

# INTRODUCTION

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Consider the greatest challenges that we face today:

- ▶ Combating global climate change
- ▶ Establishing renewable energy sources
- ▶ Ensuring that food, water, and housing resources exist for a growing human population
- ▶ Challenging the systemic injustices embedded in society
- ▶ Advancing human rights and democracy across the globe
- ▶ Decreasing conflict among individuals and between nations
- ▶ Advocating for public health and preventing new pandemics from emerging
- ▶ Seeking cures for diseases, such as cancer, dementia, HIV/AIDS, malaria, and cardiovascular diseases
- ▶ Countering misinformation that pervades traditional and social media platforms
- ▶ Seeking new frontiers in space exploration.

Each challenge is complex, and the solutions to these problems will require people to work together across multiple disciplines to effect change. Solutions will need to draw on scientific, political, physical, and social knowledge and understanding, as well as the application of mathematics and statistics (see Tate, 2019). Like these global challenges, many problems we confront in our everyday lives require us to coordinate knowledge from multiple disciplines. Few real-world problems are solely mathematical. However, curriculum and instruction in K–12 schools primarily positions core disciplines into silos. Mathematics often is taught independent of science, social studies, the arts, or languages, even though it is a necessary tool in these other disciplines. One might wonder, “How can we create an educational system in which the problems students encounter in school reflect the multidisciplinary problems they will encounter in life?”

Historically, school curriculum has focused on the three “R”s of reading, writing, and arithmetic. Language arts and mathematics continue to play a dominant role in the elementary curriculum and are emphasized in middle and high school: in recent years, these subjects have become the primary focus of high-stakes standardized tests in K–12 schools. Many educators, however, recognize that global citizens of the 21st century and beyond need skills and knowledge that transcend the three “R”s. As students progress into high school, they need opportunities to synthesize ideas from multiple disciplines and reason about mathematics and how it relates to real-world issues. There have been repeated calls



to modernize the current K–12 mathematics curriculum, including by placing a greater emphasis on mathematical modeling, working with data, and applying mathematics. The tasks in this book are designed to make use of mathematical problem-solving in rich contexts and develop thinking across multiple disciplines.

Disciplinary literacies must be integrated with mathematics in schools to prepare students to be productive global citizens. The term *literacy* has been used in many different ways, and it has become a more prominent concept over the past 50 years (see Lankshear & Knobel, 2011, for a full discussion of perspectives on literacy). When students develop a disciplinary literacy, they are able to demonstrate competencies or knowledge in a specific field that they can apply across many situations (Organisation for Economic Co-operation and Development, 2009; Wolsey & Lapp, 2017). Essentially, being literate in a subject means one has the ability to communicate, synthesize concepts, and interpret results within a particular context in order to complete tasks. For example, scientific literacy could be described as one’s ability to read, reason, and interpret data in a manner similar to that of a scientist.

Students in Grades 9 through 12 are developing the ability to think abstractly and solve complex problems. High school students are exposed to global issues in social studies and science classes. Many high school students are developing their self-identity and beginning to consider possible career paths. Students in Grades 9 through 12 are becoming more independent by taking on new roles and learning new tasks, such as seeking employment and learning how to drive. The tasks in this book are designed to develop students’ disciplinary literacies, such as sociopolitical literacy, health literacy, and historical literacy. The tasks prepare students to transform the world by addressing some of the global challenges mentioned earlier. Many of the tasks are intended to develop students’ transformational thinking by requiring them to think about how they might use mathematical ideas to elicit change in their communities or the world at large (Matthews et al., 2013). Students’ disciplinary literacies emerge as they learn about a context and its connections to mathematics. Mathematical knowledge is powerful, and students need to know that mathematics and mathematical thinking have the potential to help us address important problems in our world (Gutstein, 2006).

*Modernizing mathematics* refers to the idea that teachers should rethink how mathematics has traditionally been taught in schools by making rich tasks and collaboration the focus of instruction and promoting opportunities for active learning. Providing students with contextualized problems motivates them to learn and focuses instruction on their thinking and work. Furthermore, teachers can leverage the power of collaborative thinking to solve mathematical problems in the 9–12 classroom.

## CONTEXT IN THE MATHEMATICS CLASSROOM

Experienced mathematics teachers have undoubtedly had to respond to the question “When are we ever going to use this?” When we use contexts to teach mathematics, this question is often answered through engagement with tasks. Students begin to see the natural connections that mathematics has with other disciplines, such as science, social



studies, language arts, and fine arts (Wolf, 2015). In *Catalyzing Change in High School Mathematics*, the National Council of Teachers of Mathematics (NCTM; 2018, p. 1) posits that high school students need to be able “to be mathematically and statistically literate consumers, if not producers, of information. Furthermore, NCTM suggests that the high school mathematics curriculum should empower students by broadening their career opportunities, teaching them how to critique their world, and helping them experience the joy and beauty of mathematics. These recommendations counter many students’ mathematics experiences, in which mathematics is taught without addressing its relevance to their lives. When students are not aware of the applicability of the mathematics they learn, they may develop unproductive beliefs about mathematics as a discipline and fail to experience its utility, which can lead to a negative disposition toward learning mathematics. In contrast, the use of rich problems can help students learn about their world by giving them opportunities to apply mathematics and creating more humanizing experiences for students, especially historically marginalized youth (Gutiérrez, 2018b).

Some of these contexts could be difficult to discuss with students and might address topics that make students feel uncomfortable, such as subjects involving their personal beliefs or identities that are still forming. Although teachers should be conscientious about choosing age-appropriate contexts to cover, they also have an obligation to help students become critical consumers of information and to teach about complex social issues that ultimately have an impact on students’ lives. Rather than avoid challenging conversations, teachers should help students learn how to discuss differences in perspectives with civility and respect, while valuing individuals’ views (Hauver, 2017).

## Equitable Practices in the Classroom

As teachers consider their teaching, they should ask themselves some important questions: Do they teach all students, or do they teach each and every student? Is there a difference? When we say we teach *all* students, the phrase does not account for the fact that each learner has unique and individual needs. The word *all* focuses on the collective, rather than the individual. The phrase *each and every student* suggests that policies or practices should enable individual students to succeed in the classroom.

The needs of each and every learner are addressed in NCTM’s (2014a, p. 1) position statement on access and equity, which suggest that “creating, supporting, and sustaining a culture of access and equity requires being responsive to students’ backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program and assessing its effectiveness.” Scholars (e.g., Gutiérrez, 2018a; Martin, 2019) have challenged the extent to which this framing of access and equity is being fully realized in classrooms and advocate for a liberatory and rehumanizing mathematics education, which uses approaches to mathematics teaching and learning that do not position any particular way of thinking as dominant. A critical outcome of building such a culture is that students will develop positive mathematical identities (Aguirre et al., 2013).

*Mathematical identity* refers to how individuals tend to think of themselves in relation to the subject of mathematics. When students have positive mathematical identities, they tend to feel empowered by mathematics, see mathematics as relevant to their lives, and believe that they can successfully engage in mathematical work (NCTM, 2018). They consider themselves as individuals who can create and do mathematics and not only as consumers of mathematics. Teachers should strive to be critically conscious, which requires understanding the socio-historical-political context of schooling, an awareness of how this context may manifest in negative stereotypes about particular groups of students, and recognition of how these stereotypes may have impacted or influenced students' perceptions of themselves as learners of mathematics (Seda & Brown, 2021). Maintaining this critical consciousness requires teachers to continuously identify and disrupt tacit and explicit practices (including speech and actions) that perpetuate deficit and discriminatory ideas and policies that harm particular groups of students.

Two different frameworks have been used to help teachers think about how their practices can support students' development of positive mathematical identities. NCTM's (2014b) *Principles to Actions* outlined eight mathematical teaching practices (MTPs) that are critical to creating a culture that embraces equity and helps students develop positive mathematical identities:

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Teachers who regularly engage in these practices cultivate a classroom in which learners feel valued and, as a result, develop identities as doers of mathematics. Aguirre et al. (2013) also proposed a set of five equity-based practices (EBPs):

1. Going deep with mathematics: teaching using cognitively demanding tasks that involve conceptual understanding, problem solving, and reasoning
2. Leveraging multiple mathematical competencies: appreciating that every student has strengths in mathematics that can support teaching and learning and using tasks with multiple entry points to engage students with varying skills in making important contributions
3. Affirming mathematics learners' identities: structuring instruction to encourage persistence and utilizing mistakes and incorrect answers as sources of learning

4. Challenging spaces of marginality: creating a culture in which students' contributions are valued, status is diminished, and mathematical authority is distributed across students, teachers, and texts
5. Drawing on multiple resources of knowledge: recognizing and utilizing the range of knowledge and experiences that students bring to the classroom, such as those related to mathematics, culture, language, family, and community

Aspects of these MTPs and EBPs have guided the development of the activities in this book, which focus on student thinking and ensure that mathematical authority resides within the classroom community, not the teacher alone (EBP 4). Contextualized problems that require productive struggle and depth of mathematics form the basis of instruction (MTP 2, MTP 7, EBP 1, and EBP 2) when these problems are aligned to both mathematical and other discipline-specific objectives (MTP 1) that draw on students' lived experiences (EBP 5). Teachers can support students in discussions (MTP 4) by posing meaningful questions (MTP 5) in a way that values students' contributions and positions them as meaningful contributors to teaching and learning (EBP 4). These questions can form the basis for supporting students as they work (MTP 7) and assessing how they are thinking in order to guide instruction (MTP 8).

## Implementing Rich Tasks in the Mathematical Classroom

MTP 2 foregrounds implementing tasks that focus on reasoning and problem-solving. One of the most important instructional decisions teachers make is determining what tasks to give students (Hiebert et al., 1997). Tasks form the foundation for instruction and provide students with an opportunity to think about the nature of mathematics. Tasks are part of every mathematics classroom, although the quality of tasks may vary.

One might wonder, “What makes a task high quality?” Stein and Smith (1998) identified non-algorithmic, complex tasks as “doing mathematics” tasks, which require sustained cognitive effort to complete and enable the student to explore the nature of mathematical concepts and processes. Another way to describe high-quality tasks is to suggest they have a “low floor” and a “high ceiling” (Boaler, 2016). These terms refer to the fact that many good tasks are accessible and easy to understand by each and every learner (low floor) but have the potential to add complexity or challenge advanced learners (high ceiling). A third way to identify high-quality tasks is to embed cultural relevance by providing opportunities for students to make personal or social changes (Gutstein, 2006; Matthews et al., 2013).

Tasks should encourage students to reflect and communicate mathematically, use tools to work on mathematical tasks, and leave behind important mathematical residue (Hiebert et al., 1997). As students engage with high-quality tasks, they may come to have insights about the nature of mathematical thinking and strategies for solving problems. However, rich contextual problems also have the potential to help students learn about the problem's context as well.

Lessons that foreground students' thinking as they complete problem-solving tasks are often implemented using a three-phase process. Some curriculum materials name these three phases Launch, Explore, and Summarize (van De Walle et al., 2019). The relationship among the students, the teacher, and the task is foregrounded during each of these phases. The lesson outline for each task in this book provides guidance for engaging students in each phase.

The practices in the book *5 Practices for Orchestrating Productive Discussions* (Smith & Stein, 2018) have been widely adopted as a framework to support teachers in coordinating classroom discourse that focuses on rich mathematical tasks; these practices are aligned with the Launch-Explore-Summarize lesson format (Smith et al., 2008). Prior to engaging in these five practices, teachers must first set goals or objectives for instruction and select tasks that align with those goals, a process sometimes referred to as Practice 0 (Smith et al., 2020). After identifying the goals and tasks, teachers who want to develop rich discussions around students' mathematical work should engage in the following five practices (Smith & Stein, 2018, pp. 9–10):

1. **Anticipating** likely student responses to challenging mathematical tasks and questions to ask to the students who produce them
2. **Monitoring** students' actual responses to the tasks (while students work on the task in pairs or small groups)
3. **Selecting** particular students to present their mathematical work during the whole-class discussion
4. **Sequencing** the student responses that will be displayed in a specific order
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas

These five practices provide a starting point for teachers to plan and frame mathematical lessons that foreground student thinking throughout all phases of a three-phase lesson. The following sections describes the anticipating practice in more depth, followed by descriptions of the three-phase lesson plan.

## 🔍 Before the Lesson: Anticipating Students' Responses

To create a learning environment in which student thinking is valued, teachers must plan how they will interact with their students. As teachers engage in the anticipating practice, they should reflect on the range of ways students might understand and react to the task and what strategies students are likely to use. Here are some questions teachers should consider before they ask students to investigate a task:

- ▶ What mathematical content and practices will students have the opportunity to learn as a result of completing this task?
- ▶ Could students interpret the task in different ways?



- ▶ What are common strategies that students could use to solve this task?
- ▶ How are the possible student strategies similar to and different from one another?
- ▶ What difficulties might students encounter while solving the task, and what questions might help students get started on or engage more with the task?

By thinking about how students are likely to approach the task, teachers can organize their lesson to support a range of mathematical reasoning and ensure clear connections to learning goals. In addition, anticipating a range of student reasoning prior to a lesson can help teachers monitor students' work during a lesson. The teacher could also plan for selecting and sequencing practices by considering what types of student solutions they could feature during a whole-class discussion and in what order the solutions should be presented.

## ▶ Launching Mathematical Tasks

The first phase of the lesson involves the launch of the task. A launch is not the same as an introduction to a lesson. Launching a task should not just consist of reviewing topics from a prior class or expecting students to complete a bell-ringer activity. Just as launching a boat means setting it off to begin a voyage, launching a task means the teacher is getting students started on their own voyage of problem-solving. As you prepare to launch a task, ask yourself this important question (Smith & Stein, 2018, p. 102): “How will I introduce and set up the task to ensure that students understand the task and can begin productive work, without diminishing the cognitive demand of the task?” A launch of a task may include several steps:

- ▶ *Activate prior mathematical knowledge.* Students do not begin a task without any prior knowledge. A launch may provide opportunities for students to review or reflect on previously learned knowledge they may need as they solve the task. The goal of the launch is to activate thoughts and ideas that students can later connect to the new idea they construct during the lesson. However, teachers should be cautious that providing too much information beforehand can lower the cognitive demand of the task.
- ▶ *Build contextual knowledge.* When students are unfamiliar with the context, the teacher may use the launch as an opportunity to introduce students to the context and draw connections to their personal lives. Many creative actions can lead to building contextual knowledge, such as watching videos, reading a children's book or news article, or observing physical objects or phenomena. By talking about what they notice in videos, articles, or observations, students can share what they know about the context with their classmates, and the teacher can ensure that each student has sufficient background information to engage in the task.

- ▶ *Provide clarity about the task.* The task should be clearly stated verbally and in writing. Providing a written description on a board (e.g., whiteboard, blackboard) or on paper allows students to refer back to it when necessary. Teachers should develop a shared meaning of any ambiguous vocabulary and ask questions to ensure students understand the task. These steps are particularly important for students who are English language learners.
- ▶ *Prepare the students for exploration.* If a task requires tools (e.g., computers, manipulatives, measurement devices), students need to know where to access these tools and how to use them appropriately. Before students get to work, teachers should provide expectations about the use of tools, how to collaborate with others, and what products (e.g., calculations, explanations, representations) are expected at the end of the exploration.

Other aspects of launches may be specific to a task's context or mathematical objectives. For example, when launching a statistical investigation, teachers should take time to introduce aspects of the data set, such as the number of cases, the meaning of attributes, and how the data were collected.

## Supporting Students' Explorations

During the Explore phase, students are working. Teachers should be thoughtful about how to organize students to optimize learning. They could use a combination of individual, small-group, and whole-class groupings to support thinking and discussion. During this time, teachers should “let go” of the direction of the class while students solve the primary task individually or in small groups. Letting go of the class does not mean the teacher has no control over what is happening; rather, letting go means students can be given space for independent work and productive struggle (MTP 7). As teachers walk around the classroom monitoring students at work, they should listen carefully to how students are thinking and pose questions that both assess what the students know and advance their work (Huinker & Bill, 2017; Smith et al., 2008). They should also document the strategies students use and consider which students' work to highlight during a whole-class discussion and the sequence in which to share the work.

## Summarizing a Task by Building on Students' Thinking and Connecting Their Solutions

Students' solutions form the basis of the Summarize phase, when the instruction shifts back to a whole-class discussion that involves students and the teacher. Clear expectations and norms need to be developed so students feel comfortable sharing their solutions in a whole-class setting (Yeh et al., 2017). All students must feel that their ideas are valued and respected by their classmates. During this phase, the teacher should be strategic about how ideas are shared. During the monitoring of students' thinking during

the Explore phase, the teacher should select particular instances of thinking to share either tacitly or explicitly, being thoughtful about how to sequence these ideas. There are often multiple ways to sequence students' ideas (e.g., simplest to most complex, most common to least common); the teacher should select the approach that optimizes learning by keeping the content of the task and students' current thinking in mind. As different students describe their thinking, the teacher should ask specific questions that can help other students make sense of various strategies; expand on strategies that are shared; draw connections between different students' strategies, as well as between these strategies and learning goals; and seek patterns and generalizations (Smith et al., 2008). To ensure that all students are engaged and participate in the discussion, the teacher may want to ask students to revoice other students' statements or describe another student's strategy using their own words.

The discussions around students' solutions should be guided by the mathematical goals for the lesson, and teachers should ensure that students understand the big mathematical ideas. The Summarize phase also provides an opportunity for the teacher to elaborate on the context of the lesson and help develop students' disciplinary literacies.

## THE TASKS AHEAD

In each chapter in this book, you will find a series of tasks that focus on a single context. Each chapter begins with an introduction that describes the disciplinary literacy that is critical for students to develop and includes a brief overview of each task in the chapter, highlighting connections to the disciplinary literacy within each task. In Chapter 1, the focus is on mathematical design literacy as students are introduced to the process of mathematical modeling and learn design literacy skills such as prototyping and iteration. By designing animations and graphics, students will learn to apply their mathematical knowledge of linear functions, coordinate geometry, and geometric transformations. Dynamic geometry tools play a central role in the Chapter 1 tasks. Political and social literacies are addressed in Chapter 2 as students learn about the connections between census data and redistricting of congressional districts. Students will mutually define fair representation and consider how U.S. congressional districts can be formed to be representative of the people and communities within the district. Students will have opportunities to make sense of data representations and use their knowledge of functions to make predictions about future population changes. Disease and disinformation are the focus of Chapter 3. Teachers will use the COVID-19 pandemic as a springboard to discuss how viral spread can be modeled using exponential functions and how applying probability concepts can help leaders make public health decisions. Students will have the opportunity to develop media literacy by critiquing others' reasoning using two questions that are often used in mathematical contexts: "How do you know that is true?" and "Could you possibly be wrong?" Historical literacy connects the three tasks

in Chapter 4, in which students explore algebraic problems from an ancient Egyptian papyrus, analyze a data visualization made by Florence Nightingale about sanitary conditions for wounded soldiers, and consider connections between astronomy and trigonometry. These tasks demonstrate how mathematics has historically been used as a problem-solving tool, and the two disciplines can be taught in ways that complement each other. To build students' civic and financial literacies, Chapter 5 focuses on food insecurity. Students will learn to explain how food insecurity impacts our society and consider how to model using a quadratic function. Finally, students will cultivate their artistic literacy and learn about engineering practices in Chapter 6. After exploring artwork inspired by geometry, students will design and build robots to create similar artwork.

Each chapter begins with an introduction that provides a description of the disciplinary literacy of focus and how mathematics is used to develop that literacy. Although the series of tasks within each chapter were designed to build on one another, many tasks can be used independently. Each task has a similar structure and includes

- ▶ a description of the mathematical task that provides a snapshot of the context and overview of the math content included in the task;
- ▶ learning objectives for the task and alignment with the Common Core State Standards and any other discipline-specific standards (e.g., the *Next Generation Science Standards* or the *College, Career, and Civic Life [C3] Framework for Social Studies State Standards*);
- ▶ a section called *Situating the Task* that provides essential information for the teacher about the mathematical content and the context of the task;
- ▶ the ways that students may think about or approach the problem in the task, including possible strategies or difficulties;
- ▶ a description of how the teacher should launch the task;
- ▶ a description of how the teacher should support students as they collectively or individually explore the task, including a focus on monitoring students as they work and asking questions that advance students' thinking;
- ▶ a description of how the teacher should organize a summary of the lesson that foregrounds students' solutions; and
- ▶ ideas on how to differentiate instruction by extending students' thinking.

The tasks throughout this book provide rich opportunities for students to have thoughtful conversations about how mathematics applies to other contexts. The tasks and additional resources are available electronically through NCTM's More4U portal, which can be accessed using the access code on the book's title page. As you read through the tasks, consider how you can help students become thinkers who seek opportunities to transform their world!

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