
**A collaborative classroom, an open-ended problem, and a what-how-who structure can build students' reasoning skills and allow teachers to recognize all classroom contributions.**

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# Who: Developing Mathematical Discourse

Kyra and Jamea closely examine the numbers and letters in four rectangles (see **fig. 1**). They point to different areas, share what they notice, and—without prompting from their seventh-grade teacher, Ms. Boyana—make conjectures about what their observations mean.

Kyra: The Q just has a letter and no numbers. All the others have letters and numbers.

Jamea: Maybe it means there is just one. But, there is a 1 next to the N in the other square.

Kyra: Maybe because it has other letters and numbers in it?

Jamea: Maybe Q means "questions" because we do questions in here and P means "people." Do we have 25 people?

Kyra: [Counting] No, we only have 19.

Jamea: Maybe it's days for D and nights for N. Like one of these is 2 days and 1 night and another one is 6 nights.

Kyra: But that doesn't make sense. Q doesn't fit. What other things start with Q? Queen? Quiz? Question? Quarter? Quail?

Jamea: Wait! Quarter? A quarter is 25 cents! But, that doesn't fit. There's no C.

Kyra: [Takes the paper] Cents are pennies, so maybe the P is pennies. Do you think the other ones are money, too?

**Fig. 1** Students were asked to analyze these numbers and letters in the first “What Do You Notice?” task.

| | |
|---|---|
| Q | 2D 1N |
| 6N | 25P |

With an increased focus on using social discourse to enhance students’ mathematical thinking and reasoning (NCTM 2014, Staples and King 2017), teachers are looking for discussion strategies that encourage middle-level students to make sense of mathematical concepts. However, structuring these valuable discussions is complex. “Mathematical discourse should build on and honor student thinking, and provide students with opportunities to share ideas, clarify understandings, develop convincing arguments, and advance the mathematical learning of the entire class” (Smith, Steele, and Raith 2017, p. 123). In other words, teachers must carefully consider *what* tasks provide meaningful opportunities to explore ideas, generate hypotheses, and promote questions within a collaborative environment. Then, teachers need to consider *how* to structure the activity to encourage discussions and incorporate responses that contribute to understanding specific mathematical objectives. Additionally, teachers must select *who* will speak to “advance the mathematical storyline of the lesson” (NCTM 2014, p. 30). By intentionally focusing on these elements in mathematics instruction, middle-grades teachers can develop a classroom culture that not only emphasizes sense making but also values the intellectual capacity that students bring to the classroom (Gutiérrez 2013; Lemons-Smith 2008). In this article, we describe how teachers can promote meaningful discussions using the what-how-who structure while giving students opportunities to make sense of mathematical ideas within a social context.

## WHAT TASKS STIMULATE MEANINGFUL DISCOURSE?

One of the greatest contributions to students’ opportunity to learn is the selection of tasks (Lappan and Briars 1995). Mathematics teachers must analyze the standards to determine what content to teach and identify which tasks embody the desired content and skills because different tasks promote different kinds of thinking (Stein et al. 2000). Thus, to provide a strong foundation for what mathematics students will learn (Hiebert et al. 1997), it is imperative that teachers intentionally identify *what* tasks

(a) provide relevant connections to students’ funds of knowledge,
(b) stimulate meaningful opportunities to explore mathematical situations,
(c) encourage students to generate questions, and (d) promote sense making through collaborative discussions.

An open-ended task, such as “What Do You Notice?” discussed in the opening vignette, is a logical format for removing barriers (Sullivan 2003). By providing multiple entry points, each and every student can gain access to and engage in discussions of mathematical content. The task, adapted from Danielson’s (2016) book *Which One Doesn’t Belong?* prompts students to investigate the similarities and differences among each representation and discuss their observations in a social context. For example, after making several observations and hypotheses to make sense of the letters and numbers, Kyra and Jamea concluded that the initials related to money: Q represented quarters, P symbolized pennies, and the N and D represented nickels and dimes. Finding this connection

**Fig. 2** In this second “What Do You Notice?” task, only numbers appeared.

| | |
|---|---|
| 9 | 16 |
| 25 | 43 |

prompted the students to generate additional observations: “All the coins make 25 cents except this one; it’s 30” (i.e., 6N represented 6 nickels, or $0.30), “the quarter is the one that makes 25 cents with the least number of coins,” and “6 nickels is the only one with an even number of coins *and* an even number of cents.” Not all students identified that the letters represented coins and their values. Some students speculated that the letters were abbreviations, whereas others argued that they were variables. Some students also noted the lack of numbers in the Q frame or noticed the color of the coins—“pennies are the only [coins] that are not silver.”

Such open-ended tasks not only provide access for a diverse group of students to engage in conversation but also offer flexibility to teachers to adapt them to a range of mathematical concepts, such as number sense (see **fig. 2**) and geometry, and use them at different times. Implementing a task during the initial phase of a lesson can also serve as a formative assessment, review, or introduction to a new concept. For example, the opening task could be used to introduce the differences among coefficients, abbreviations, and variables; to show how variables represent quantities; and to solve simple equations (CCSSI 2010).

This example can also be extended into problem-solving explorations that can last the entire class period. “What Do You Notice?” can be extended to (a) justify an “imposter” by identifying which representation does not belong (Danielson 2016; Wyborney 2015) and (b) create new problems containing multiple “imposters.” When Boyana integrated an extension with her seventh graders using the numbers-only task (see **fig. 2**), new conversations and mathematical discussions emerged. Students discovered that there could be multiple reasons why different numbers did not belong. During small-group discussions, she encouraged continuing investigations, such as identifying how each number could be the “imposter,” which then generated additional questions referring to mathematical relationships (e.g., “Does 25 not fit because it’s the only number that can be represented with a single coin?” “Is it because 25 is the only number in the set that is a factor of 100?”). Structuring classroom activities using open-ended tasks provides a flexible foundation that can positively contribute to developing discussions that enhance students’ mathematical reasoning.

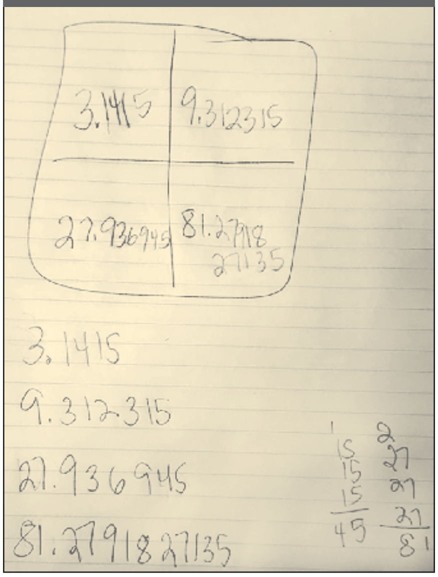
**Teachers must consider what tasks provide meaningful opportunities to explore ideas, generate hypotheses, and promote questions.**

## HOW DO WE STRUCTURE TASKS TO ENCOURAGE DISCOURSE?

The variable nature of open-ended tasks can stimulate mathematical conversations and allow students to negotiate a shared meaning and understanding of the mathematics. However, it is the teacher who takes an active role in purposefully facilitating activities to promote social discourse. Applying the “wonder” component to open-ended problems like “What Do You Notice?” serves as a pedagogical strategy. It can pique students’ curiosity and encourage new questions and inquiries as students make sense of representations without risk of failure. For example, when Boyana gave her students **figure 2**, she overheard them ask their partners: “Are these numbers supposed to all fit together?” “I wonder why they are all together. Do they follow a pattern?” She then asked her students to take two to three minutes to examine the four numbers closely and write down as many observations as they could. Once time was up, she asked the students to turn and share with a partner. Some students noticed that 9 was the only single-digit number and the only number with digits not totaling 7. They also noticed that 16 was the only even number. As students shared their observations, they clarified their thinking and generated questions to negotiate meaning of the mathematics embedded in the task.

Presenting tasks by first asking students to make individual observations allows them time to make sense of the representations. Following the individual reflection, students can discuss their observations with partners or in small groups. Students in Boyana’s class willingly shared their observations. By first prompting students to record what they noticed

**Fig. 3** A student generated this "What Do You Notice?" task.




about the numbers in the task, she provided a safe environment in which students were more comfortable sharing their thoughts without extensive pressure on identifying a solution. Approaching tasks by first making observations, then sharing what questions emerge encourages students to construct mathematical knowledge through social interactions with meaningful problems.

## WHO IS SPEAKING?

Such tasks as "What Do You Notice?" allow teachers to engage students in meaningful conversations that "develop language to express ideas, represent evidence, and clarify their reasoning" (Staples and King 2017, p. 38). Therefore, it is critical that each and every student is given an opportunity to engage in the classroom's social discourse. Without explicitly and purposefully attending to whose voice is represented in classroom conversations or valuing the out-of-school knowledge that students bring, teachers are not giving students the support, confidence, or opportunities necessary to reach their highest levels of mathematical success (NCTM 2014). Thus, teachers must be cognizant of who answers questions, solves tasks, or shares mathematical strategies while implementing instructional decisions that recognize a variety of students' contributions.

The design of the problem allows students to take risks because they recognize that all contributions are valued and that each and every voice is heard. While listening to her students' conversations during the number-only version of "What Do You Notice?" Boyana heard one student share that 16 was the age of her sister, and another student commented that 25 was his favorite number. At the other end of the room, she overheard another student wondering about the relationships among the numbers as he noticed characteristics such as 43 being the only prime number and 16 being the only even number. She asked each student to share one observation with the class. Sharing simple observations, such as "there are four numbers and they are all different," allowed students to feel more comfortable in the social setting because each response was valued. Boyana not only valued students' voices but also empowered students by exploring student-generated questions. As the class continued to share, one student noticed the equations $3 \times 3 = 9$, $4 \times 4 = 16$, and $5 \times 5 = 25$ on a peer's paper. She exclaimed, "I did that, too! Three squared, four squared, and five squared are all perfect squares! I wonder if 43 is a perfect square, too?" Boyana encouraged students to work in small groups to explore this question. Michalla stated, "Well, $6 \times 6$ is 36 and $7 \times 7$ is 49, so no. Forty-three can't be a perfect square." Boyana asked Stefan if he agreed with Michalla's conjecture. Stefan said yes. She continued to push Stefan's thinking and asked why he agreed. Stefan replied, "Well, 43 is between 36 and 49, and those numbers are perfect

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**She not only valued students' voices but also empowered students by exploring student-generated questions.**

squares. But, you would have to square a number between 6 and 7 to get 43." By first identifying and sharing what students noticed, and then exploring the questions students generated, Boyana provided valuable opportunities for students to negotiate meaning as they analyzed the reasonableness of different mathematical arguments (Staples and King 2017).

Finally, Boyana extended the activity by asking students to create their own "What Do You Notice?"

task to share with the class to further encourage and validate each and every student's voice.

J. T.: Remember when all of the digits added to 7? What if we make a grid where the digits add to a different number, like 9?
Christy: OK, what numbers will work? I like 18, 27, 63, and 90. But in the first problem, we did not have any three-digit numbers. What if we add 108?

The group initially decided to use the numbers 5, 18, 45, and 63 but were not satisfied with the task they created. Trevon commented, "This will be way too easy for them because they just did the other problem." The group finally decided to use pi because they thought it would be more challenging for their peers to figure out the pattern.

J. T.: So, pi is 3.14, right?
Christy: Remember, we should use 3.1415 [erases boxes and records].
Trevon: OK, what if for the number in the next box we multiply pi by 3?
Christy: No, let's make it harder. Let's multiply each digit by 3.
J.T.: I don't get it. What do you mean each digit?
Christy: Well, if we take 3 × 3, that is 9. Then 1 × 3 is 3; 4 × 3 is 12; 1 × 3 is 3; and 5 × 3 is 15. So, the next number would be [Christy writes and reads] 9.312315.
Trevon: That's cool. So, if we take 3 times each digit in that number Christy just said, we would get something like 27.936945.
J. T.: [Looks at Trevon's paper] I don't see how you got 45 at the end because 1 × 3 is 3, and 5 × 3 is 15; so, shouldn't it be 27.9369315?
Trevon: No, I like it where we take the last two digits and multiply them by 3 because then people will really have to wonder where we got this number.
Christy: Oooo, that sounds so good. OK, so, the last number would be 81.2791827135 [see **fig. 3**].
Trevon: Awesome. Ms. Boyana, we are ready!

Although this student-generated example demonstrated creativity using single-digit multiplication, it also caused confusion because students assumed the pattern followed the traditional multiplication algorithm. Because the example stumped the entire class, Boyana chose to discuss this task more in depth and asked students to analyze and make sense of the underlying mathematics.

Allowing students an opportunity to create their task can elicit richer conversations. Developing this culture of learning enhances sense making and motivates each and every student to remain engaged in creative brainstorming while discussing similarities, differences, and relationships among the observations.

## FINAL THOUGHTS

"Mathematical discourse is a critical practice through which students develop mathematical communication and argumentation skills and the ability to critique the reasoning of others" (Staples and King 2017, p. 37). Boyana developed a culture of discourse using the what-how-who structure by attending to "what" tasks she selected, "how" she structured the classroom conversations, and "whose" voice was heard during the discussion. Open-ended tasks, similar to "What Do You Notice?" allow teachers to facilitate mathematical discourse using the what-how-who structure and empower students to explore mathematical content within a social context. By integrating this structure into mathematical activities, middle-level teachers can build a classroom culture that not only emphasizes sense making but also recognizes the intellectual capacity that all students bring to the classroom (Gutiérrez 2013, Lemons-Smith 2008).

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### Let's Chat about Developing Mathematical Discourse

**On Wednesday, January 16, 2019, at 9:00 p.m. ET, we will expand on "What, How, Who: Developing Mathematical Discourse" (pp. 202–8), by Kelley Buchheister, Christa Jackson, and Cynthia E. Taylor. Join us at #MTMSchat.**

**Our monthly chats fall on the third Wednesday of the month.**




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