Student Explorations in Mathematics

7 Billion . . . and Growing

Students use data, measurement, unit analysis, percentages, and statistical analysis in this exploration of population growth. In the first task, Sharing Earth's Resources, students estimate the space needed for one person and calculate how much area the entire population needs. Further questions involve water and meat consumption. Unit analysis and percentages are needed to answer many questions.

Students explore the definition of growth rate and model population in the next tasks. Data are analyzed with both linear and—for older students—exponential models. Finally, students learn about alternative models for growth that take into account the carrying capacity for a population.

The NOVA website http://www.pbs.org/wgbh/nova/earth/global-population-growth.html depicts the growth of human population over time on an interactive map and is an excellent tool to use to start this exploration. Teachers should note that some of the wording on this website is ambiguous, but the reference is included here because it can be an aid for students as they consider their answers to questions 2–5. The teacher notes include solutions to an exponential equation using tables, graphs, and symbolic analysis.

TEACHER NOTES

Common Core Connections

Standards for Mathematical Practice (SMP) 4

Suggested answers are in **red**. Instructional notes are in **blue** and are preceded by the hand icon. These two features do not appear in the student edition.

Classroom Materials

- Metersticks or yardsticks, graphing device

Supplemental Material

- Student Edition

The *SEM* Editorial Panel acknowledges the ideas and contributions of Kate Remillard that led to the development of this exploration.

Extended Teacher Notes for 7 Billion . . . and Growing

Supplement to question 24, solutions to $9{,}000 = 2{,}619.05 \cdot 1.017^x$

1. Use a table. When x is between 70 and 75, $y = 9{,}000$. Since x = the number of years after 1950, the population will be 9 billion between 2020 and 2025.

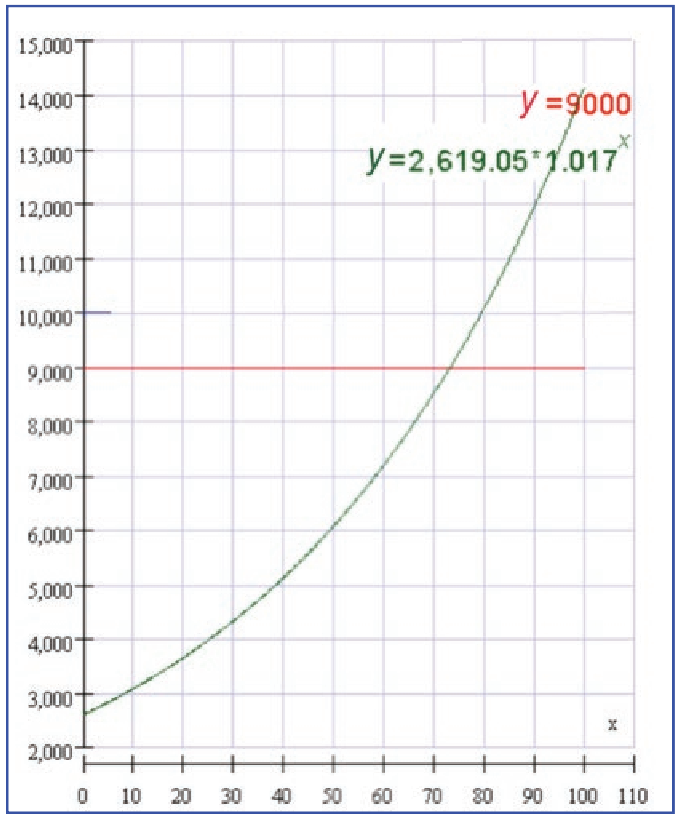
2D Functions

Remember to press return.

Min X: 0 X Step: 5

x	y
0	2619.05
5	2849.368
10	3099.94
15	3372.548
20	3669.128
25	3991.789
30	4342.825
35	4724.732
40	5140.222
45	5592.251
50	6084.031
55	6619.058
60	7201.135
65	7834.4
70	8523.353
75	9272.893
80	10088.347
85	10975.512
90	11940.694
95	12990.753
100	14133.154

2. Graph. One equation is $y = 9{,}000$. The second equation is $y = 2{,}619.05 \cdot 1.017^x$. The graphs intersect at $x = 73$.



3. Symbolic manipulation

$$9{,}000 = 2{,}619.05 \cdot 1.017^x$$

$$\frac{9{,}000}{2{,}619.05} = 1.017^x$$

$$\ln \frac{9{,}000}{2{,}619.05} = \ln\left(1.017^x\right) = x \ln(1.017)$$

$$x = \frac{\ln \dfrac{9{,}000}{2{,}619.05}}{\ln(1.017)} \approx 0.73$$

Student Explorations in Mathematics

7 Billion . . . and Growing

In 1804, the world population reached 1 billion people. In 1927, it was 2 billion. Although some of the following information may differ from source to source, the United States Census Bureau reported that the next milestones, 3, 4, 5, and 6 billion, occurred in 1959, 1974, 1987, and 1999, respectively (http://www.census.gov/population/international/data/worldpop/table_population.php).

In 2012, the world population reached 7 billion people! Demographers (people who study human populations) project that Earth will be home to 9 billion people by 2042. In this mathematical exploration, you will consider the implications of the current population growth for your lifetime. To see world population tracked in real time, visit http://ngm.nationalgeographic.com/7-billion.

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1. How old will you be in 2045? ________________

2045 – student's birth year

Note that the students in 2015 will be the world's experienced leaders in 2045.

Sharing Earth's Resources

A foremost concern of those who study population growth is "Where will everyone fit?" Suppose you wanted to gather all 7 billion people on Earth together in one place for a "get to know you" party. Where could you hold your party? Will your party be in a city? County? State? Country?

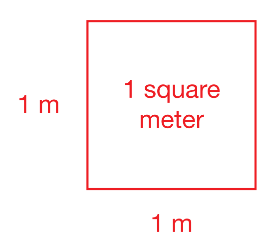
2. Take a guess: Where would be a reasonable location to hold a party for 7 billion people? Discuss your reasoning with your classmates.

Answers will vary. Accept all reasonable answers. For example, a school would be too small, but an entire continent will be too large.

Guests should not be crowded together. You will want each party guest to have enough "elbow room" to be comfortable. They will no doubt want to talk and joke with one another and maybe even do a little dancing.

3. Use a meterstick or yardstick to determine a reasonable amount of *area* for each guest. Sketch a rectangle to represent the space you are allotting per person at the party. Label the rectangle's length, width, and area.

Possible area



Students will need space to lay out possible rectangles. Consider going outside and using sidewalk chalk to draw rectangles.

4. Calculate the total area required for 7 billion party guests. Convert the units in your answer to square miles and round to the nearest mile.

Example: 7,000,000,000 × 1 square meter =
7,000,000,000 square meters

A square mile has 2,590,000 square meters, so 7,000,000,000 sq. meters ÷ 2,590,000 sq. meters per square mile ≈ 2,703 square miles.

The unit conversion may be challenging for students, particularly with the large numbers. To facilitate understanding, scaffold with smaller numbers and more easily understood units. For example, if you have 100 clowns, and cars that carry 25 clowns each, how many cars do you have? Some students might be ready for unit analysis and multiplication, but using division, which is an algorithm about sharing items, is a more natural approach.

5. Use the following websites or other resources to identify a suitable location for your party. (Conversions: 1 km = 0.6214 mile and 1 mile = 1.60934 km)

Areas by country: https://www.cia.gov/library/publications/the-world-factbook/rankorder/2147rank.html

Areas by U.S. state: http://www.worldatlas.com/aatlas/populations/usapoptable.htm

Areas by city: http://www.citymayors.com/statistics/largest-cities-area-125.html

Possible answer: Everyone could gather in Tokyo/Yokohama (2,700 square miles) or for a tight squeeze in Delaware (2,026 square miles).

6. To survive, each person will need more land than the area allotted for the party. Why is land necessary for use in sustaining life? What kind of issues does the increasing population present? What types of creative solutions are necessary?

Possible answer: We need land to grow food and a place to build shelter. However, the more people there are, the less land is available for each of these purposes. More people create more pollution, waste, and demand on natural resources. We will need to consider, for example, vertical structures and better food technologies.

7. No matter where or when you live on Earth, you have essential needs. Generate a list of what is absolutely needed to sustain life and discuss why each item on the list is essential for every one of the 7 billion people currently living on Earth.

Possible answers include clean air, water, food, shelter, energy.

The form of energy in use in 2045 may be different from the fossil-fuel–based energy sources that are used today. Consider a discussion with students about possible forms that energy could take in the future. Teachers may also use Maslow's Hierarchy of Needs chart to prompt discussion: (Physiological → Safety → Social → Esteem → Self-Actualization). See http://www.simplypsychology.org/maslow.html

WATER FOOTPRINT

All human activities use water: drinking, cooking, washing, and any form of production. The water footprint is a way of measuring direct and indirect water use.

The water footprint is the total volume of water used to produce the goods and services consumed by an individual, community, or business.

(Adapted from Food and Agriculture Organization of the United Nations 2012)

You probably considered that we will need land to grow crops to feed 7 billion people. You may also have discussed the need for every person to have access to clean drinking water. (In fact, each person needs 2–4 liters per day.) But did you ever consider how much water is needed to produce the food that you eat? You might be surprised to learn that producing one person's daily food requires between 2000 and 5000 liters of water!

The water footprint of animal food products is particularly high. Producing feed crops for livestock, slaughtering livestock, and processing meat and dairy products all require large quantities of water.

8. How many liters of water are needed to produce one person's food for one year if she uses 3500 liters every day?

$$3500\,\frac{\text{liters}}{\text{day}} \times 365\,\frac{\text{days}}{\text{year}} = 1,277,500\,\frac{\text{liters}}{\text{year}} \text{ for one person}$$

9. An Olympic-size swimming pool is 50 meters long, 25 meters wide, and 2 meters deep. How many liters of water can an Olympic-size swimming pool hold? (1 cubic meter = 1,000 liters).

$V = 2\text{ m} \times 25\text{ m} \times 50\text{ m} = 2,500\text{ m}^3$

$$\frac{2,500\text{ m}^3}{1} \times \frac{1,000\text{ liters}}{1\text{ m}^3} = 2,500,000\text{ liters}$$

10. Approximate what fraction of the pool's water is required to produce your food for one year. Ignoring evaporation tor the purpose of this question, project how many years' worth of food could be produced using one Olympic-size pool of water.

Half of an Olympic-size swimming pool would contain 1,250,000 liters. Thus, the yearly requirement of water (1,277,500 liters) is slightly more than half (50%) of the swimming pool. An Olympic-size swimming pool would supply the water required to produce a little less than 2 year's food supply for 1 person.

11. One Olympic-size swimming pool would supply enough water to produce food for how many people in one day? Round your answer to the hundreds place.

$$2{,}500{,}000 \text{ liters} \div 3{,}500 \frac{\text{liters}}{\text{person}} \approx 714 \text{ persons} \approx 700 \text{ persons}$$

12. Use the rounded answer to question 11 to estimate how many Olympic-size swimming pools are needed to supply water to produce food for the world's 7 billion people for one day.

$$7{,}000{,}000{,}000 \text{ persons} \div 700 \frac{\text{persons}}{1 \text{ pool}} \approx 10{,}000{,}000 \text{ pools}$$

13. Consider: It takes about 1500 liters of water to produce 1 kg of wheat and 10 times as much (15,000 liters) to produce 1 kg of beef. In the year 2000, Americans consumed, on average, 64.4 pounds of beef per person. How much water was required to produce the beef consumed by an average American in the year 2000?

1 kg ≈ 2.2 pounds

$$\frac{64.4 \text{ pounds}}{1} \times \frac{1 \text{ kg}}{2.2 \text{ pounds}} \approx 29.27 \text{ kg}$$

$$29.27 \text{ kg} \times \frac{15{,}000 \text{ liters}}{1 \text{ kg}} \approx 439{,}050 \text{ liters}$$

This exercise provides an opportunity for students to discuss putting an answer into a familiar context. For example, the amount of water required is slightly less than 1/5 of an Olympic-size swimming pool.

In 2014 the United States population was roughly 300,000,000. For real-time United States population estimates, visit the following "population clock" website: http://www.census.gov/popclock/

14. Determine what percentage of the world population lives in the United States.

$$\frac{300{,}000{,}000}{7{,}000{,}000{,}000} \approx 4.3\%$$

This exercise presents a nice opportunity to reinforce *percent* as a "number out of 100." Encourage students to think about this answer this way: If there were only 100 people on Earth, just 4 of them would be American.

15. In 2009, Americans consumed 12,239,000 metric tons of beef and veal. The overall world consumption for the same year was 56,668,000 metric tons. What percentage of all beef and veal did Americans consume in 2009?

$$\frac{12{,}239{,}000}{56{,}668{,}000} \approx 22\%$$

16. The latest data available on consumption of livestock products are in the table below. The average U.S consumption of livestock products is what percentage *more* than the world average?

Per capita consumption of livestock products, 2005	
Location	Consumption (kg/person/year)
World	41.2
United States	126.6

207%. U.S. consumption is 85.4 kg/person/year more than world consumption (126.6 – 41.2). As a percentage of world consumption, this is 85.4 ÷ 41.2 ≈ 2.07. American consumption of livestock is more than triple the world consumption of livestock.

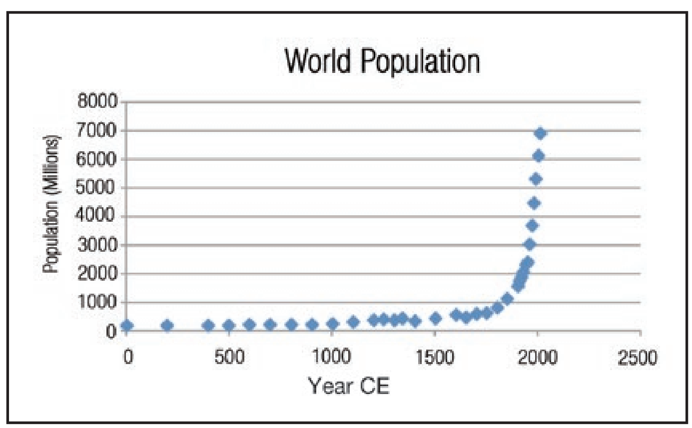
17. Why should we be aware of our water footprint? What can individuals, communities and nations do to decrease their water footprint?

Sample response: Americans make up just over 4% of the world population. Our consumption of beef is about 22% of the world's consumption. This suggests that the U.S. water footprint due to animal consumption is much larger than those in other parts of the world. Ideas to decrease water footprint will vary; for example, less reliance on animal food, more use of recycled water.

See http://www.unwater.org/wwd12/ for more information on water use and the UN World Water Day.

Here We Grow!

18. Study the World Population Growth graph below. Describe population growth over time. To what might you attribute the drastic growth in recent history?



Answers will vary. For example, increased health care has decreased infant mortality and extended average length of life. Access to safer water supplies and machines doing dangerous work once done by hand have also decreased deaths.

See chapter 3, "World Population Growth," in Soubbotina and Sheram (2000) for background on population growth: http://www.worldbank.org/depweb/english/beyond/beyondco/beg_03.pdf

Consider a class discussion comparing the U.S. growth to that of other countries. China has concerns about population growth and laws about limiting the number of children per family. Some areas of Africa have concerns about disease and health.

19. The growth rate of a population for a given time period is defined as the difference between the birth and death rates of the population. Birth and death rates are not the only factors that impact the overall growth of a population of a country. List other factors.

Immigration/emigration is an additional phenomenon that affects a country's growth rate. However, migration does not affect the overall world growth rate.

20. In the United States in 2010, there were 13.7 births per 1,000 people and 8.4 deaths per 1,000 people. What was the growth rate in the U.S. that year? Express your answer as a ratio and as a percentage.

$$\text{2010 Growth Rate} = \frac{13.7}{1{,}000} - \frac{8.4}{1{,}000} = \frac{5.3}{1{,}000} = 0.53\%$$

Help students understand the context of these numbers. During 2010, for every 1,000 people alive, 13.7 more were born. In 2010, for every 1,000 people alive, 8.4 people died. Finally, for every 1000 people that the U.S. already had in 2010 (because more people were born than died per 1000), the population increased by 5.3 persons. How does 0.53% compare to recent years? To 50 years ago?

21. Find the 2010 *growth* rates for each of the countries listed below. Compare and contrast the growth rates. Discuss the numbers in the context of your knowledge of world events, culture, and other factors.

China and India both have growing populations, with India growing faster than China. Russia has a declining population, indicated by the negative growth rate. The absolute value of Belgium's growth rate is relatively close to zero, indicating a relatively stable population.

You can obtain international birth and death rate information by visiting the U.S. Census Bureau: https://www.census.gov/population/international/data/idb/region.php?N=%20Results%20&T=7&A=separate&RT=0&Y=2010&R=-1&C=CH When selecting a report, choose "Components of Population Growth" from the drop-down menu.

22. The overall world population is increasing, but birth rates are actually decreasing. How is this possible?

Death rates are decreasing. People are living longer. Advancements in medicine and technology have contributed to both decreasing birth rates and decreasing death rates.

Where Do We Grow From Here?

Mathematical models help us represent real-world phenomena and make predictions about unknown outcomes. If a prediction does not come true, then we must both learn more about the phenomena and adjust the model to yield more accurate predictions. Mathematical modeling is used widely across many fields of study, including medicine, computer science, business, social sciences, economics, engineering, biology, chemistry, physics, and more. Mathematical modeling is an important tool in understanding population growth.

23. Population growth since 1950 has been the most dramatic in history. The table on the next page shows the population over this time period. Complete the table for "Years since 1950" and then do the following:

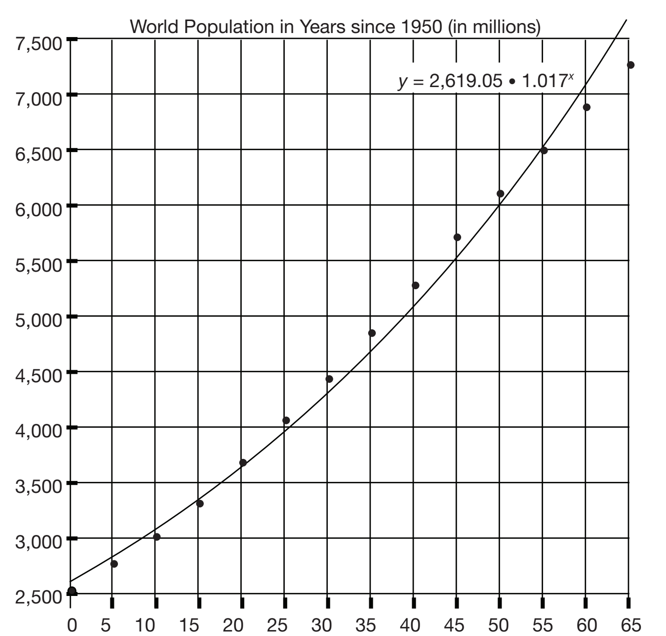
Country	Birth Rate (per 1000)	Death Rate (per 1000)	Growth Rate (expressed as a ratio)	Growth Rate (expressed as a percentage)
China	12.2	6.9	$\frac{5.3}{1{,}000}$	0.53%
Russia	11.1	16.0	$\frac{-4.9}{1{,}000}$	−0.49%
Belgium	10.1	10.5	$\frac{-0.4}{1{,}000}$	−0.04%
India	21.3	7.5	$\frac{13.8}{1{,}000}$	1.38%

- Create a scatterplot using the data.
- Find the equation of the line of best fit for the data.
- Find the exponential equation that best models the data.

Linear regression: $y = 75{,}658x + 2{,}298{,}793$

Year	Years since 1950 (x)	World Population (y)
1950	0	2,532,229
1955	5	2,772,882
1960	10	3,038,413
1965	15	3,333,007
1970	20	3,696,186
1975	25	4,076,419
1980	30	4,453,007
1985	35	4,863,290
1990	40	5,306,425
1995	45	5,726,239
2000	50	6,122,770
2005	55	6,506,649
2010	60	6,895,889
2015	65	7,284,296

Exponential regression, y-axis is in thousands of people



The data appear linear, so students who have not studied exponential growth will still be able to work on this question. Whether the students work on the exponential equation or not, the class could discuss why a linear model is only representative of the data for short periods of time. This is an opportunity to point out the similarities and differences between linear and exponential growth models, including the growth rates (75,658 people per year for the linear and 1.7% yearly for the exponential).

24. If the population continues growing according to this exponential model, when will the world population reach 9 billion?

Around 2023

This is an opportunity to introduce or discuss several solution methods, including using tables, graphs, and symbolic analysis. The teacher notes include details for these three methods.

The exponential model predicts a population of 9 billion much earlier than the demographers' estimation of 2045. Models for population growth must take into account an environment's carrying capacity, the maximum population the environment can support for indefinite periods of time. The rate of growth of a population decreases as the population approaches the carrying capacity of its environment. One of the factors affecting the carrying capacity is the consumption of resources. The more resources we consume per person, the lower the carrying capacity. Consuming fewer resources per person increases the maximum population that the environment can support.

Can you . . .

- find a reasonable place to host a party for the projected 9 billion people on earth in 2045? If you use the footprint of your school, how tall would the building need to be to fit everyone inside?

- investigate another growth model? In 1845, Belgian mathematician Pierre Verhulst developed the logistic model for population growth. Population growth is initially rapid and then gradually slows as it approaches a carrying capacity. For a video explaining more about logistic growth, explore the following website: http://www.bozemanscience.com/logistic-growth/

- make predictions about the distribution of wealth rather than population? View and analyze data from national and global statistics about social, economic, and environmental topics: http://www.gapminder.org/

- learn more about water use? Are we using more water than we are gaining? If so, when will we "run out"? Here is a website with rainfall information for several U.S. cities: average-rainfall-cities.findthebest.com

Did you know that . . .

- the Population Division of the United Nations Department of Economic and Social Affairs projects that the population will peak at 9.22 billion in 2075, dip slightly to 8.43 billion in 2175, and rise gradually to 8.97 billion by 2300? (United Nations 2004)
- *National Geographic* magazine did a special year-long series on population? To learn more, click here and be sure to watch the three-minute YouTube video: http://ngm.nationalgeographic.com/7-billion
- you can experiment with exponential and logistic growth by using an interactive applet that shows you the growth pictorially and graphically? Go to http://www.otherwise.com/population/
- innovation is necessary for confronting today's rapid population growth? Vertical farming is one such innovation being discussed. Click on the link below for a short YouTube video describing the concept: http://www.youtube.com/watch?v=1clRcxZS52s.

Sources

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Data Sources

World Population 1950–2050:

http://www.census.gov/population/international/data/worldpop/table_population.php

http://esa.un.org/unpd/wpp/index.htm

Historical Estimates of World Population:

http://www.census.gov/population/international/data/worldpop/table_history.php

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