Exploring Representations for Multiplication

The Case of Mr. Harris and the Band Concert Task

Mr. Harris wanted his third-grade students to understand the structure of multiplication and decided to develop a task that would allow students to explore multiplication as equal groups through a familiar context—the upcoming spring band concert. He thought that the Band Concert Task (shown below) would prompt students to make or draw arrays and provide an opportunity to build conceptual understanding toward fluency in multiplying one-digit whole numbers by multiples of 10 using strategies based on place value and properties of operations—all key aspects of the standards for third grade students. He felt that the task aligned well with his math goals for the lesson and supported progress along math learning progressions, had multiple entry points, would provide opportunities for mathematical discourse, and it would challenge his students. As students worked on the task he would be looking for evidence that his students could identify the number of equal groups and the size of each group within visual or physical representations, such as collections or arrays, and connect these representations to multiplication equations.

> The third-grade class is responsible for setting up the chairs for the spring band concert. In preparation, the class needs to determine the total number of chairs that will be needed and ask the school's engineer to retrieve that many chairs from the central storage area. The class needs to set up 7 rows of chairs with 20 chairs in each row, leaving space for a center aisle. How many chairs does the school's engineer need to retrieve from the central storage area?

Mr. Harris began the lesson by asking students to consider how they might represent the problem. "Before you begin working on the task, think about a representation you might want to use and why, and then turn and share your ideas with a partner." The class held a short conversation sharing their suggestions, such as using cubes or drawing a picture. Then the students began working individually on the task.

As Mr. Harris made his way around the classroom, he noticed many students drawing pictures. Some students struggled to organize the information, particularly those who tried to represent each individual chair. He prompted these students to pause and review their work by asking, "So, tell me about your picture. How does it show the setup of the chairs for the band concert?" Other students used symbolic approaches, such as repeated addition or partial products, and a few students chose to use cubes or grid paper. He made note of the various approaches so he could decide which students he wanted to present their work, and in which order, later during the whole class discussion.

In planning for the lesson, Mr. Harris prepared key questions that he could use to press students to consider critical features of their representations related to the structure of multiplication. As the students worked, he often asked: "How does your drawing show the seven rows?" "How does your drawing show that there are 20 chairs in each row?" "Why are you adding all those twenties?" "How many twenties are you adding and why?"

He also noticed a few students changed representations as they worked. Dominic started to draw tally marks, but switched to using a table. When Mr. Harris asked her why, she explained she got tired of making all those marks. Similarly, Jamal started to build an array with cubes, but then switched to drawing an array. Their initial attempts were valuable, if not essential, in helping each of these students make sense of the situation.

Before holding a whole class discussion, Mr. Harris asked the students to find a classmate who had used a different representation and directed them to take turns explaining and comparing their work, as well as their solutions. He encouraged them to also consider how their representations were similar and different. For example, Jasmine who had drawn a diagram compared her work with Kenneth who had used equations (see reverse for copies of their work). Jasmine noted that they had gotten the same answer and Kenneth said they both had the number 20 written down seven times. Molly, in particular, was a student who benefited from this sharing process because she was able to acknowledge how confused she had gotten in drawing all those squares (see reverse side) and had lost track of her counting. Her partner helped her mark off the chairs in each row in groups of ten and recount them. The teacher repeated this process once more as students found another classmate and held another sharing and comparing session.

Written by DeAnn Huinker (University of Wisconsin-Milwaukee), drawing on experiences with teachers and students in the Milwaukee area. This case is intended to support the Guiding Principle for Teaching and Learning in *Principles to Actions: Ensuring Mathematical Success for All* (Reston, Va.: National Council of Teachers of Mathematics 2014).

During the whole class discussion, Mr. Harris asked the presenting students to explain what they had done and why and to answer questions posed by their peers. He asked Jasmine to present first since her diagram accurately modeled the situation and it would likely be accessible to all students. Kenneth went next as his approach was similar to Jasmine's but without the diagram. Both clearly showed the number 20 written seven times. Then Teresa presented. Her approach allowed the class to discuss how skip counting by twenties was related to the task and to multiplication, a connection not apparent for many students. Below is an excerpt from this discussion.

Mr. H: So, Teresa skip counted by twenties. How does this relate to the Band Concert situation?

Connor: She counted seven times like she wrote on her paper.

Mr. H: I'm not sure I understand, can someone add on to what Connor was saying?

Grace: Well each time she counted it was like adding 20 more chairs, just like what Kenneth did.

Mr. H: Do others agree with what Grace is saying? Can someone explain it in their own words?

Mason: Yeah, the numbers on top are like the 7 rows and the numbers on the bottom are the total number of chairs for that many rows.

Mr. H: This is interesting. So what does the number 100 mean under the 5?

Mason: It means that altogether five rows have 100 total chairs, since there are 20 chairs in each row.

Mr. H: Then what does the 140 mean?

Mason: It means that seven rows would have a total of 140 chairs.

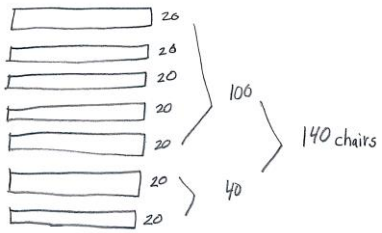
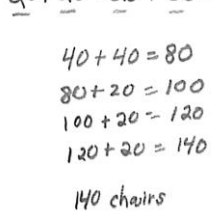
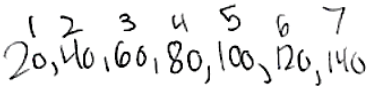
[Mr. Harris paused to write this equation on the board: 7 x 20 = 140.]

Mr. H: Some of you wrote this equation on your papers. How does this equation relate to each of the strategies that we have discussed so far? Turn and talk to a partner about this equation.

[After a few minutes, the whole class discussion continued and Grace shared what she talked about with her partner.]

Grace: Well, we talked about how the 7 means seven rows like Jasmine showed in her picture and how Teresa showed. And the 20 is the number of chairs that go in each row like Jasmine showed, and like how Kenneth wrote down. Teresa didn't write down all those twenties but we know she counted by twenty.

Toward the end of the lesson, Mr. Harris had Tyrell and Ananda present their representations because they considered the aisle and worked with tens rather than with twenties. After giving the students a chance to turn and talk with a partner, he asked them to respond in writing whether it was okay to represent and solve the task using either of these approaches and to justify their answers. He knew this informal experience with the distributive property would be important in subsequent lessons and the student writing would provide him with some insight into whether or not his students understood that quantities could be decomposed as a strategy in solving multiplication problems.

Jasmine	Kenneth	Teresa
		
Molly	**Tyrell**	**Ananda**