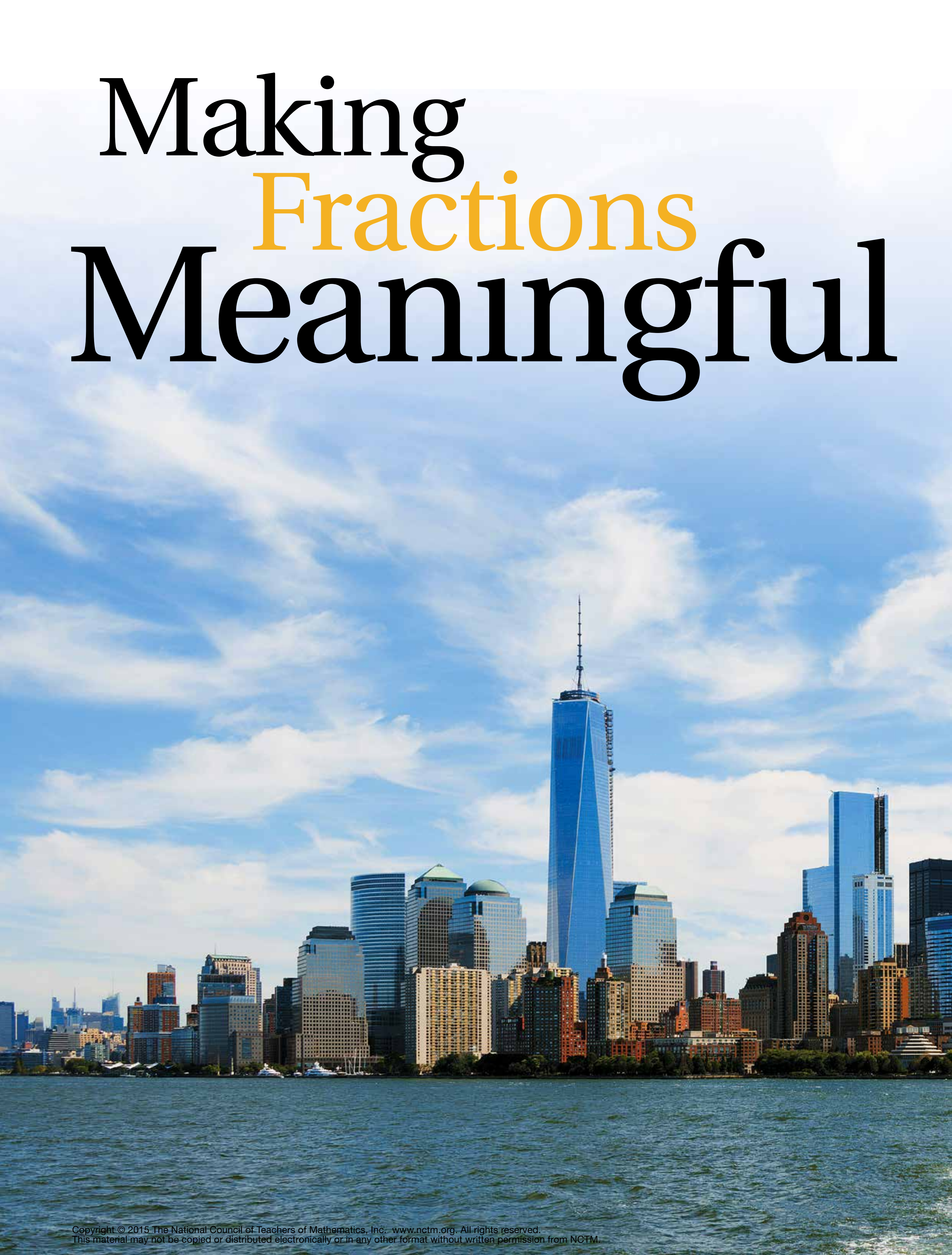
Making Fractions Meaningful



Sharing submarine sandwiches while touring New York City provides a powerful problem scenario for both elementary school students and preservice teachers to deepen their understanding of foundational concepts.

Kelly K. McCormick

Research and experience have shown us that we cannot afford to teach children the way we were taught; all children deserve to experience mathematics as a participatory sport, working hard to understand, represent, and make connections among significant concepts. To be able to support meaningful mathematical experiences, preservice elementary school teachers (PSTs) must learn mathematics in deep and meaningful ways (Ma 1999). They need to experience investigating and making sense of the mathematics they will be called on to teach. To expand their own—often limited—views of what it means to teach and learn elementary school mathematics, they also must observe and experience elementary school children investigating and making sense of important mathematical ideas.

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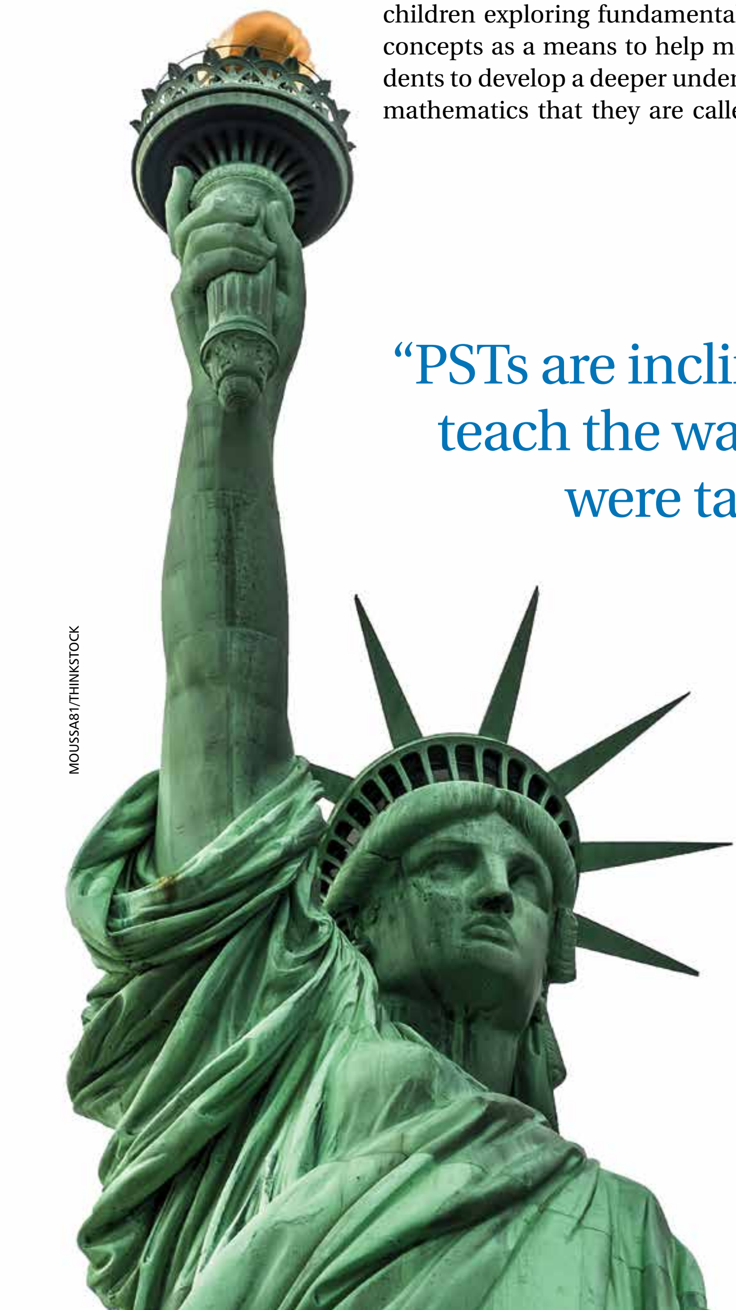
> Typically, PSTs believe that teaching elementary school mathematics is a process of transmitting, presenting information, or explaining clearly; information should be rigidly sequenced and structured; it is straightforward; and it is easy. (Chval et al. 2009, p. 99)

Filled with memories and past experiences, PSTs are inclined to teach the way they were taught (Ball 1990); therefore, teacher educators must take every opportunity to transform, nurture, and support PSTs' beliefs about mathematics teaching and learning.

In my content course for PSTs, I use videos of children exploring fundamental mathematical concepts as a means to help motivate my students to develop a deeper understanding of the mathematics that they are called on to teach. Moreover, the videos and the act of experiencing the same rich tasks and parallel discussions challenge and shape PSTs' beliefs about mathematics teaching and learning by situating their learning within contexts that occur in real elementary school classrooms (Lannin and Chval 2013). Because the goal of content courses is to build a deeper understanding of mathematics, providing opportunities for PSTs to investigate and make sense of fundamental mathematical ideas is a powerful way to challenge their knowledge, skills, attitudes, and beliefs, so that the next generation of children does not experience mathematics as a subject focused solely on memorization and rote learning of procedures (Chval et al. 2009).



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"PSTs are inclined to teach the way they were taught."

This article describes how I launch a PST content course using a video entitled *Sharing Submarine Sandwiches, Grades 5–8: A Context for Fractions* (Dolk and Fosnot 2006), in which fourth and fifth graders explore fundamental fraction concepts. Fractions are one of the most difficult topics in elementary school mathematics (Fazio and Siegler 2011). However, developing an "understanding of fractions as numbers" (CCSSI 2010, p. 22) and extending the "understanding of fraction equivalence and ordering" (CCSSI 2010, p. 28) are key concepts in elementary school mathematics, as Common Core State Standards for Mathematics (CCSSM) notes. CCSSM also states that this understanding should include the ability to compare fractions with the same numerator and recognize that comparisons are "valid only when two fractions refer to the same whole" (CCSSI 2010, p. 30). Teachers must be able to support this foundational understanding of fractions; however, most of the PSTs in my courses have limited and often erroneous knowledge of fractions.

Introducing the task

When I introduce the unit on fractions, I begin with a brief discussion about the meaning of the numerator and denominator. I ask my students for examples of how they have used fractions during the past week. The most common examples come from cooking or food (e.g., 2/3 cup of flour or 1/2 of a pizza). Unpacking their examples, we are able to come to a shared understanding of the meaning of the numerator and the denominator; that is, the numerator tells us the number of parts, and the denominator tells us what type of parts are being created (e.g., thirds or halves), or the number of equal parts that the whole has been divided into.

I ask my students to put themselves in the place of the fourth graders they are about to observe. I tell them that the teacher in the video, Ms. Carol Mosesson Teig, will introduce the task that they will investigate. Teig presents the problem context to her fourth- and fifth-grade class by telling about a field trip she took with her former class when she taught in New York City. She explains how her former students were upset after the trip because they believed that the way the submarine sandwiches were shared for lunch was unfair and that children in some groups were given more to eat than others. She explains how different groups of children went to different landmarks in New York City because they were studying the city as a class:

> Four students went to the Museum of Natural History, five went to the Museum of Modern Art (MOMA), eight went … to Ellis Island and the Statue of Liberty, and the five remaining students went to the Planetarium. (Fosnot and Dolk 2002, p. 2)

Teig describes how the school cafeteria staff had made seventeen submarine sandwiches for all the children to share for lunch, so she divided the sandwiches in the following way. She gave—

- three sandwiches to the four children going to the Museum of Natural History;
- four sandwiches to the five children going to the MOMA;
- seven sandwiches to the eight children going to Ellis Island and the Statue of Liberty; and
- three sandwiches to the five children going to the Planetarium.

Tieg then poses three questions for her current class to investigate:

- “Was the sharing of submarine sandwiches among the different groups of children fair?”
- “How much of a sandwich did each child in each group get?”
- “Which group got the most?”

“PSTs find that putting themselves in the place of fourth graders is difficult; observing the children’s initial discussion allows them to consider the children’s reasoning.”

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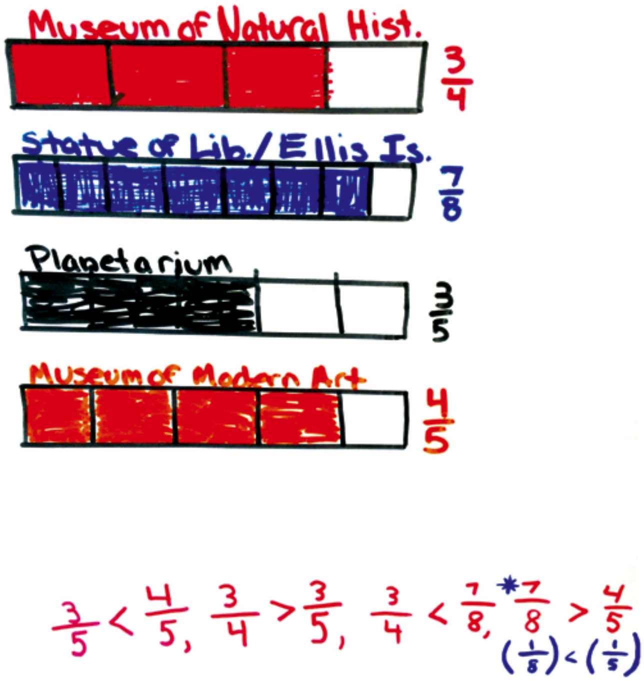
After Teig poses the first question, she gives her students time to reflect on the question and share their ideas with the child sitting next to them. In the video, the children then share their initial ideas and discuss their thoughts as a class. Because putting themselves in the place of fourth graders is difficult for the PSTs, observing the children’s initial discussion allows them to consider the children’s reasoning about the problem. Introducing the problem in this way allows the PSTs to observe an exemplar of an elementary school teacher posing a mathematically rich problem to her students. This gives my students the opportunity to observe how the problematic context engages the children. Similarly, introducing the task to the PSTs in this way also engages them in the task and the mathematics. Just as this experience fosters big ideas and supports the development of powerful mathematical strategies for the elementary school children (Fosnot and Dolk 2002), the experience supports the development of powerful ideas for the PSTs as well.

Before the PSTs observe the children’s methods of solving this task, the PSTs work on the task in groups of three or four students. On the

FIGURE 1

Although this problem context lends itself to visual models, comparing the fractions was the more challenging part of the task for the first group of preservice teachers, especially comparing 7/8 to 4/5.



board, I post the directions that are essential to making this task so powerful: "Remember to put yourself in the place of a fourth grader and use reasoning about the meaning of the numerator and denominator to help you solve this problem." I also remind my students that the submarine sandwich context and the problem are real to the students, and that, likewise, they should consider them real too when solving it. I explain that the fourth and fifth graders would want to know how to physically divide and share each sandwich. I tell them,

> Be sure to show and be ready to explain how you would literally divide or share the sandwiches for each group on the field trip. The elementary children would want to know this. To them, this is a real problem with real sandwiches that need to be shared among children.

Unlike the children, many of the PSTs want to begin to solve this problem using common denominators because this is the only way that they know how to compare fractions. I express to them,

> Even though this is a mathematically correct procedure, you should focus on the problem—sharing sandwiches—and also on making sense of the meaning of the numerator and denominator. Use the meaning of the numerator and denominator to help you think about the problem.

Other students go directly to a diagram of the sandwiches, try to visually cut the two-dimensional sandwiches into equal pieces, and compare the size of the pieces visually. I remind these students to also—

> use the meaning of the numerator and denominator to help make sense of your diagrams when comparing the fractions. The diagram and meaning should complement one another.

Sharing submarine sandwiches: The role of context

One of the powerful aspects of the context of sharing sandwiches is that it lends itself to being solved in multiple ways. While they solve this problem, I observe my students making sense of mathematics, conjecturing, finding patterns, communicating their mathematical ideas to one another, and using multiple representations to make sense of the mathematics. The context and structure of fair sharing and comparing are also powerful because they support understanding big ideas in elementary school mathematics, such as understanding fractions as division and the importance of the whole when comparing fractions (Fosnot and Dolk 2002).

Because this context lends itself to being modeled visually, students use diagrams to represent their thinking. The group of students whose work is represented in **figure 1** shared that each child in each group would get the majority of a sandwich; however, they also explained that "in each group, one poor child would get the ends of the sandwiches." They found that—

- the Museum of Natural History children would get 3/4 sandwich;

- the Statue of Liberty and Ellis Island children would get 7/8 sandwich;
- the Planetarium children would get 3/5 sandwich; and
- the MOMA children would get 4/5 sandwich.

Comparing the fractions was the more challenging part of the task for this group. As their diagram shows (see **fig. 1**), they began by comparing 3/5 to 4/5 and explained, "If your parts are the same size—fifths—you would want more of the parts: 4/5 of the sandwich, not 3/5."

They next noticed that two of the fractions shared the same numerators, 3/4 and 3/5, and reasoned,

> If one sandwich is being divided into four equal pieces and another is being divided into five equal pieces, the sandwich divided into four equal pieces will have bigger pieces. Since both groups get the same number of pieces, three pieces, the group getting 3/4 will get more than the one getting 3/5 of a sandwich.

They then compared the fractions 3/4 and 7/8 and said that they knew they could "cut each of the fourths in half and make eighths," so that they could view the part of the sandwich that was 3/4 of a sandwich as 6/8 of a sandwich. They "knew that 6/8 of a sandwich, or 3/4 of a sandwich, is less than 7/8, because if you have same-size pieces, the one with more pieces, or the greater numerator, is more."

And last, they noted that the most difficult comparison was between 7/8 and 4/5. One student in the group commented,

> We noticed that 7/8 was 1/8 away from a whole sandwich, and 4/5 was 1/5 away from a whole sandwich, so we compared the 1/8 to the 1/5. We knew that 1/5 is greater than 1/8, so that means that the group that got 7/8 of the sandwich, the Statue of Liberty and Ellis Island group, actually got more. Since 1/5 is greater than 1/8 and that is what was missing from a whole sandwich, the fraction with less missing is actually greater, or it is closer to the whole."

This "comparing the missing piece" as the class later came to call it, took some unpacking to do as a class. To help the rest of the class, these students introduced other contexts (e.g., sharing pizzas) and drew additional diagrams to make sense of this new method.

The second group of PSTs decided to use a number to represent each child in each group. For example, five children went to the Planetarium group, so they numbered the children 1, 2, 3, 4, and 5. Each child would get one piece of each of the subs that they shared, so children in the Planetarium group would get three 1/5 of a sub, or 3/5 of a sub" (see **fig. 2**). These students compared the fractions in a fairly similar

FIGURE 2

The second group of preservice teachers decided to represent each child in each group with a number, comparing fractions in a manner fairly similar to the first group's method. They were also excited to share that they had discovered a pattern and were later delighted to see that the children had discovered the same pattern.

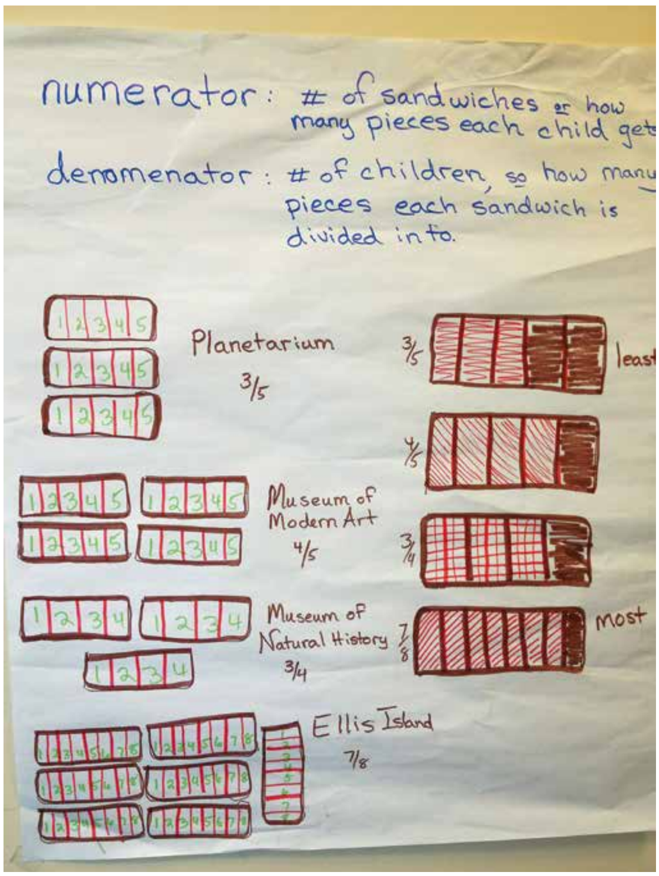


FIGURE 3

Another group decided to give each child half of a whole sandwich and "didn't compare the halves. . . . just compared 'the extra' that everyone got."

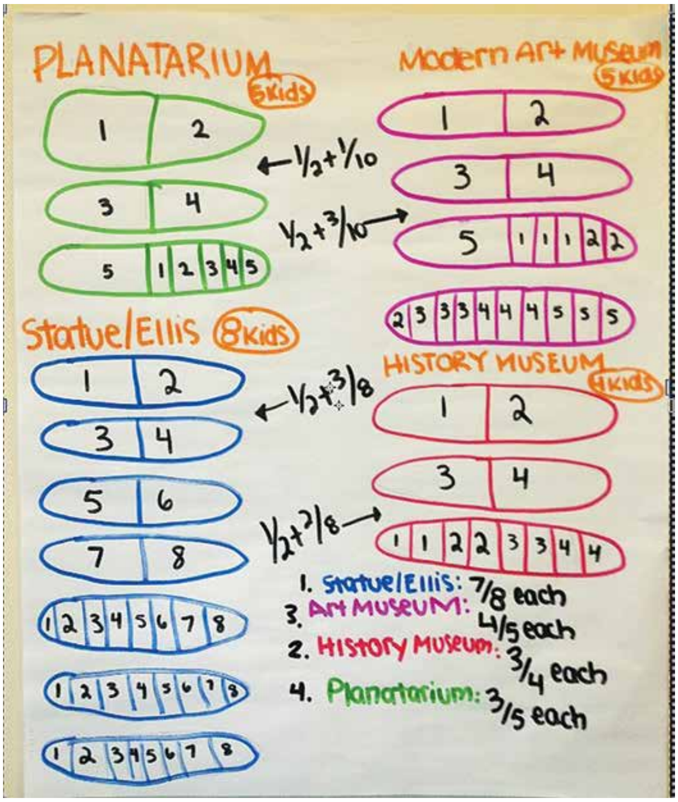
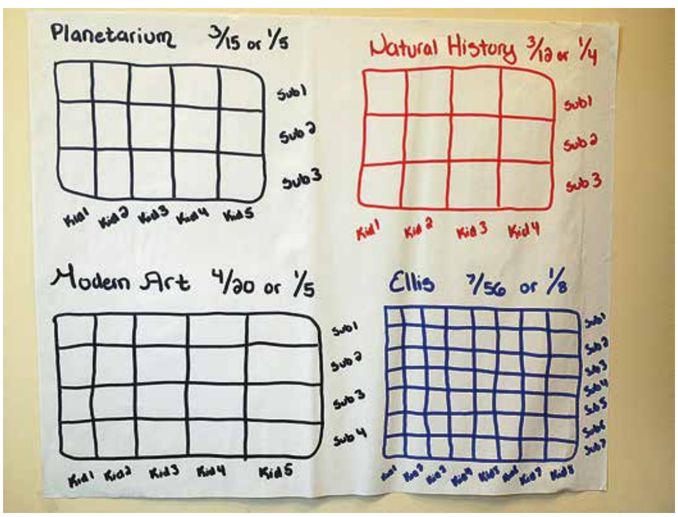


FIGURE 4

After listening to other adults discuss their work, PSTs in the final group appeared confused. Prompted to explain their work, they claimed how it helped them to "put all the subs in a group together." Their peers recognized that the wholes were not the same, a necessity for comparing fractions.



way to the previous group's method but were also excited to share that they had discovered a pattern:

> The numerator was the number of sandwiches—or how many pieces each of the children gets—and the denominator is the number of children—or the number of pieces each sandwich is divided into.

Later, observing the fourth and fifth graders discussing their work, the PSTs were delighted to see that the elementary school children discovered the same pattern; that is, both the children and the PSTs were beginning to see—

> the connection between multiplication (3 × 1/5) and division (3/5). . . . This is a big idea on the landscape of learning for fractions, decimals, and percents. (Hersch, Fosnot, and Cameron 2006, p. 20)

Another group decided to give each child 1/2 of a whole sandwich (see **fig. 3**). One student explained, "We wanted everyone to have half of a sub, instead of just lots of little pieces; so we cut them in half and then divided up the rest, so everyone got half and then some more." They also said that they "gave each child in each group a different number."

One member of this group stated, "We didn't compare the halves. We just compared 'the extra' that everyone got. For both the Planetarium and the MOMA groups, we had to divide a half into five equal parts, which gave us tenths." They "knew that 3/10 is greater than 1/10 and 3/8 is greater than 2/8." They then compared 3/10 to 3/8. Even though they had never been taught a method for comparing fractions with common numerators, as with the previous groups, they were able to reason that if "we have the same number of pieces, we just compare the size of the pieces, and we know that eighths are greater than tenths, so 3/8 is greater than 3/10." The experience of constructing a new method—using common numerators—to compare fractions empowered the PSTs.

After listening to the other students discuss their work, the PSTs in the final group appeared confused and stated, "Our fractions are different. We got—

- 3/15 or 1/5;
- 3/12 or 1/4;
- 4/20 or 1/5;
- 7/56 or 1/8."

When prompted to explain their work and how they shared the sandwiches among the children, they explained how it helped them to "put all of the subs in a group together" (see **fig. 4**).

"There were five kids in the Planetarium group, so we had each one getting 3/15, or 1/5 of the group's subs. The other groups [in our class] got 3/5."

Although their reasoning seemed to make sense to them, I gave everyone in the class time to consider why the "fractions are different." As Emily, a student in a different group explained, "The wholes are different. The wholes are not the same."

Meredith, who was also in Emily's group, followed up: "Yes, our wholes are one sandwich. You have a different whole for each group. Your 1/5 for the Planetarium is not the same as your 1/5 for the MOMA." She added, "You can't put your subs together. You need the same whole when comparing fractions. Your whole should be the same for each group if you want to compare them."

This was a mathematically momentous time for our class that we often referred to later in the unit when we needed to be reminded of the importance of having the same whole when comparing fractions.

Making mathematics meaningful

Investigating the Equal Sharing problem, observing the children solve the same rich task, and experiencing similar discussions shaped the PSTs' knowledge and beliefs. For example, one of my adult students commented, "This is the first time fractions ever made sense to me."

Several students agreed, and one stated, "I wish I was taught about fractions this way when I was in elementary school."

When asked why this was such a powerful experience, a student explained that both the children and the PSTs worked with their respective peers to investigate this challenging, engaging task and to build on their previous knowledge to construct these new methods for comparing fractions.

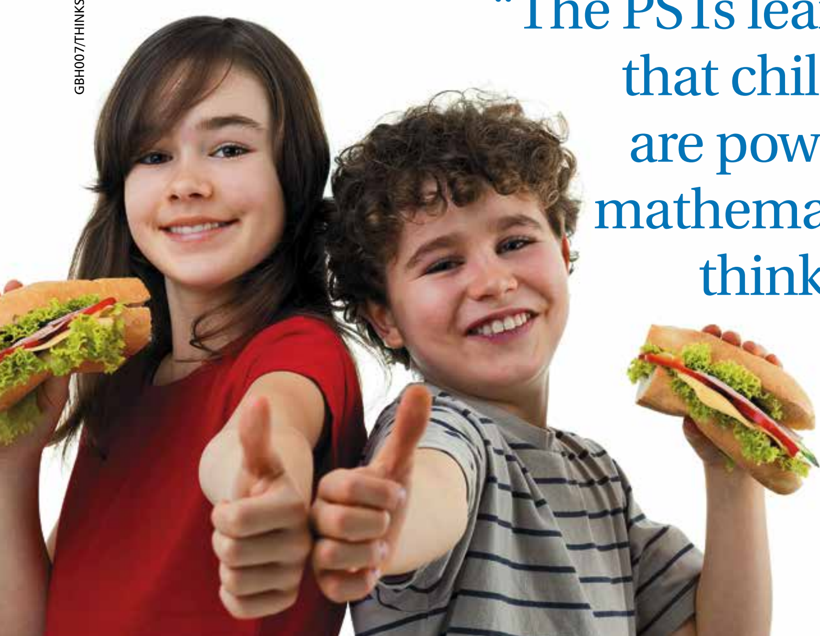
Another noted, "The children learned much more than if the teacher had simply shown them the new procedures."

The PSTs were able to construct the new mathematics and deepen their understanding of fractions in the process. They experienced the powerful mathematical learning that takes place when one is actively involved in a community of learners investigating and making sense of important mathematical ideas. Moreover, this experience also shaped the PSTs' beliefs in positive ways about children's ability to do mathematics. They learned that children are powerful mathematical thinkers.

PSTs need numerous experiences, such as the one described in this article, to help shape their strong beliefs about teaching and learning mathematics. To change the detrimental beliefs they hold, they must learn mathematics in deep and meaningful ways. Through such investigations as this one, PSTs learn what it means to do mathematics; and that is, to investigate, conjecture, make sense, represent, communicate, reason, make connections, generalize, and solve cognitively demanding problems. They must understand that teaching mathematics is *not* a process of transmitting knowledge, presenting information, or explaining clearly and should *not* be rigidly sequenced, structured, or straightforward, and that teaching elementary school mathematics is a cognitively demanding task; it is *not* easy (Chval et al. 2009, p. 99).

Common Core Connections
3.NF.A.3d
4.NF.A.2

"The PSTs learned that children are powerful mathematical thinkers."



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