

PREFACE

This book is the fourth volume in a series of four. The first three books in the Young Mathematicians at Work series were on number and operation and were co-authored with Maarten Dolk. This volume extends that work to encompass the development of algebra in children between the ages of six and twelve.

The series is a culmination of a long and fruitful journey characterized by collaboration, experimentation, reflection, and growth. Mathematics in the City (www.mitccny.org) was established in 1995 with initial funding from the Exxon-Mobil Foundation and the National Science Foundation. Today, funded by many sources—including continued funding from NSF—it is a large center of in-service and research for mathematics education, K–8. Over the past twenty years, many researchers, mathematicians, teacher educators, and classroom teachers have contributed to our work. Besides the Young Mathematicians at Work series, we have published professional development materials (comprising digital materials and facilitator guides) and a K–6 curriculum (comprised of twenty-four units, colorful posters, and eight trade books). For further information on these, please see www.newperspectivesonlearning.com.

THE IMPORTANCE OF NUMERACY AND ALGEBRA . . .

Numeracy and algebra are critical issues. In today's world children need a strong understanding of number and operation. They need good mental arithmetic strategies and a deep enough understanding of operations that the transition to algebra is easy. From our perspective, none of the curricula we were working with treated computation sufficiently. Some focused on developing a repertoire of pencil/paper strategies; some designated focus algorithms; others focused primarily on learner-invented strategies; and some primarily made use of hands-on materials such as base blocks to teach the standard algorithms. None really pushed children to generalize or

transition to algebra. In fact, when algebra was taught, it was usually seen as a separate strand characterized by analysis of patterns and functions.

To strengthen computation, we began to design minilessons with strings of related problems to develop deep number sense and a repertoire of strategies for mental arithmetic. Our goal in designing these was to encourage children to look at the numbers first before they decided on a strategy, and to have a deep enough sense of landmark numbers and operation that a toolbox of strategies could be used to calculate efficiently and elegantly—like mathematicians.

Over the years, to help our teachers develop vibrant math communities, rather than developing isolated hands-on activities, we built *sequences of investigations* to ensure progressive mathematics development. Several of the sequences we designed focused on developing mathematical modeling, for example, the open number line and the open array. Once these models are developed they can be used as powerful tools for thinking—for generalizing, proving, and even doing algebra.

For three years we ran a think tank on the emergence of algebra. Building on our initial work on number and operation, we field-tested sequences using the open number line to develop equations and strategies for solving for unknowns. We worked on encouraging children to develop conjectures and proofs. We helped teachers find the moments to push for generalization—to extend the work on number to algebra. Along the way we began to gain an understanding of how algebra might be taught in the elementary school.

This book is a culmination of that work. In it we describe our views on algebra and its development. We provide a “landscape of learning”—a trajectory of big ideas, strategies, and models for algebra—depicting the landmarks or milestones to be supported and celebrated. We tell the stories of many talented teachers and their students hard at work exploring and structuring their lived worlds algebraically.