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mathematics

Algebra

1

S-**P**attern Task

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Please note: The lesson is not meant to be a script to follow, but rather a set of questions that target specific mathematical ideas which teachers can discuss together in professional learning communities.

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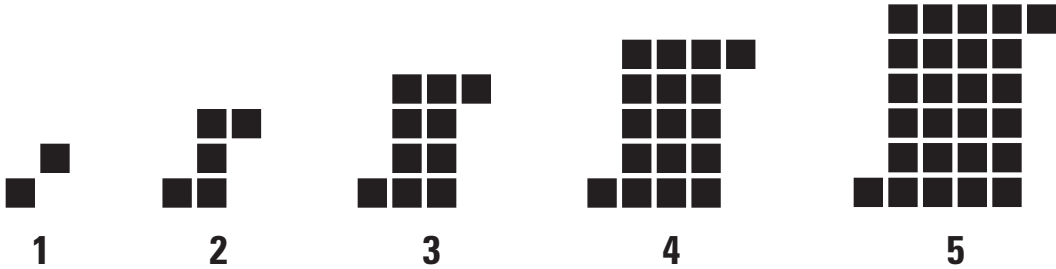
S-Pattern Task

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TASK

Name _____ Date _____

S-Pattern Task¹



1. What patterns do you notice in the set of figures?

2. Sketch the next two figures in the sequence.

¹ Adapted from Foreman, L.C. & Bennett, A.B., Jr. (1995). *Visual Mathematics: Course II, lessons 1-10*. Salem, OR: Math Learning Center.

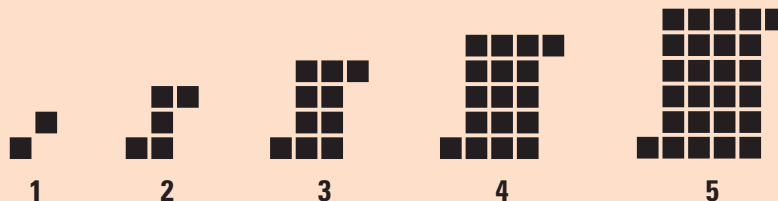
**TASK**

3. Describe a figure in the sequence that is larger than the 20th figure without drawing it.
4. Determine an equation for the total number of tiles in *any* figure in the sequence. Explain your equation and show how it relates to the visual diagram of the figures.
5. If you knew that a figure had 9802 tiles in it, how could you determine the figure number? Explain.
6. Is there a linear relationship between the figure number and the total number of tiles? Why or why not?

S-Pattern Task

Rationale for Lesson: In this lesson students write the general expression for a pattern that is growing quadratically. This requires students to look for the underlying mathematical structure of the pattern and use this structure to represent the general case for any step in the pattern. There are several different equivalent expressions that represent the general case, and students need to relate each to the physical arrangement of tiles and to each other.

Task: S-Pattern Task



1. What patterns do you notice in the set of figures?
2. Sketch the next two figures in the sequence.
3. Describe a figure in the sequence that is larger than the 20th figure without drawing it.
4. Determine an equation for the total number of tiles in *any* figure in the sequence. Explain your equation and show how it relates to the visual diagram of the figures.
5. If you knew that a figure had 9802 tiles in it, how could you determine the figure number? Explain.
6. Is there a linear relationship between the figure number and the total number of tiles? Why or why not?

Common Core State Standards for Mathematical Practice²

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.

| | | |
|--|--|---|
| <p>Common Core State Standards for Mathematical Content²</p> | <p>HSA.SSE.A.1</p> <p>HSA.SSE.A.1.A</p> <p>HSA.SSE.B.3</p> <p>HSA.CED.A.2</p> | <p>Interpret expressions that represent a quantity in terms of its context.</p> <p>Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> |
| <p>Essential Understandings</p> | <ul style="list-style-type: none"> • Expressions can be used to model quadratic relationships that occur in real-world or mathematical contexts. The term(s) in an expression that models a quadratic relationship can be related directly to the situation that is being modeled, whether the situation is real-world, geometric, or mathematical. • Two or more expressions are equivalent if each can be transformed into the others through a series of successive uses of the distributive property of multiplication over addition and/or combining (collecting) like terms. • While the distributive property of multiplication over addition and/or combining (collecting) like terms changes the looks of an expression, it does not change its value. Therefore, applying the distributive property and/or collecting like terms to one expression in an equation does not affect the solution set of the equation. | |
| <p>Materials Needed</p> | <ul style="list-style-type: none"> • Student reproducible task sheet • Scientific or graphing calculators should be made available, but students should be allowed to decide whether or not they need to use this tool. • Two colors of square tiles, approximately 30 per color per group • You may want to have students put their solution methods on posters in order to display them side-by-side for comparison. If so, you will need poster paper, markers, and tape. | |

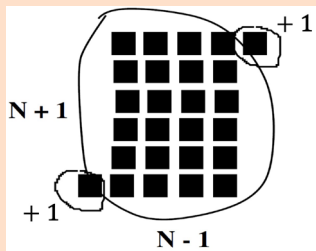
² National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). (2014). Mathematics. *Common core state standards for mathematics*. Retrieved from <http://www.corestandards.org/Math>

SET-UP PHASE

Distribute the task and display the diagram of the five figures in the pattern in the front of the classroom. Have students independently read through the task, and then read through the task with students. Have several students explain to the class what they are trying to find when solving the problem in the task. Stress to students that they will be expected to explain how and why their expressions model the pattern by referring and making connections to the figures in the diagram of the pattern. Also, stress that they are to model the situation with at least two different expressions and then show that the expressions are equivalent.

NOTE: Make about 50 square tiles available to each group so that they have the option of physically manipulating the figures. Also, consider having each group put their solutions methods on posters in order to display them side-by-side for comparison as you discuss the various equivalent expressions in the Share, Discuss, and Analyze phase.

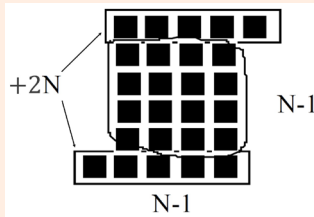
EXPLORE PHASE

| Possible Student Pathways | Assessing Questions | Advancing Questions |
|--|---|---|
| <p>Group can't get started.</p> | <p>What do you notice about the figures in the pattern?</p> <p>How many tiles are in the first figure? The second figure? The fifth figure?</p> | <p>How are the figures growing? What is changing and what is remaining the same?</p> <p>Have students use the square tiles and ask them to construct the 6th figure. Then ask:</p> <ul style="list-style-type: none"> • How did you know how to construct the figure? • How would you construct the 7th figure? |
| <p>In the diagram or with manipulatives, sees a rectangle with the dimensions $(n + 1) \times (n - 1)$ that always has 2 additional tiles at the opposite corners. Writes the expression:</p> $(n + 1)(n - 1) + 2$  | <p>How did you get this expression?</p> <p>Explain what each term of your expression represents in the drawing (model).</p> | <p>Can you simplify your expression? Can you relate your simplified expression to the picture?</p> |

Possible Student Pathways Assessing Questions Advancing Questions

In the diagram or with manipulatives, sees each figure as a square with dimensions $(n - 1) \times (n - 1)$ with rows of tiles size n both above and below the square. Writes the expression:

$$(n - 1)(n - 1) + 2n$$



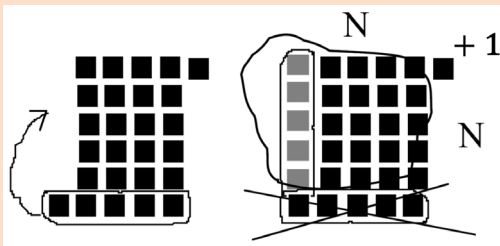
How did you get this expression?

Explain what each term of your expression represents in the drawing (model).

Can you simplify your expression? Can you relate your simplified expression to the picture?

In the diagram or with manipulatives, moves a row to make the row a column to form an $n \times n$ square plus 1 tile. Writes the expression:

$$n^2 + 1 \text{ or } (n)(n) + 1$$



How did you get this expression?

Explain what each term of your expression represents in the drawing (model).

You made a square by moving the tiles around. Can you see any other way to relate the shape to the figure number without moving tiles? Are your expressions equivalent?

Ask if group cannot write another expression: Another group wrote $(n + 1)(n - 1) + 2$ to model the pattern. Simplify their expression. I'll be back to see what you find.

Creates a table by counting tiles. Rules out a linear relationship because there is not a constant rate of change. Finds a numeric pattern, but has trouble translating the pattern into an algebraic expression that models the pattern of growth of the figures.

| Figure # | # of Tiles | Pattern |
|----------|------------|---------|
| 1 | 2 | 1 + 1 |
| 2 | 5 | 4 + 1 |
| 3 | 10 | 9 + 1 |
| 4 | 17 | 16 + 1 |
| 5 | 26 | 25 + 1 |

How did you get the values in your table?

Explain the pattern that you found in your table.

How can you represent the pattern you found algebraically?

SHARE, DISCUSS, AND ANALYZE PHASE

EU: Expressions can be used to model quadratic relationships that occur in real-world or mathematical contexts. The term(s) in an expression that models a quadratic relationship can be related directly to the situation that is being modeled, whether the situation is real-world, geometric, or mathematical.

$$(n + 1)(n - 1) + 2$$

- I saw groups use a lot of different strategies to investigate this problem. Let's look at your work together and make some comparisons. When I call on your group, please post your work in the front of the room.
- Let's start with Group 1. Explain your solution. *[We got $(n + 1)(n - 1) + 2$. We checked it and it gave all the right number of tiles for each figure.]* What do you mean it gave all the right number of tiles for each figure? *(For Figure 1, the expression gave us 2 and there are 2 square tiles in Figure 1. For Figure 2, the expression gave us 5 and there are 5 square tiles in Figure 2. This works the same for each figure.)* Can someone else explain how Group 1 tested their expression? So, the expression that you wrote to model the pattern of growth of the figures can tell us how many square tiles are in Figures 1 through 5. We just substitute the figure number in for n and then evaluate the expression to get the number of tiles in the figure. **(Revoicing)**
- Will the expression tell us how many square tiles are in Figure 100? **(Challenging)** *(Yes, the expression should tell us the number of tiles for any figure number.)* Let's take a closer look at why this may be true. Group 1, explain how your expression represents the figures. Please point to the figure when explaining how your expression represents it. *[Take a look at what we did here to Figure 5. We saw a rectangle here with the one side always equal to the figure number + 1 and the other side always equal to the figure number - 1, plus there's always these 2 additional tiles in the corners. So, we wrote the expression $(n + 1)(n - 1) + 2$. n is the figure number.]* What does $(n + 1)(n - 1)$ represent? Can someone else explain how Group 1's expression represents the figures?
- So, does each term in the expression represent something in the figure? **(Challenging)** *(Yes, in Group 1's expression $(n + 1)$ represents how many squares tall that rectangle is and $(n - 1)$ represents how many squares wide that rectangle is. That's why $(n + 1)(n - 1)$ represents the total number of tiles in the rectangle. And, the plus two is because there are always 2 extra squares, one in each corner.)* **So, I'm hearing that expressions can be used to model or represent relationships that occur in geometric contexts, and that the terms in an expression each represent a quantity that occurs in the context. (Recapping the EU)** Can someone say in their own words what I just said using Group 1's expression and their Figure 4?

$$(n - 1)(n - 1) + 2n$$

- Now, let's look at Group 2's work. Group 2, please display your work and explain to us how your expression models the figures. *[We saw that each of the figures has a square with each side equal to $(n - 1)$ and that there is always rows of tiles size n both above and below the square. So, we wrote the expression $(n - 1)(n - 1) + 2n$.]*
- So, does each term in Group 2's expression represent something in the figure? **(Challenging)** *(Yes, in Group 2's expression $(n - 1)(n - 1)$ represents how many tiles are in the square they circled in the middle of each figure, and the $+ 2n$ represents the two rows that are n tiles long that are always above and below that square.)* **So, again I'm hearing that expressions can be used to represent relationships that occur in geometric contexts, and that the terms in an expression each represent a quantity that occurs in the context. (Recapping the EU)**

EU: Two or more expressions are equivalent if each can be transformed into the others through a series of successive uses of the distributive property of multiplication over addition and/or combining (collecting) like terms.

$$n^2 + 1$$

- So, now we've seen 2 different expressions that we feel both correctly model the pattern of growth of the figures. Let's take a look at one more, and I want you to think about what it means that more than one expression correctly models the pattern of growth of the figures.
- Group 3, explain your solution. *(We got $n^2 + 1$. We checked it and it also gave all the right number of tiles for each figure just like how Group 1's expression did.)* Remind us again what you mean it gave all the right number of tiles for each figure?
- Explain to us how your expression models Figure 5. *(We moved the bottom row of tiles over here to become the left side of the figure. This gave us a 5 by 5 square, plus this one square tile here in the upper right corner. This worked for each of the figures. We also noticed that the dimensions of the square is always the figure number by the figure number. So, we wrote the expression $n \times n + 1$, where n is the number of tiles in the side of the square and 1 is the extra square tile that is always up here. Then, since $n \times n$ is n^2 , we just used $n^2 + 1$.)* Do we agree or disagree that Group 3's expression correctly represents the pattern of growth of the figures?
- So, now we have 3 different expressions that we've said correctly represent the pattern of growth of the figures. Are these 3 expressions equivalent? Take a few minutes to discuss this with your group members. Please share what your group discussed. *(We said all 3 are equivalent because they all have terms that correctly represent the figures and they all give the correct number of tiles for any figure number.)* Can someone explain what she just said? So, I'm hearing that when expressions are equivalent, the same input will always give the same output for any of the equivalent expressions. **(Revoicing and Marking)** Can someone explain what I just said using the context of the problem?
- Group 6, show us what you did to prove to yourselves that two of the expressions are equivalent. *(We simplified $(n + 1)(n - 1) + 2$ and got $n^2 + 1$, so we said the expressions are equivalent.)* Please display your work and explain each step of how you simplified $(n + 1)(n - 1) + 2$. *[First we used the distributive property on $(n + 1)(n - 1)$.] What do we call $(n + 1)$ and $(n - 1)$? *(These are factors.)* Yes, please continue and use 'factor' in your explanation. *(We distributed the n in the first factor like this to the n and the $- 1$ in the second factor. Then we distributed the $+ 1$ the same way to the n and the $- 1$. So, then we got to here with $n^2 - 1n + 1n - 1 + 2$.)* Hold on for a moment, can someone else explain what Group 6 did to get from $(n + 1)(n - 1) + 2$ to $n^2 - 1n + 1n - 1 + 2$? So, I'm hearing that when we have an expression in which factors are multiplied, like Group 1's $(n + 1)(n - 1)$, we can use the distributive property to simplify expressions. **(Revoicing and Marking)***
- As I was talking to groups during the Explore phase, some of you were saying that you FOILed factors like $(n + 1)(n - 1)$. Can someone explain why the FOIL method works here? How is the FOIL method connected to the distributive property? The FOIL method is a way of remembering to multiply each term in each factor to each of the terms in a second factor when simplifying two binomials by multiplying them like Group 6 did with $(n + 1)(n - 1)$. Can someone explain what I mean by binomial? Sometimes you will simplify by multiplying factors that are not binomials, in other words one or both factors might have more or less than two terms. So, FOIL will not accurately describe the process. However, remember that what you're really doing when you simplify factors by multiplying them is the distributive property where you multiply each term in each factor to each of the terms in a second factor just like Group 6 did here. **(Marking)** Can someone explain what I just said?

LESSON GUIDE

- Now, let's get back to Group 6's process of simplifying $(n + 1)(n - 1) + 2$ to see if it's the same as $n^2 + 1$. Please continue. *(We combined all of the like terms in $n^2 - 1n + 1n - 1 + 2$ and got $n^2 + 1$.)* Say more about what you mean by "combined all of the like terms." *(Well, we only had one n^2 term, so there was nothing to do there. There were these two n terms, and they canceled each other out.)* Can someone else explain what Group 6 means that the n terms canceled each other out? Can someone else explain why Group 6 didn't combine the n^2 and n terms?
- So, I'm hearing that another way to simplify an expression is to combine or collect like terms and that the sign in front of a term matters because the sign tells you if the term is positive or negative. **(Revoicing and Marking)** Can someone explain again how Group 6 simplified by combining like terms.
- So, we've seen that Group 1's $(n + 1)(n - 1) + 2$ can be simplified to $n^2 + 1$. What does this mean? *(The expressions both represent the figures and they're equivalent.)* Say more about how we were able to show that the expressions are equivalent? *(Well, since we could simplify $(n + 1)(n - 1) + 2$ using the distributive property and then collecting like terms to get to $n^2 + 1$, we showed that the expressions are equivalent.)* Can someone say what she just said in their own words.
- **So, I'm hearing that expressions are equivalent if each can be transformed into the other through a series of uses of the distributive property and/or combining like terms. (Recapping the EU)**
- Earlier, we also said that when expressions are equivalent, the same input will always give the same output for any of the equivalent expressions. These are two very important methods for testing whether two expressions are in fact equivalent. **(Marking)** Are two expressions also equivalent when the expressions represent the same phenomenon? So, all the expressions represent the same figures. Does this count too? **(Challenging)**
- Now, who can explain to us how they determined that $(n - 1)(n - 1) + 2n$ is equivalent to $n^2 + 1$?
- What if we only wanted to determine if $(n + 1)(n - 1) + 2$ and $(n - 1)(n - 1) + 2n$ are equivalent? What could we do?

EU: Expressions can be used to model quadratic relationships that occur in real-world or mathematical contexts. The term(s) in an expression that models a quadratic relationship can be related directly to the situation that is being modeled, whether the situation is real-world, geometric, or mathematical.

EU: While the distributive property of multiplication over addition and/or combining (collecting) like terms changes the looks of an expression, it does not change its value. Therefore, applying the distributive property and/or collecting like terms to one expression in an equation does not affect the solution set of the equation.

Creates a table by counting tiles

- Now, let's take a look at Group 4's work. Display your work and explain to us what you did? *(We made a table. We knew it wasn't a linear relationship because there isn't a constant rate of change. Then we added a third column to show a pattern that we were seeing.)*
- Let's stop here and think about what Group 4 just said about this relationship not being linear. What can we say about the type of relationship between the figure number and the number of square tiles? *(It's quadratic.)* Say more about that, why is it quadratic and not linear? *(The number of tiles does not go up by a constant rate from figure to figure, so it's not linear.)* Can you show us that you mean? *(Figure 1 has 2 tiles, Figure 2 grows to 5 tiles, then Figure 3 grows to 10 tiles. The rate of growth in tiles is not constant, so it's not linear.)* How do we all feel about what she just said?

- Say more about how you know it's quadratic based on Group 3's expression? ($n^2 + 1$ is quadratic because of the n^2 .) So, I'm hearing that algebraically we can tell that the relationship grows quadratically because of the n^2 term. **(Marking and Revoicing)** Can someone show us what's happening on the figures that's causing the figures to grow quadratically? **(Challenging)** (From one figure to the next, the figures get wider and taller. So, the figures are growing in two ways.) Can someone say more about how these expressions represent figures that are growing in two ways? [Group 3's $n^2 + 1$ represents a square with both dimensions (n) and (n) growing.] So, do all of the expressions we discussed represent figures that are growing in two dimensions?
- **In past lessons and problems, we have represented lots of different linear relationships with expressions. Today I've heard that expressions can also be used to represent quadratic relationships, again, as long as the terms in an expression each represent a quantity that occurs in the context of the situation. (Recapping the EU)**
- Now, take a minute with your group and discuss the pattern that Group 4 has in that last column.
- Tell us what your group discussed. (The pattern shows the figure number squared + 1. We think that is the same as Group 3's expression $n^2 + 1$.) What does everyone else think about what he just said?
- So, we have three expressions that we've said represent the number of tiles for any given figure number, but all of the expressions look different. Does that mean the expressions might give different value for the same figure number? Write down your thoughts for a minute, then I'll have you discuss this with your group members.
- So, what did you discuss in your groups? (All of the expressions will give the same correct value of the number of tiles for any figure number because they are all equivalent to $n^2 + 1$.) So, are you saying that Group 1's $(n + 1)(n - 1) + 2$ will give the same value for the number of tiles for a given figure number before and after we simplify it to $n^2 + 1$? **(Challenging and Revoicing)** (Yes, because the expressions are equivalent. All we did was use the distributive property and collect like terms to get from the original expression to the simplified expression.) Do we all agree with what she just said?
- **So, I'm hearing that when we simplify an expression using the distributive property and combining like terms we do change how the expression looks, but that this process of simplifying the expression does not change the expression's value.** Can someone give us an example of what I just said using one of the expressions we discussed today?



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