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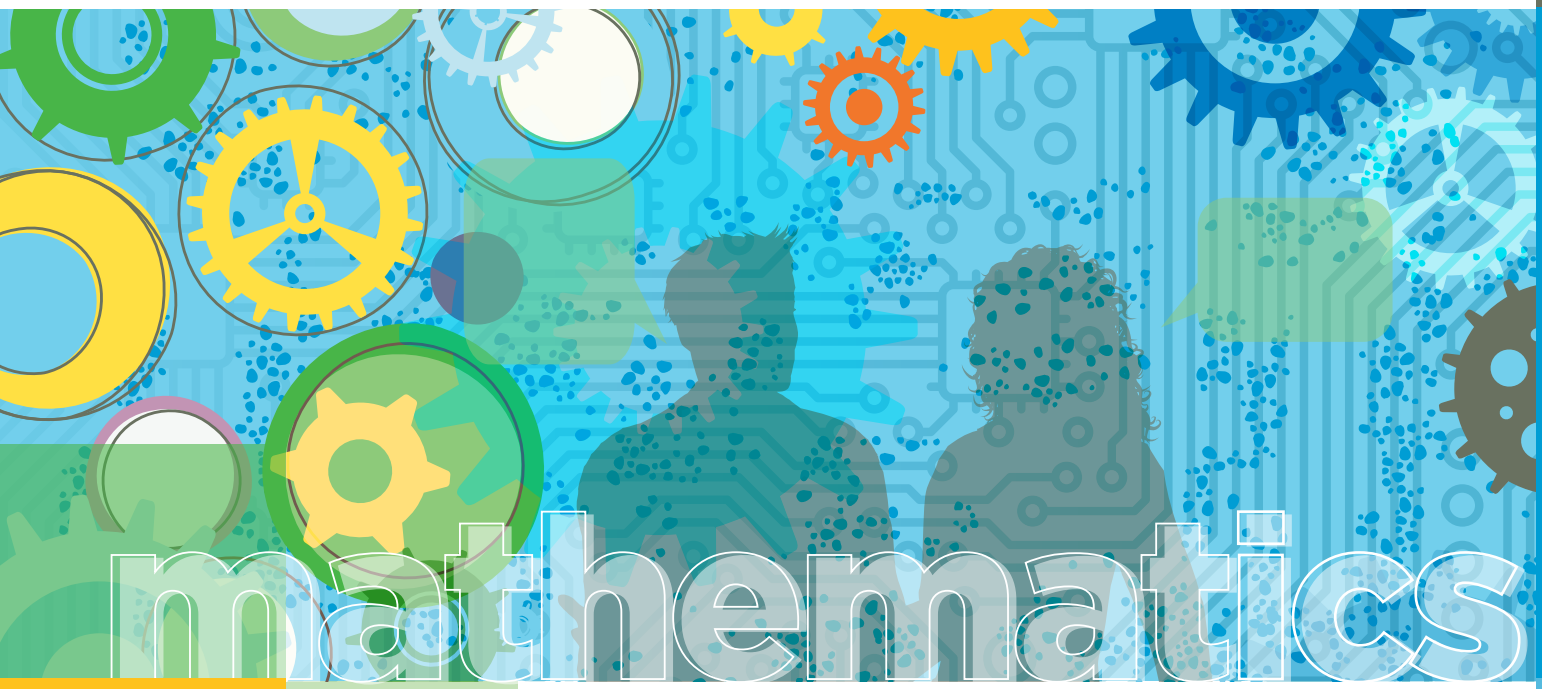

mathematics

Grade 8

Counting Cubes Task

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This is a single lesson. Full sets of related lessons, including accompanying student materials, that are developed by the Institute for Learning are available at:

ShopIFL.com

Please note: The lesson is not meant to be a script to follow, but rather a set of questions that target specific mathematical ideas which teachers can discuss together in professional learning communities.

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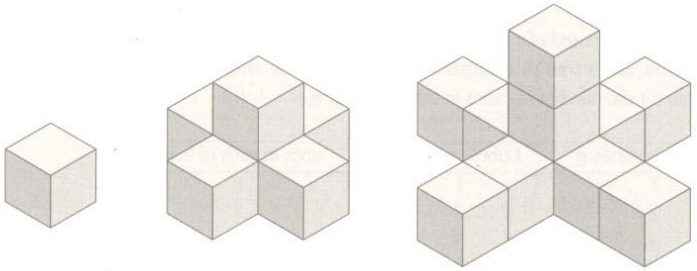
TASK

Name_____Date_____

Counting Cubes



Building 1 **Building 2** **Building 3**

1. Describe a pattern you see in the cube buildings.

2. Use your pattern to write an expression for the number of cubes in the n^{th} building.

Adapted from "Counting Cubes", Lappan, Fey, Fitzgerald, Friel, & Phillips (2004). Connected Mathematics™, Say it with symbols: Algebraic reasoning [Teacher's Edition]. Glenview, IL: Pearson Prentice Hall. © Michigan State University

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TASK

3. Use your expression to find the number of cubes in the 5^{th} building.
   Check your results by constructing the 5^{th} building and counting the cubes.

4. Look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the n^{th} building.

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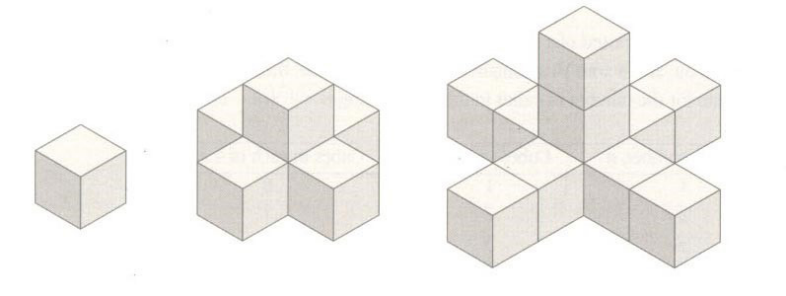
LESSON GUIDE

Counting Cubes Task

Rationale for Lesson: The Counting Cubes task provides an opportunity for students to explore, extend, and describe a visual, linear growth pattern. Variables are used to represent quantities that change, which allows students to represent the pattern algebraically. Students first describe a pattern that they see in a sequence of three cube buildings and figure out a way to represent that pattern as an algebraic expression that can be used to find the number of cubes in any building. After checking to see if their expression works by building the fifth building in the pattern, they then try to identify another way to represent the pattern, and thus generate a different expression to represent this growth pattern. Several different expressions can be generated, providing an opportunity for students to discuss how each of these expressions represents the pattern, and to justify why these expressions are equivalent. Students can then use algebraic methods to prove that they are equivalent. The task also provides an opportunity for students to extend what they know about expressions to begin to explore functions and how to write equations that represent function rules.

Task: Counting Cubes Task



Building 1 **Building 2** **Building 3**

1. Describe a pattern you see in the cube buildings.
2. Use your pattern to write an expression for the number of cubes in the n^{th} building.
3. Use your expression to find the number of cubes in the 5^{th} building.
   Check your results by constructing the 5^{th} building and counting the cubes.
4. Look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the n^{th} building.

Adapted from "Counting Cubes", Lappan, Fey, Fitzgerald, Friel, & Phillips (2004). Connected Mathematics™, Say it with symbols: Algebraic reasoning [Teacher's Edition]. Glenview, IL: Pearson Prentice Hall. © Michigan State University

Common Core State Standards for Mathematical Practice[2]	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP7 Look for and make use of structure.

2 National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). (2014). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math

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Common Core State Standards for Mathematical Content[2]	8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
Essential Understandings	• Expressions can be used to model linear relationships that occur in real-world or mathematical contexts. The term(s) in an expression that models a linear relationship can be related directly to the situation that is being modeled, whether the situation is real-world, geometric, or mathematical. • Two or more expressions are equivalent if each can be transformed into the other(s) through a series of successive uses of the distributive property of multiplication over addition and/or combining (collecting) like terms. • A function is a rule that assigns to each input exactly one output for a real-world, geometric, or mathematical situation.	
Materials Needed	• Student reproducible task sheet • Three to four copies of the set of buildings, either on chart paper, transparencies, or on a piece of paper, that students can use to explain their solutions at the front of the class • Bags of 25 small cubes for each student or pair of students • Scientific or graphing calculators should be made available (Students should be allowed to decide whether or not they need to use this tool.) • Document camera or overhead projector	

1 Function notation is not required in Grade 8.

2 National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). (2014). Mathematics. *Common core state standards for mathematics.* Retrieved from http://www.corestandards.org/Math

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SET-UP PHASE

Distribute the task and display the diagram of Buildings 1, 2, and 3 in the front of the classroom. Have students independently read through the task, and then ask someone to read it aloud. Make sure students understand the terminology in the task. Use the following prompts:

- Who can remind us what we mean by a "pattern"?
- Who can explain what the task means when it asks you to "write an expression for the number of cubes in the n^{th} building"?

Stress to students that they will need to explain how their expressions represent the pattern that they see in the cube buildings.

Have students work on the task individually for five minutes before working with a partner or small group. Provide bags with 25 cubes for each pair/small group so that they can build the first few buildings.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Has trouble getting started.	What are you trying to find? What do you notice about the buildings? How many cubes are in each of the buildings?	How are the buildings growing? What is changing and what is remaining the same? Use the cubes to build the first four buildings and discuss what you need to do to build each of them. Try to explain how the buildings grow. I'll be back to hear what you come up with.
Creates a table. Determines the number of cubes in the first three buildings (1, 6, 11) and identifies the recursive pattern "add 5", but does not see a pattern based on the building number. ***Building #*** / ***# of Cubes*** 1 / 1 2 / 6 3 / 11	How did you get the values in your table? Explain the pattern that you found in your table.	You said that the pattern is "add 5". How could you figure out the number of cubes in the 5^{th} building if you didn't know the number of the cubes in the 4^{th} building? Can you see a relationship between a building number and the number of cubes?

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Possible Student Pathways	Assessing Questions	Advancing Questions
Defines *n* as the length of each arm and creates the expression $5n + 1$, where 1 represents the hidden cube in the middle.	How did you get this expression? Explain what each term of your expression represents in terms of the buildings.	How does the length of an arm relate to the building number?
Finds a pattern and creates the expression $5(n - 1) + 1$ by relating the number of cubes in each of the five visible arms, $(n - 1)$, to the building number, *n*, and then adds 1 for the center cube.	How did you get this expression? Explain what each term of your expression represents in the buildings.	Can you simplify your expressions? Can you relate your simplified expression to the buildings?
Finds a pattern and creates the expression $4(n - 1) + n$ where $4(n - 1)$ represents the number of cubes in the four visible arms and n is the number of blocks in the center tower.	How did you get this expression? Explain what each term of your expression represents in the buildings.	Can you simplify your expressions? Can you relate your simplified expression to the buildings?
Finds a pattern and creates the expression $5n - 4$ that includes the center cube once for each of the five arms, and then compensates for "over-counting" the center cube by subtracting 4.	How did you get this expression? Explain what each term of your expression represents in the buildings.	Without actually making the graph, discuss what a graph of the relationship between building numbers and the number of cubes in a building will look like. I'll be back to hear about what you discuss.

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SHARE, DISCUSS, AND ANALYZE PHASE

EU: Expressions can be used to model linear relationships that occur in real-world or mathematical contexts. The term(s) in an expression that models a linear relationship can be related directly to the situation that is being modeled, whether the situation is real-world, geometric, or mathematical.

- Groups saw several different patterns and created at least four different expressions to represent the number of blocks in the nth building. When you are called to the front, please show your work and be prepared to share your thinking. Explain where you see the pattern, and your expression, in the diagrams of the buildings.
- Let's start with Group 2. What did you find? *(We made a table and then we figured out the number of blocks in each building. We saw that you add five each time.)*
- Show us where the five is added each time. Demonstrate to us using the blocks. *(One is added on top in the middle and one is added on each of the four sides. So that is plus five.)*
- Several other groups also found this pattern. So how can you use this rule to find the number of cubes in the fourth building? The fifth building? What if I ask you for the number of cubes in the 123rd building? How would you use this rule?
- You found a recursive rule that tells you how the number of cubes in each building relates to the number of cubes in the building before it **(Revoicing)**, but you see that it is impossible to use a recursive rule to find the number of cubes in any given building. Using a variable to represent the building number, and creating an expression that uses that variable will make it easier to find the number of cubes in larger buildings. **(Marking)**
- Let's move on to Group 3's solution. Write your expression on the board and explain how you arrived at it. *[We found the expression $5(n-1)+1$. We see that the building has five arms and each arm has one less cubes in it than the building number. So $(n-1)$ represents the number of cubes in each arm.]* Hold on there for a second. Can someone tell us what n represents in their $(n-1)$? *(The building number.)* Ok, continue Group 3. *[Well, there are five arms so we multiplied by five. Our first expression was just $5(n-1)$ but then we saw that it didn't give the right number. It always gave us one less, so we added one to get our expression.]*
- So you added one so that it worked? *(Yes.)* Who can explain what the +1 represents in the building? Why does adding one produce the correct number of cubes? *(There's a cube inside that is hidden.) (It's the one cube in Building 1.) (That one cube is in all of the buildings even though you can't see it so that's what +1 represents.)*
- Will this expression tell us the number of cubes in the 100^{th} building? Who can convince us that it will work? **(Challenging)** *(Like the other group said, each time you go to the next building you add five, or you add 1 cube to each arm each time. So each time the building number goes up one the number of cubes in each arm goes up one so it will keep working.)*
- Who can explain it another way? *[All of the buildings will have one less cube in each arm than the building number–it's like you add one to the building number and one to each arm so there's always one less. So it will still be $5(n-1)$ and you will still have the cube in the center. So it will work.]*
- **So, I'm hearing that expressions can be used to model relationships, and that the terms in an expression relate directly to the situation, in this case a geometric pattern of growth of the buildings. (Recapping the EU)**

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- A few groups created the expression $5n + 1$ instead of $5(n - 1) + 1$. Who can figure out how they might be seeing the pattern? For example, let's look at Building 3. How do you think they used this expression $5n + 1$ to determine the number of cubes in Building 3? *[I think they thought of n as the number of cubes in each arm, which is 2. So 5(2) + 1 = 11. But since this is the third building and n stands for the building number, that's why we came up with the expression 5(n – 1) + 1.]*
- So we see it is really important to define what our variable represents. **(Marking)** If n represented the number of cubes in each arm then $5n + 1$ would work perfectly fine. What is it about the way that the task is worded that lets you say what n stands for? Is there any benefit to defining your variable one way or the other to represent the pattern of growth of these buildings? **(Challenging)**
- There were two other expressions that groups came up with. $5n - 4$ and $4(n - 1) + n$. I'll write them on the board. Select an expression that your group didn't come up with. Talk at your tables. What do the components of each of these expressions represent in the buildings? **(Challenging)**
- (Student name) your group raised an excellent question. Share what you were wondering. *(We see how these expressions work starting with Building 2. The expressions all talk about five arms or four arms and a center tower. But Building 1 doesn't have five arms. So we think that we just have to say that Building 1 has one cube and then we can use the expressions for all of the other buildings.)*
- (Student name) your group talked about the same issue. What was your conclusion? *(We thought the same thing, but then we saw that when you used each of these expressions for Building 1, and put in 1 for n, all of the expressions give you one. So they work.)*
- Who can explain why it makes sense that the expressions will still work? *(We think that n – 1 represents the number of cubes in each arm. So in the first building there are n – 1 or zero cubes in each arm. So it makes sense. Five times zero equals zero.)*
- **So, I'm hearing that when you write an expression to model a situation, the expression will always give the correct output for any given input. (Revoicing and Marking)** Who can explain in their own words what I just said using the context of the problem? OK, let's hold onto this idea of inputs and outputs for a moment and we'll come back to it later.

EU: Two or more expressions are equivalent if each can be transformed into the other(s) through a series of successive uses of the distributive property of multiplication over addition and/or combining (collecting) like terms.

- We've come up with three different expressions in which n was defined as the building number: $5(n - 1) + 1$, $4(n - 1) + n$ and $5n - 4$. They all look very different. Are these three expressions equivalent? If so, how do you know? Take a few minutes to talk about this at your tables.
- Group 4, share what your group came up with. *(We used the three expressions to find the number of cubes in Buildings 10, 35, and 100 and each time we got the same number. So we're pretty sure that they are equivalent.)*
- I saw several other groups doing this. What other numbers did groups try? Did you always get the same number of cubes?
- How do you know that if you tried another building, say Building 537, the expressions won't produce a different number of cubes? **(Challenging)** *(We said that these expressions have to be equivalent because they all describe the same situation. We can see how each of these can be used to describe any of the buildings, so they must work.)*

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- Group 6, you had another approach. Share what you did. *(We remembered how we learned to simplify expressions, so we tried it with these expressions, We used the distributive property and then combined like terms and the first two expressions both became 5n – 4.)* Please come to the board and show us how you simplified the first expression. (Student name) come up also and show us how you simplified the second expression. Justify each of your steps.
- **So, we've seen that different expressions can represent the same situation. But when two or more expressions represent the same situation, these expressions can be shown to be equivalent by using the strategies that we've learned for simplifying expressions, including the distributive property and collecting like terms. (Recapping the EU)** Can someone say in their own words how we can use the distributive property and collecting like terms to determine if expressions are equivalent?

EU: A function is a rule that assigns to each input exactly one output for a real-world, geometric, or mathematical situation.

- Earlier we discussed that for each of our equivalent expressions, the same input gave a corresponding equivalent output. How did we see these inputs and outputs in Group 2's table? *(Their table lists the inputs of the building numbers on the left and the corresponding outputs as the numbers of cubes in a building on the right.)*
- Mathematicians use the term "function" to describe a rule that assigns to each input exactly one output. **(Marking)** How could these expressions we discussed today help us construct a function to model the pattern of growth of these buildings? *(The expressions are what gives us the output for each of the inputs.) (The expression seems like it is the rule for what to do to the input to get the output.) (We put the building number/input into the expression to get the number of cubes in the building/output.)*
- Someone say more about what would be the inputs and what would be the outputs? *(The inputs would be the building number that we put into the expression for n.) (The output would be the number of cubes that our expression tells us are in a building.)*
- Take a look at this equation: $c = 5n - 4$. What is the same and what is different between this equation and the expression $5n - 4$? *(They both include 5n – 4.) (5n – 4 represents the growth of the buildings for both the expression and the equation.) (They both have the variable n for the input.) (The equation provides the variable c for the output, but the expression doesn't provide anything for the output.) (The equation represents a statement of equality that the output equals whatever 5 times the input minus 4 equals.)*
- So, how does the equation $c = 5n - 4$ represent a function rule? Turn and talk. *(The equation shows a rule that tells you what happens to the building number and what number of cubes it will equal for what the expression does to the building number.) (We said that it shows that the number of cubes equals something for each building number and what needs to be done to that building number in order to get the right number of cubes.)*
- **So, let me try to summarize your ideas outside of the context of today's problem. What I'm hearing is that an equation can represent a function rule that assigns to each input exactly one output, in this case a geometric pattern of growth of the buildings. (Recapping the EU)** So, we've modeled lots of different situations with expressions. If I ask you to write an equation to represent a function rule that models a situation, how is this different and how is this the same as if I had asked you to write an expression to model the situation? **(Challenging)**

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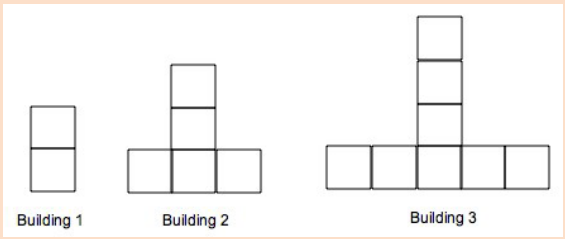
- How does Group 2's table represent each part of the equation $c = 5n - 4$? *(All the values of n are in the left column and all their corresponding values of c are in the right column.)* Can someone explain what she just said using the terms input and output? **(Challenging)** So, can someone summarize how a table represents a function rule?
- Take a moment to write an equation that represents a function rule for each of the expressions we discussed today. What did you get? What would the tables for each of these function rules look like? **(Challenging)**
- So, earlier we said that all three of the expressions were equivalent and correctly modeled the pattern of growth of the buildings. Does this mean that there are three correct equations that represent the same function rule? Go back to the definition. Does each of these assign to the same inputs a corresponding equivalent output? Turn and talk.

Application	 1. Describe a pattern that you see in this set of "buildings" built of squares. 2. Assuming the pattern continues, use your pattern to write a equation that represents a function rule for the number of squares in the nth building. 3. Describe how your equation represents a function rule that explains the pattern that you see in the buildings.
Summary	How did you use the diagram to help you write an equation that represents a function rule for the number of cubes in each building?
Quick Write	What are the similarities and what are the differences between expressions and equations that represent function rules?

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