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# Changes through the Years: Connections between Psychological Learning Theories and the School Mathematics Curriculum

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During the twentieth century, and extending to the present day, the teaching of mathematics in American schools has experienced six identifiable phases with differing emphases: (1) drill and practice, (2) meaningful arithmetic, (3) new math, (4) back to basics, (5) problem solving, and—currently—(6) standards and accountability. Each of these phases is worthy of attention because each corresponds to a period when American education in general was going through radical and fundamental changes, and each introduced new and innovative practices to mathematics education.

In fact, the existence of these historical phases would seem to belie the old cliché that there's nothing new under the sun. The field of education seems to be continually passing through cycles of change. On closer examination, however, many educational innovations are actually recycled in slightly modified forms. Not only is it difficult for newcomers to the profession to tell which aspects of mathematics education are recent innovations and which represent established practice, but it is often difficult for veteran teachers as well to recognize old ideas when they come around again, perhaps in new clothing. Thus, another reason for looking back at major phases of mathematics education over the past century or so is to discover the extent to which certain contemporary practices have their roots in the educational changes of yesteryear.

Finally, and most important, through historical analysis we may gain some perspective on the forces and issues that contribute to change in education. Historical perspective helps us avoid tunnel vision about the uniqueness of the educational problems we face today and suggests options to be considered as we ponder their solutions.

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This paper is a revision and update of an article entitled "Connections between Psychological Learning Theories and the Elementary Mathematics Curriculum," written by the first author under her former name (Diana Lambdin Kroll), that was published in NCTM's 1989 Yearbook, *New Directions for Elementary School Mathematics.*

During any era, numerous diverse factors play a part in directing and influencing educational practice. Indeed, the factors surrounding the emergence of the phases listed above were complex: a constellation of mathematical, political, psychological, and sociological elements. This article focuses attention on the influence of one significant factor—psychological learning theories—both on earlier and on contemporary phases in the teaching of mathematics, but it also highlights the influences of other important factors. Although numerous strands of psychological theory and educational practice can be discerned during each phase, major psychological trends are generally recognizable, and the work of particular theorists can usually be considered central. Table 1.1 gives a brief overview of each of the phases, including their main theorists, classroom focus, and primary teaching methods.

## The Drill-and-Practice Phase

In the early years of the twentieth century, drill and practice, which had long been one component of mathematics instruction, became its primary focus. Prior to 1900, the aim of schooling had been to confront students with difficult mental exercises to build up their powers of reasoning and thought. However, implicit faith in "mental discipline" as a theory of learning had begun to wane by the early 1900s, when Edward Thorndike proposed a new theory—a theory that became known in its several forms as "connectionism," "associationism," or "S-R bond theory." Thorndike claimed that learning is the formation of connections or bonds between stimuli (events in the environment) and responses (reactions of an organism to the environment). His theory maintained that through conditioning, specific responses are linked with specific stimuli.

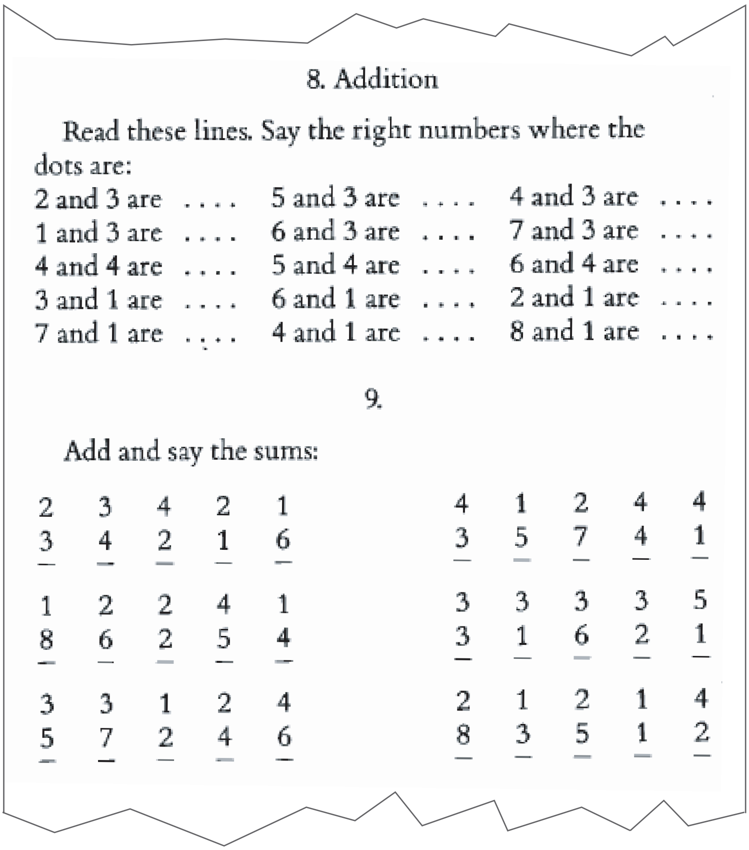
In 1922 Thorndike published *The Psychology of Arithmetic*, in which he demonstrated how his theory applied to the teaching of arithmetic. He explained that teachers needed to recognize and make explicit the essential bonds that constitute the subjects they teach. As an example of what he meant by bonds, Thorndike listed seven separate S-R bonds in "simple two-column addition of integers." Among the list were bonds such as "learning to keep one's place in the column as one adds," "learning to add a seen [number] to a thought-of number," and "learning to write the figure signifying units rather than the total sum of a column." The list concluded with the statement that "learning to carry also involves itself at least two distinct processes" (1922, p. 52). According to Thorndike, the teacher's aim was to arrange for students to receive the right type of drill and practice on each of the right bonds for the right amount of time.

That Thorndike's theory did, in fact, influence mathematics education can be seen by examining yearbooks of professional societies and textbooks of the time. For example, in the introduction to the 1930 Twenty-ninth Yearbook of the National Society for the Study of Education (entitled *Report of the*

Table 1.1
*Relationships between Phases of Mathematics Education and Psychological Learning Theories*

| Phase | Main Theories and Theorists | Focus | How Achieved |
|---|---|---|---|
| Drill and practice (approx. 1920–1930) | Connectionism or Associationism (e.g., Thorndike) | Facility with computation | Rote memorization of facts and algorithms<br>Break all work into series of small steps |
| Meaningful arithmetic (approx. 1930–1950s) | Gestalt Theory (Brownell, Wertheimer, van Engen, Fehr) | Understanding of arithmetic ideas and skills<br>Applications of math to real-world problems | Emphasis on mathematical relationships<br>Incidental learning<br>Activity-oriented approach |
| New math (approx. 1960–1970s) | Developmental psychology, socio-cultural theory (e.g., Bruner, Piaget, Dienes) | Understanding the structure of the discipline | Study of mathematical structures<br>Spiral curriculum<br>Discovery learning |
| Back to basics (approx. 1970s) | (Return to) connectionism | (Return to) concern for knowledge and skill development | (Return to) learning facts by drill and practice |
| Problem solving (approx. 1980s) | Constructivism, cognitive psychology, and sociocultural theory (Vygotsky) | Problem solving and mathematical thinking processes | Return to discovery learning, learning *through* problem solving |
| Standards, assessment, and accountability (approx. 1990s to present) | Cognitive psychology, sociocultural theory<br>vs. renewed emphasis on experimental psychology (NCLB) | Math wars: concern for individual mathematical literacy<br>vs. concern for administration of educational systems | NSF-developed student-oriented standards-based curricula<br>vs. focus on test preparation for state-specified expectations |

*Society's Committee on Arithmetic*), the editor presents the overall perspective of the volume: "Theoretically, the main psychological basis is a behavioristic one, viewing skills and habits as fabrics of connections" (Knight 1930, p. 5). As another example, figure 1.1 shows a portion of a page from Thorndike's (1924) arithmetic text for third graders, a page showing drill on basic addition facts. Since Thorndike did not provide any prior instruction to encourage pupils to relate the basic facts to one another, pupils working on this page would probably view the exercise 3 + 1 as totally unrelated to the exercise 1 + 3.



8. Addition

Read these lines. Say the right numbers where the dots are:

| | | |
|---|---|---|
| 2 and 3 are .... | 5 and 3 are .... | 4 and 3 are .... |
| 1 and 3 are .... | 6 and 3 are .... | 7 and 3 are .... |
| 4 and 4 are .... | 5 and 4 are .... | 6 and 4 are .... |
| 3 and 1 are .... | 6 and 1 are .... | 2 and 1 are .... |
| 7 and 1 are .... | 4 and 1 are .... | 8 and 1 are .... |

9.

Add and say the sums:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 4 | 2 | 1 | 4 | 1 | 2 | 4 | 4 |
| 3 | 4 | 2 | 1 | 6 | 3 | 5 | 7 | 4 | 1 |
| 1 | 2 | 2 | 4 | 1 | 3 | 3 | 3 | 3 | 5 |
| 8 | 6 | 2 | 5 | 4 | 3 | 1 | 6 | 2 | 1 |
| 3 | 3 | 1 | 2 | 4 | 2 | 1 | 2 | 1 | 4 |
| 5 | 7 | 2 | 4 | 6 | 8 | 3 | 5 | 1 | 2 |

Fig. 1.1. A portion of a page from a 1924 third-grade text: Thorndike's *The Thorndike Arithmetics*

One major effect of Thorndike's theories was the segmentation of the curriculum into many disjoint bits. Teachers attempted to be certain that an entire collection of individual bonds was established and exercised in order for each higher-level skill to be mastered. Since each bond was believed to exist in isolation, it was thought that mixed, unorganized drill was perhaps even more effective than practice on a systematic arrangement of facts; with mixed drill the problem of interference between similar bonds was avoided. Another effect of connectionist theory was the prescriptive teaching methods that it encouraged. Teachers tended not to permit unorthodox algorithms or novel solutions, since the most efficient way to direct pupils to form correct bonds was to carefully keep them from forming incorrect bonds. Thus, in the curriculum of the drill-and-practice era, mathematics was taught by concentrating on drill with skills that had been segmented into small, distinct, easily mastered units.

Although reformulated types of drill and practice returned during the "back to basics" era of the 1970s and continue to be used to this day, in the years immediately following the drill-and-practice era the focus of mathematics education shifted considerably. Parents and educators of the 1930s and 1940s began to question the extreme emphasis on drill for drill's sake and to wonder whether the mathematics learned in school was of any practical use. The focus of instruction during those years—the progressive era—became an attempt to ensure that the skills students had mastered constituted "meaningful" arithmetic.

## The Meaningful Arithmetic Phase

With the 1930s came the Great Depression. High unemployment and other social pressures encouraged more adolescents to remain in school and to continue their education through high school (Bidwell and Classen 1970, p. 531). From approximately 1930 to 1950, when the progressive education movement was influential in the United States, there was a new emphasis on "learning for living." The major change in mathematics education was a shift from an emphasis on drill for drill's sake to a focus on attempting to develop mathematics concepts in a "meaningful" way. This vague term meant different things to different educators of the progressive era.

For some progressives, mathematics was meaningful when it was encountered in the context of practical activity. Mathematics was learned for social utility—to acquire the tools for dealing with problems that would be encountered in later life. These educators recommended an activity-oriented approach. Some believed that students (particularly at the elementary school level) would learn all the mathematics they needed—and learn it better—through incidental experience rather than by systematic teaching. As a writer in the 1935 Tenth Yearbook of the National Council of Teachers of Mathematics (NCTM) commented:

> A large part of the most efficient learning is incidental, that is, learning a special subject with reference to some broader interest or aim without realizing it: Learning number relationships in connection with telling time or making change;

> learning baseball averages (without effort) through sheer interest in big league contests (Wheeler, p. 239).

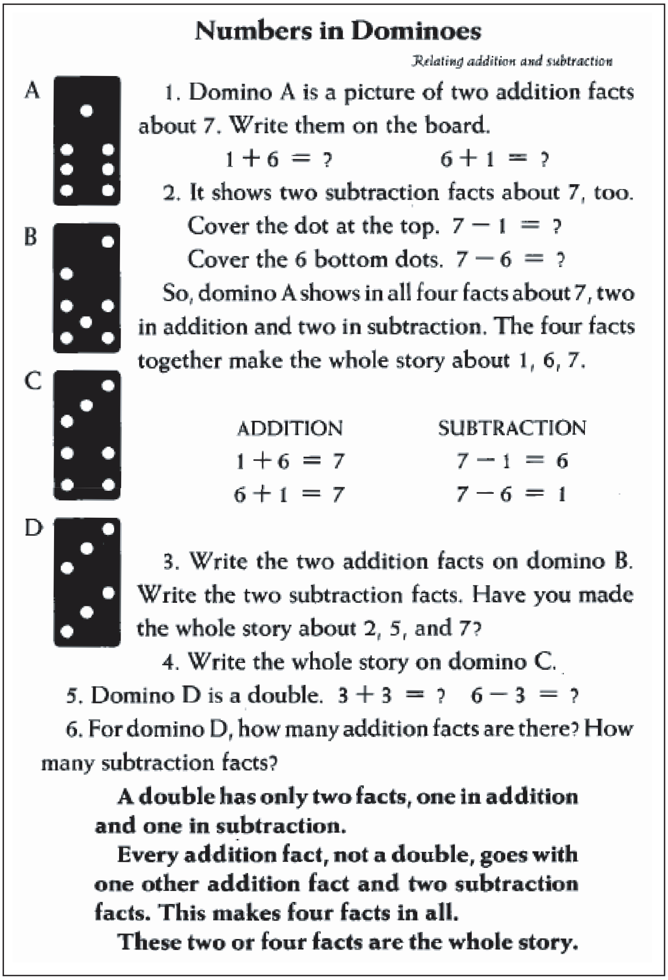
However, incidental learning was often haphazard, slow, and time-consuming, and according to William Brownell—a psychologist and influential mathematics educator of the time—the arithmetic learned under such circumstances was "apt to be fragmentary, superficial, and mechanical" (Brownell 1935, p. 18). When children were left to do whatever they felt like, their experiences were often so diverse and unstructured that they were unable to relate the different bits and pieces.

As a result, many mathematics educators of the era debated the merits of incidental learning. They proposed a different interpretation of "meaningful arithmetic," claiming that "to learn arithmetic meaningfully it is necessary to understand it systematically" (McConnell 1941, p. 281). In a 1938 *Mathematics Teacher* article, Buckingham attempts to clarify this view of meaningful arithmetic by emphasizing what he saw as an important distinction between the significance and the meaning of arithmetic. For him, the significance of number was functional: "its value, its importance, its necessity in the modern social order," whereas the meaning of number was mathematical—embodied in the structure of the number system (p. 26). In general, although proponents of this type of "meaning theory" acknowledged the motivating and enriching value of number experiences that emanate from student-initiated activities, they emphasized as well the notion of "arithmetic as a closely knit system of understandable ideas, principles, and processes" (Brownell 1935, p. 19).

This emphasis on mathematical relationships is clearly illustrated in figure 1.2, a reproduction of a page from Buswell, Brownell, and Sauble's (1959) third-grade text. In introducing basic subtraction facts to pupils, these textbook authors emphasized the relationship of the subtraction facts to previously learned addition facts, explicitly pointing out the "whole story" involving two different addends, a "story" in which the student relates two addition facts and two subtraction facts into one coherent whole.

Another important factor in the emergence of the new meaning theory was the introduction to this country of the "Gestalt" or "field" theory of learning (Fehr 1953). The central idea of field psychology is expressed in the German word *Gestalt*, which means an organized whole in contrast to a collection of parts. Gestalt psychologists regarded learning as a process of recognizing relationships and of developing insights. They believed that it is only when the relationship of a part of a situation to the whole situation is perceived that insight occurs and a solution to a problem can be formulated. Rather than drill on individual skills that in sum might lead to the solution of a problem (as a connectionist would), the field psychologist would try at the start to bring all the elements of a problem together.

A writer in the 1941 Sixteenth Yearbook of the NCTM rejected the connectionist theories of the earlier era as he explained the relationship of field



**Numbers in Dominoes**

*Relating addition and subtraction*

A

B

C

D

1. Domino A is a picture of two addition facts about 7. Write them on the board.

$1 + 6 = ?$ $\quad$ $6 + 1 = ?$

2. It shows two subtraction facts about 7, too.
   Cover the dot at the top. $7 - 1 = ?$
   Cover the 6 bottom dots. $7 - 6 = ?$

So, domino A shows in all four facts about 7, two in addition and two in subtraction. The four facts together make the whole story about 1, 6, 7.

| ADDITION | SUBTRACTION |
|---|---|
| $1 + 6 = 7$ | $7 - 1 = 6$ |
| $6 + 1 = 7$ | $7 - 6 = 1$ |

3. Write the two addition facts on domino B. Write the two subtraction facts. Have you made the whole story about 2, 5, and 7?
4. Write the whole story on domino C.
5. Domino D is a double. $3 + 3 = ?$ $\quad$ $6 - 3 = ?$
6. For domino D, how many addition facts are there? How many subtraction facts?

**A double has only two facts, one in addition and one in subtraction.**

**Every addition fact, not a double, goes with one other addition fact and two subtraction facts. This makes four facts in all.**

**These two or four facts are the whole story.**

Fig. 1.2. A page from a 1959 third-grade text: Buswell, Brownell, and Sauble's *Arithmetic We Need*

psychology to the new meaning theory of arithmetic. Arguing that "meaning inheres in relationships," McConnell (1941, p. 280) claimed that the connectionists had "mutilated" arithmetic by decomposing it into numerous unrelated facts and by emphasizing "discreteness and specificity" (p. 275). The new "meaning theory," however, emphasized "understanding and relating the many specific items which are included [in our decimal number system]" (p. 280).

The meaning theory of arithmetic attempted to combine the progressive idea of activity learning with the ideas of the Gestalt psychologists. Teaching meaningful arithmetic demanded a very different kind of instruction from what had been the norm during the earlier drill-and-practice phase. Rote memorization was de-emphasized, and activity-based discovery was used to help students see connections among the many discrete skills and concepts they were learning. A new era in mathematics education was born. In fact, one important aspect of the "new math"—a focus on clarifying mathematical structures that was threaded throughout the curriculum from the earliest years right through high school—might be seen as a refinement and an extension of the meaningful arithmetic programs initiated during the progressive era.

## The "New Math" Phase

The profound and far-reaching changes introduced by what became known as the "new math" during the 1960s were actually stimulated by severe criticism of American education in the years immediately following the Second World War. These changes received additional attention and support when the USSR's launch of the *Sputnik* satellite in 1957 fueled concerns that U.S. schools were not doing an adequate job of preparing graduates capable of understanding those science and mathematics concepts necessary to compete in a new, elite, technology-driven national workforce (Ferrini-Mundy and Graham 2003). In an article in the *Saturday Review*, an education writer blamed outdated teaching methods for the students' difficulties:

> In many classrooms teachers still rely exclusively on the drill-and-memory system of the past. Others teach arithmetic "incidentally," as it is needed in other subjects and school activities. (Dunbar 1956, p. 54)

In response to such criticisms, mathematicians, educators, and psychologists joined together for the first time in a massive effort to restructure the teaching of mathematics at all levels.

In September 1959, thirty-five scientists, scholars, and educators gathered at Woods Hole on Cape Cod to discuss how science and mathematics education might be improved. A major theme for the Woods Hole conference was that a knowledge of the fundamental structures of a discipline was considered crucial. The conference participants believed that "an understanding of fundamental principles and ideas ... appears to be the main road to adequate 'transfer of training'" and that the learning of "general or fundamental principles" ensures that individuals can reconstruct details when memory fails" (Bruner 1960, p. 25).

Thus, in the early 1960s, at least a dozen curriculum development projects were established to write new mathematics textbooks. Many of these efforts were funded by the recently established National Science Foundation.

The new math texts differed considerably from traditional texts both in organization and in content. Most of the new math curricula for secondary school evolved from the course of study proposed by a Commission on Mathematics appointed by the College Entrance Examination Board (CEEB 1959).