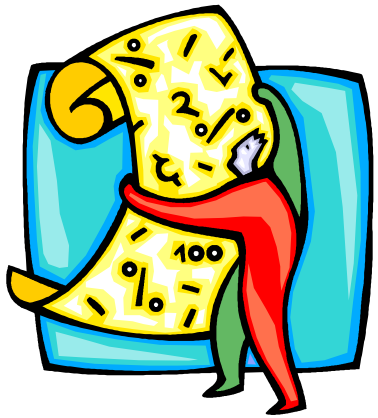
Raccoons



15 campers for 9 pizzas

Comparing Ratios

Using multiplicative comparisons is a powerful proportional reasoning strategy which is an important element in algebra.



Multiplicative Comparisons

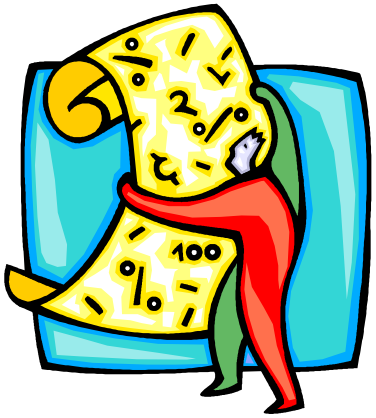
Bear Camp	
# of Campers	# of Pizzas
3	2
1	$\frac{2}{3}$
$\frac{3}{2}$	1

Raccoon Camp	
# of Campers	# of Pizzas
5	3
1	$\frac{3}{5}$
$\frac{5}{3}$	1

Comparing Ratios

Ratio table

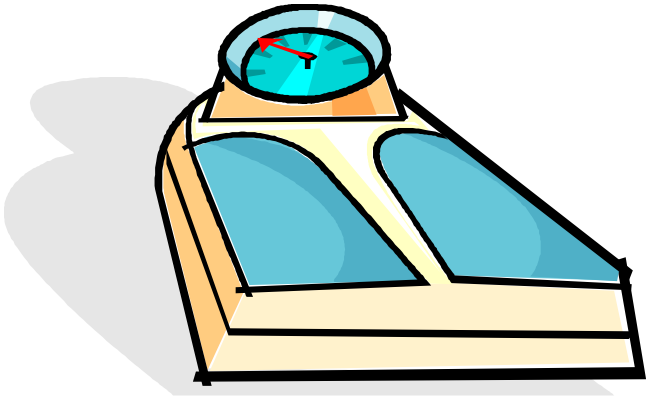
Relationship of two variable quantities



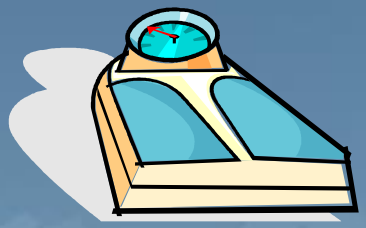
Ratio Table

A person who weighs 160 pounds on Earth will weigh 416 pounds on the planet Jupiter.

How much will a person weigh on Jupiter who weighs 120 pounds on earth?



Ratio Table

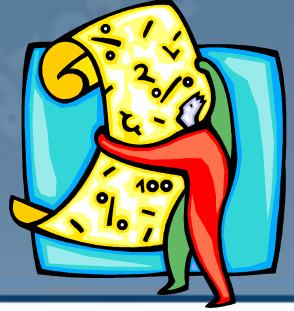


÷ 2 ÷ 2 X 3

Earth weight	160	80	40	120
Jupiter weight	416	208	104	312

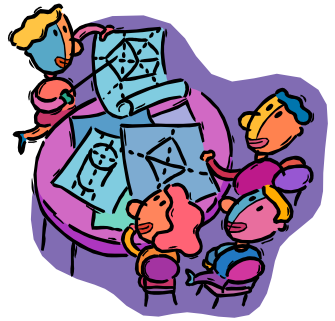
Add

Earth weight	160	80	40	120
Jupiter weight	416	208	104	312



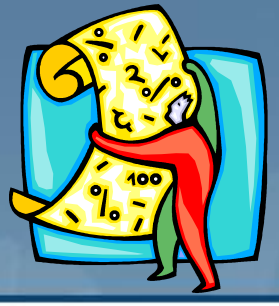
Ratio Table

- At your table, use a ratio table to solve your assigned problem.
  - Find a variety of ways to use the ratio table with the problem.
  - What success/challenges might students encounter using the ratio table?



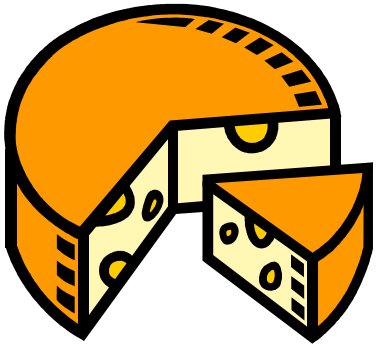
- Share your problem with large group.

Ratio Table



Cheese is $4.25 per pound.

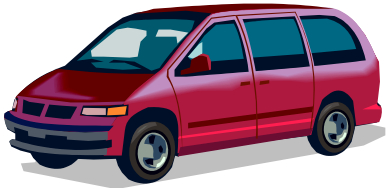
How much will 12.13 pounds cost?



	Lbs	Cost	Notes
A	1	4.25	Given
B	10	42.50	A x 10
C	2	8.50	A x 2
D	0.1	0.425	A ÷ 10
E	12.1	51.125	B + C +D
F	0.01	0.0425	D ÷ 10
G	0.03	0.1275	F x 3
H	12.13	51.5525	E + G

Compare Ratios

Use a tape diagram to solve the problem:

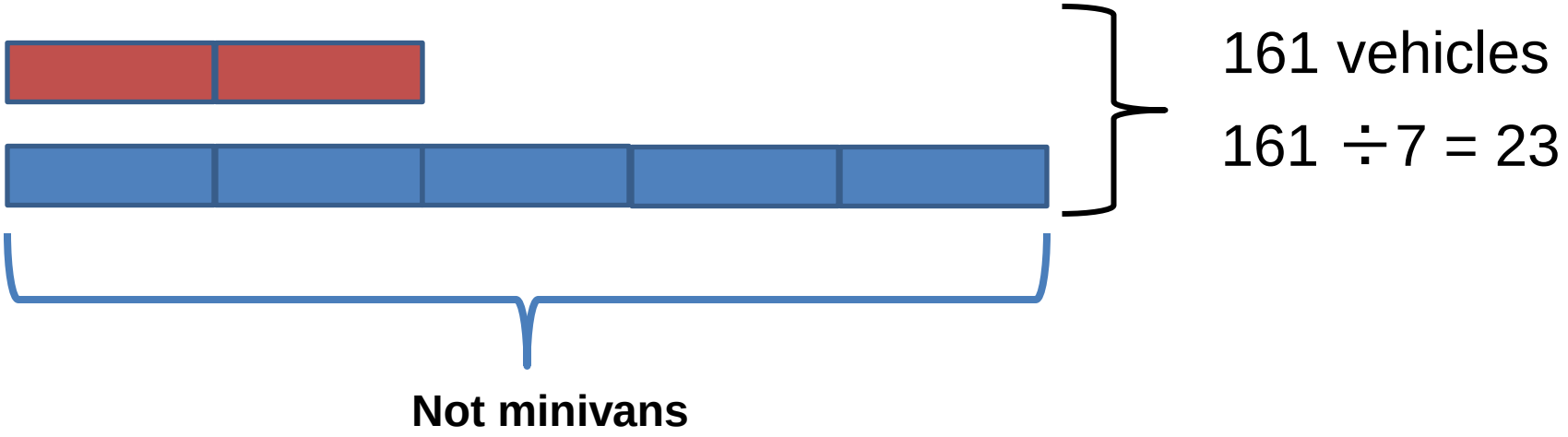

The school parking lot holds 161 vehicles.

When Carla looked at the filled parking lot at school, she noticed there were 2 minivans for 5 other types of vehicles.

How many of the vehicles are not minivans?

Tape Diagram

161 vehicles in ratio of 2:5

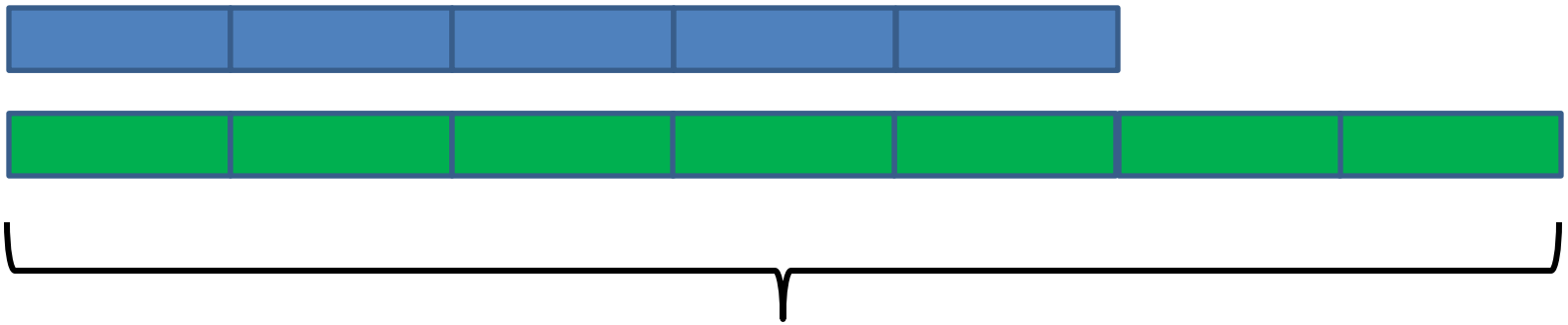


Vehicles that are not minivans

23 X 5 = 115

Tape Diagram

161 vehicles in ratio of 5:7



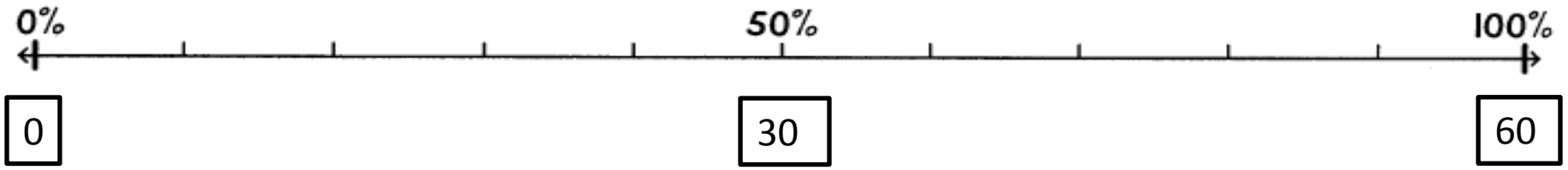
161 total vehicles

$161 \div 7 = 23$

Vehicles that are not minivans

23 X 5 = 115

Solve Percent Problems Using Double Number Line



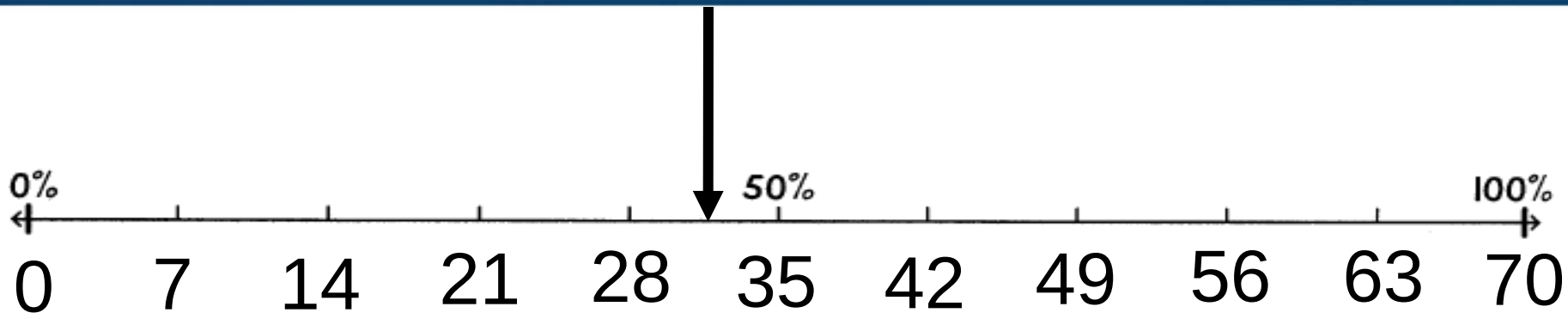
Explore percent problems

(1) 45% of 70 = _____

(2) 30% of _____ = 75

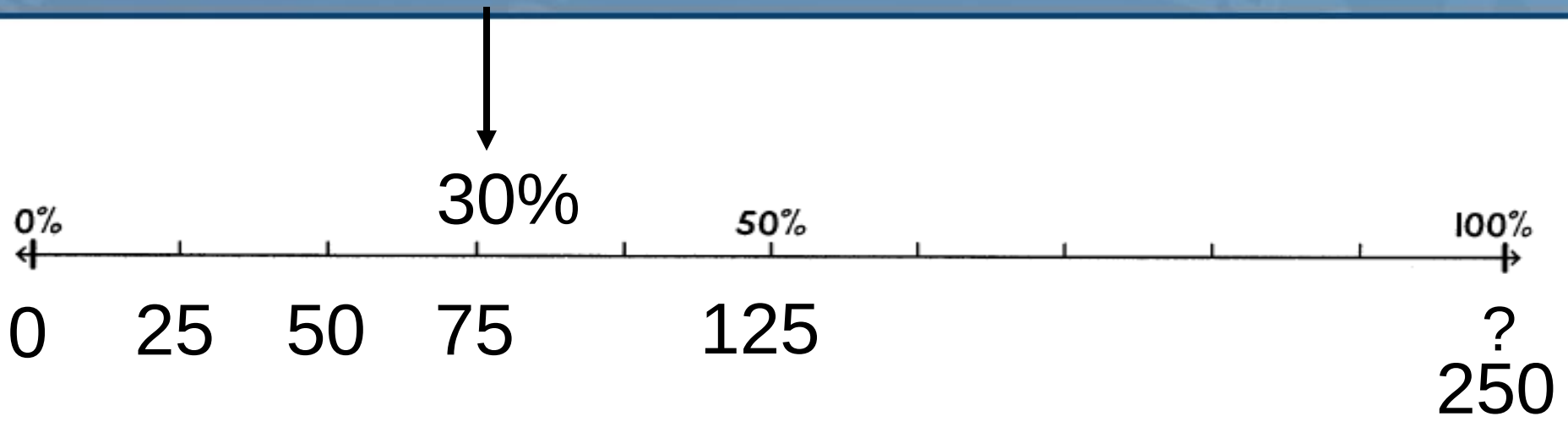
(3) _____% of 75 = 30

Solve Percent Problems Using Double Number Line



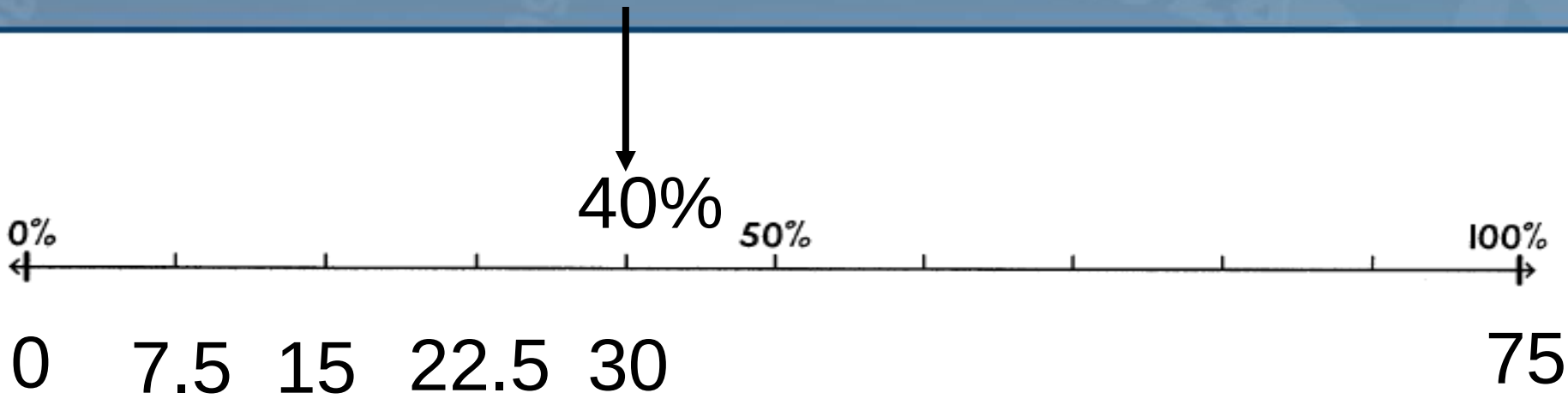
(1) 45% of 70 = 31.5

Solve Percent Problems Using Double Number Line



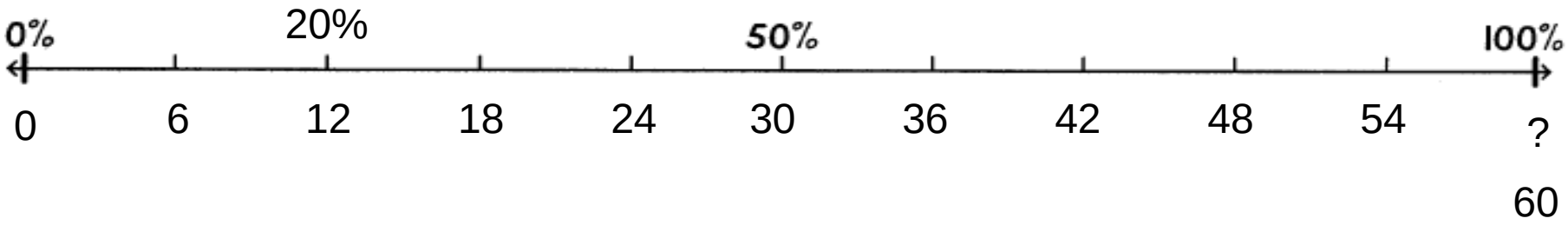
(2) 30% of **250** = 75

Solve Percent Problems Using Double Number Line



(3) 40 % of 75 = 30

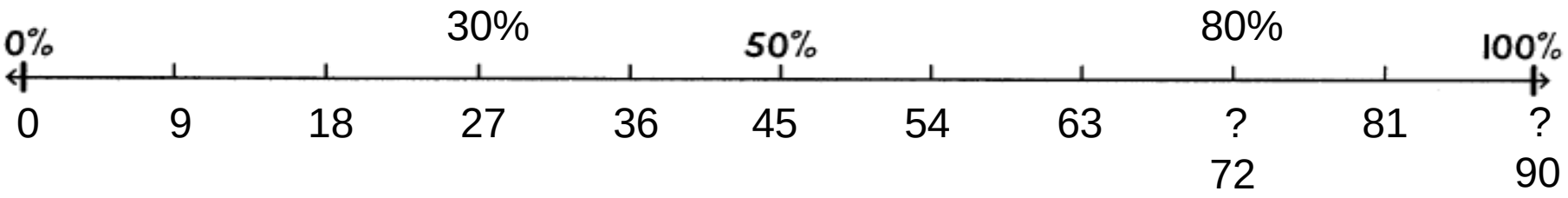
Double Number Line for Percents



20% of Ms. Thompson's show dogs are Labradors. She has 12 Labradors.

How many show dogs does she have?

Double Number Line for Percents



Jan has completed 27 items which is 30% of the test. She needs to complete 80% to move on.

- How many items does she need to complete to move on?
- How many items are there on the test?

Multiple Representations

A car magazine is writing a story about four different cars, reporting the number of miles driven for different amounts of gas.

- With your **Expert group**, describe the gas mileage for your assigned car using multiple representations (words, table, equation, and graph).




Multiple Representations

- Each **Home group** will have one member from the **Expert group** to discuss their assigned car.
- Use the various representations to decide:
  - The ordering of the cars from greatest number of miles per gallon to least number of miles per gallon.
  - The car Krystal likely bought if she drove 924 miles and used 28 gallons of gas.

Summary

There are several ways the same collection of equivalent ratios can be represented. These include:

- unit ratio,
- ratio tables,
- tape diagrams, and
- double number lines.

Reflection

Use and connect mathematical representations Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

(*Principles to Actions: Ensuring Mathematical Success for All* [NCTM 2014], p. 29)

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Exit Ticket

Describe how the various representations might contribute to the learning of ratios by students.

- Unit ratio
- Ratio table
- Double number line diagram
- Tape diagram

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