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The Power of Stories

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Stories are powerful tools for learning and developing personal and organizational narratives. Learning to listen to, tell, and interpret stories helps decision-makers to maximize sense making and a sense-giving role. Stories provide the context for understanding, feeling, and interpreting. It is through stories that people and organizations define themselves and establish their identities. Stories have the potential to provide decision-makers with the concrete tools to help them imagine and understand passageways through the ambiguity-opportunity cycle to catalyze change at the classroom, school, district, state, and provincial levels. The recommendations made in the Catalyzing Change series might be ambiguous to some readers because they may find imagining the opportunities to create impactful change difficult to do. Consequently, stories support making sense of ambiguity in ways that readers can see the opportunities within their contexts. The Catalyzing Change series pushes organizations to be reflective of their practices and policies to create change. Stories provide the power for organizations to—

- imagine themselves in the change they aspire to;
- unpack ambiguity to realize possibilities;
- change mindsets;
- plan and move toward attainable action-oriented goals; and
- learn from others.

The collection of stories in this book highlights the recommendations of the Catalyzing Change series across various contexts, including individual teachers' classrooms, school-level change, district-level impacts, and state initiatives. Additionally, there are rural, suburban, and urban contexts. For example, detracking and creating a common pathway for school mathematics is a recommendation that many decision-makers understand provides equitable opportunities but grapple with the complexities for creating such opportunities. This book contains stories of detracking and establishing a common pathway in rural, urban, and suburban school districts. Although the goals of detracking and a common pathway are similar, the stories unpack the complexities for supporting teaching, building buy-in among stakeholders, and making decisions about curriculum.

Three stories unpack statewide initiatives, highlighting the partnerships among PK–12 education, postsecondary education, industry, and other stakeholders. They emphasize that developing common pathways, developing innovative curricula, and supporting equitable teaching are communal efforts. These stories embed several recommendations, including common pathways and essential mathematics concepts and practices supporting wonder, joy, and beauty. These stories allow us to imagine common mathematics pathways from PK–12 through postsecondary education and the vertical alignment of essential mathematics concepts.

The Catalyzing Change series made recommendations focused on equitable teaching practices. Stories in this book focus on supporting teachers in changing their practices to be more equitable. For example, teachers engaged in co-planning and observations of one another's teaching, participated in professional development opportunities focused on leveraging tasks and problem solving to create learning opportunities, unpacked ways they engaged in assessing learners, and partnered with math coaches. A common theme among these stories focuses on creating space for teacher collaboration and supporting equitable teaching practices through continuous improvement. Additionally, equitable teaching supports learners' mathematical identity and sense of agency and builds a sense of professional collectivism. Teachers, coaches, leaders, and other stakeholders support one another to create high-quality opportunities for each and every student.

LEARNING FROM THE STORIES

Change is a process, not an event. It begins with awareness and acknowledgment of the need to change and slowly and steadfastly make progress, often along a shaky path, toward implementation of new structures and realities. Catalyzing Change calls for all stakeholders to confront inequitable policies and practices, including challenging our own beliefs. The Catalyzing Change series, with books focused on early childhood and elementary, middle school, and high school mathematics, is part of an ongoing long-term collaborative process to support stakeholders in ensuring that each and every student has access to high-quality mathematics teaching and learning. The work of dismantling inequitable structures, practices, and instruction will require sustained effort by stakeholders over many years.

The stories in this book represent efforts along the continuum of change, including work that is just starting, to initiatives in progress, to examples of advanced implementation. Each story shares an approach addressing one or more of the key recommendations:

1. Broaden the purposes of learning mathematics
2. Create equitable structures in mathematics
3. Implement equitable instruction
4. Develop deep mathematical understanding

The stories share efforts at the district and state levels as well as within schools or individual classrooms. Chapter 3 challenges us to consider broadening the purposes of learning mathematics. The stories in chapters 4–6 illustrate changes at the classroom and school levels. Chapters 7–10 share district-level stories of change. Finally, chapters 11–13 describe statewide initiatives. We encourage you to use these stories as a lens to interrogate your own situation with more critical eyes and to inspire new possibilities toward creating just, equitable, and inclusive mathematics education systems.

To support your learning as you read the stories of transformation in this book, we offer the following questions for your consideration. These questions are intended to support you in reflecting on your own situation or as prompts for critical conversations with colleagues, community, and stakeholders in PK–12 mathematics. We encourage you to download the more4U resource that includes this set of questions and use it as a bookmark to prompt your reflections and discussions.

Reflection and Discussion Questions—Key Recommendations

- How are the key recommendations being used as a catalyst to initiate critical conversations and make plans for long-term change?
- How are the broader purposes of learning mathematics being communicated to teachers, students, families, administrators, and policymakers?
- Which inequitable structures in mathematics are in the process of being dismantled?
- What professional learning opportunities are supporting teachers in studying and implementing research-informed and equitable mathematics teaching practices?
- How are schools prioritizing the development of students' positive mathematical identities and strong sense of agency?
- What are some ways schools, districts, or states are confronting and disrupting the rush to algorithms in the elementary graders, the rush to algebra in middle school, or the rush to calculus in high school?
- How are schools engaging in conversations about the overproceduralization of mathematics and reframing the equity agenda to focus on deep mathematical understanding, reasoning, and sense making?

Reflection and Discussion Questions—The Change Process

- Where is the mathematics initiative along the continuum of change—starting, progressing, strong implementation?
- What conditions in the classroom, school, district, or state necessitated the need for change in the mathematics education system?
- How did the mathematics improvement effort obtain stakeholder buy-in for the change?
- What key challenges had to be faced and addressed on the path toward change?
- What key action steps propelled and supported the change efforts?

- What insights can you gain about the effectiveness of the change toward improved mathematics teaching and the dismantling of inequitable structures?
- How is the change creating a more just, equitable, and inclusive mathematics education for each and every student?
- How does the story relate to your own setting? What action steps might you begin to take toward improving your mathematics education system?

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Points of View: Making Space for Wonder, Joy, and Deeper Understanding in Mathematics

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Why should a student learn mathematics? If you say, "because math is useful," most people won't disagree with you. Mathematics has helped humankind build pyramids, navigate the seas, conduct commerce, and set foot on the moon. These days, the many technological gizmos we rely on in all areas of our lives were made possible by mathematical calculations and innovations. Society derives many benefits from the applications of mathematics.

However, on a *personal* level, we don't need to understand mathematics to use those gizmos. We are content to use many inventions in our lives without knowing exactly how they work. Although it is true that a solid mathematics background is a prerequisite for highly technical scientific professions, our students may never regularly use the advanced mathematics they learn in high school. So, what might inspire a student to master a technically difficult subject? In an activity where struggling is the norm, what is it that will keep them coming back for more? What personal reasons might motivate a student to learn mathematics?

Consider this analogy. To master a sport, like tennis, you have to spend many hours in training, practicing your forehand or your serve. Although the long-term benefits of regular tennis—such as staying in tip-top physical shape—might be compelling enough for some to endure the training, one often needs some short-term inspiration as well, such as the joy of playing the game and the thrilling suspense of a particularly long rally. This is the wonder and the joy of tennis.

Similarly, in mathematics, we can hope that some will find mathematics compelling for its long-term benefits. I've argued elsewhere (Su 2020) that mathematical training can and should build many virtues —like persistence and curiosity and habits of generalization—that are far more important than specific skills, like factoring a quadratic or taking a derivative. Such virtues serve our students well the rest of their lives, whatever their profession or personal circumstance. If we, as teachers, can help students see that growth in these virtues is bona fide evidence of their growth in mathematics, students will find the long-term benefits of mathematics compelling.

But what about short-term inspiration? Where is the wonder and joy of mathematics to be found? This is a pressing question, because tending to the wonder and joy of mathematics can be easy to forget in the drive to master skills. Yet, wonder and joy appear in the first key recommendation of each of the *Catalyzing Change* books (NCTM 2018, 2020a, 2020b). For instance, *Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations* (NCTM 2020b, p. 7) recommends:

> Each and every student should develop deep mathematical understanding, understand and critique the world through mathematics, and experience the wonder, joy, and beauty of mathematics, which all contribute to a positive mathematical identity.

The other Catalyzing Change volumes echo this call, which is a broader set of purposes of mathematics than one often hears in common discourse about why students should learn mathematics. Moreover, including joy, wonder, and beauty as explicit goals also provides another route to cultivate positive mathematical identities.

In this chapter, I'll focus on one aspect that I find prominent in many experiences of mathematical wonder: the ability to see an idea from many points of view. Looking at a mathematical idea from multiple points of view is deeply tied to the idea of using multiple representations to help students build their understanding, which is identified as one of the eight Mathematics Teaching Practices in NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014). This idea of connecting representations runs throughout many of the Essential Concepts identified by *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM 2018), that links using multiple representations to equitable teaching through building identity, agency, and competence by drawing on multiple sources of knowledge.

Throughout the chapter, I discuss how points of view can evoke wonder, build connections, and spark joy.

POINTS OF VIEW THAT EVOKE WONDER

Consider the famous optical illusion in figure 3.1: an image which, depending on how you look at it, might appear to be a young lady or an old lady (Hill 1915). Which do you see?

Once you learn that there are two ways to view it, you'll try to look for both—and when you succeed in seeing them, you can't help but mentally shift back and forth between them! This gravitational tug shows that your mind enjoys dwelling on a mental correspondence between the elements of both views. For instance, the ear of the young lady is the eye of the old lady, and the mouth of the old lady is the necklace of the young lady.

Let's think a minute about what makes this illusion fun. There is a paradox of how seeing a picture one way could be so different from looking at it another way. There are the mental gymnastics you engage in by going back and forth between the two points of view. Maybe you favor one view, and you find the other one hard to see. You're almost compelled to try to reconcile them, to take in both views at the same time. This is a source of delight, but also a challenge.

FIG. 3.1. A YOUNG LADY OR AN OLD LADY

FIG. 3.2. ART BY SCOTT KIM

Optical illusions aren't the only place a shift of perspective can generate wonder. Take a look at the piece of art in figure 3.2 created by the artist Scott Kim. It appears to be a stylistic rendition of the name of my institution: Harvey Mudd College. But when viewed upside down (try it!) you'll see it spells out a completely different word. (You can find plenty more like this in Scott Kim's [1981] book *Inversions*.)

Such a figure is called an *ambigram*, and it evokes awe. It makes you wonder: How is this even possible? It compels you to study it for a while, to see how each letter maps to another letter (or letters) when inverted. You're motivated to ask: What made this ambigram work? You might even be prompted to be creative and create your own ambigram with another word pair. Many artists rely on shifting points of view to entertain or inspire. M.C. Escher's visual illusions often rely on shifting the primacy of foreground and background. The organization Alaska Geographic (2020) inspires us to view Alaska's public lands as sanctuary for both polar bear and raven by incorporating both animals into its logo—by focusing on either black or white, one sees either the raven or the bear. Photographer Thandiwe Muriu showcases patterns from Nairobi's fabric shops with fashion models who are simultaneously both hidden and highlighted, depending on your point of view (Zane 2021).

The delight of shifting points of view isn't always visual. This riddle is one of my favorite jokes.

Q. Where does a king keep his armies?

A. In his sleevies!

The answer is a pun, and what makes a pun amusing is the sudden shift in our point of view. In mathematics, the delight of shifting points of view is a *motif* that runs through mathematics at every level, from elementary school to high school to college and through research mathematics. I'll show you several examples from the K–12 curriculum that highlight the ways we can emphasize this motif to promote joy and wonder in student learning.

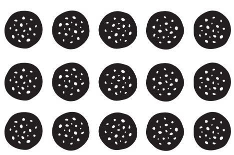
POINTS OF VIEW THAT MAKE AN EQUIVALENCE OBVIOUS

When you first learn to add numbers in elementary school, it may not be obvious that adding 3 more things to 5 things is the same as adding 5 more things to 3 things. But if you look at this grouping of cookies both forward and backward

you'll see that the forward perspective may be viewed as 5 + 3 and the backward perspective can be viewed as 3 + 5

so that indeed 3 + 5 must be equal to 5 + 3. An analogous shift of perspective enables you to see $a + b = b + a$ for any two whole numbers a and b.

Similarly, we may observe that a grid of 3 rows with 5 cookies in each row



when turned on its side, is a grid of 5 rows of 3 cookies each. Thus $3 \times 5 = 5 \times 3$. An analogous rotation of an $m \times n$ grid will show that $m \times n = n \times m$ for any whole numbers m and n. So, the commutative properties of addition and multiplication are not arbitrary rules that someone made up for us to follow; rather, they are natural mathematical laws the world participates in that we can discover. They are consequences of looking at arrangements of cookies in two different ways.

How do you get your students to experience the delight of discovering the commutative property for themselves? Draw a rectangular array of cookies and ask them to interpret it as a multiplication. Then turn it on its side and ask them to interpret it again.

To take it one step further, once your students understand these examples, see if your students can find a convincing visual demonstration of this instance of the distributive property: $3 \times (4 + 5) = (3 \times 4) + (3 \times 5)$. Then ask them to formulate a convincing argument that works more generally.

The idea that you could see an equality by counting a set of objects two different ways is an important idea even at higher levels of counting. For instance, when students learn about combinations and permutations, students may enjoy seeing that the number of ways to choose a group of 3 people from 8 people must be equal to the number of ways to choose a group of 5 people from 8 people, simply by reversing who is viewed to be in the group and who is viewed to be outside the group. This beautiful idea shows an equivalence even without resorting to the formula for permutations (that involves factorials). Such an argument is called a *combinatorial proof* and it is the backbone of a college-level discrete mathematics course.

POINTS OF VIEW THAT MAKE DEEPER CONNECTIONS

The study of fractions is rife with opportunities to shift our vantage point. Helping students see that $\frac{3}{6}$ equals $\frac{1}{2}$ is really just changing points of view. A cake cut into 6 pieces, with the first 3 pieces frosted, represents $\frac{3}{6}$ of the cake frosted.

But I can also group the pieces into two portions with three pieces each.

from which we see one of the two portions is frosted, which is half the cake. By looking at a frosted cake from two different vantage points, we grasp the equivalence of fractions.

But this cake example provides another pair of viewpoints. This geometric way of understanding fraction equivalence, grouping pieces together, seems quite different from the algebraic way that we determine equivalence by factoring and simplifying, that is,

$$\frac{3}{6} = \frac{3 \times 1}{3 \times 2} = \frac{1}{2}.$$

How are the algebraic and geometric points of view related? Making that connection is an opportunity for students to experience deeper understanding.