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### Appendix 2

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### Appendix 3

Tasks

Sands of Time

Punching Up the Recipe

You Tell Me, Which Is Larger?

Red Gumball or White? That Is the Question!

Pizza Presto: Which Chef Is Faster?

Are They Equivalent?

Measure for Measure, How Long Was That Board?

Loads of Luscious Lilikoi

Off the Top of Your Head, Which Is Larger?

Brainstorming about the Better Buy

Relating Reciprocals to 1

All the Same When Simplified

How Many Can You Find?