Elementary Modeling: CONNECTING COUNTING WITH

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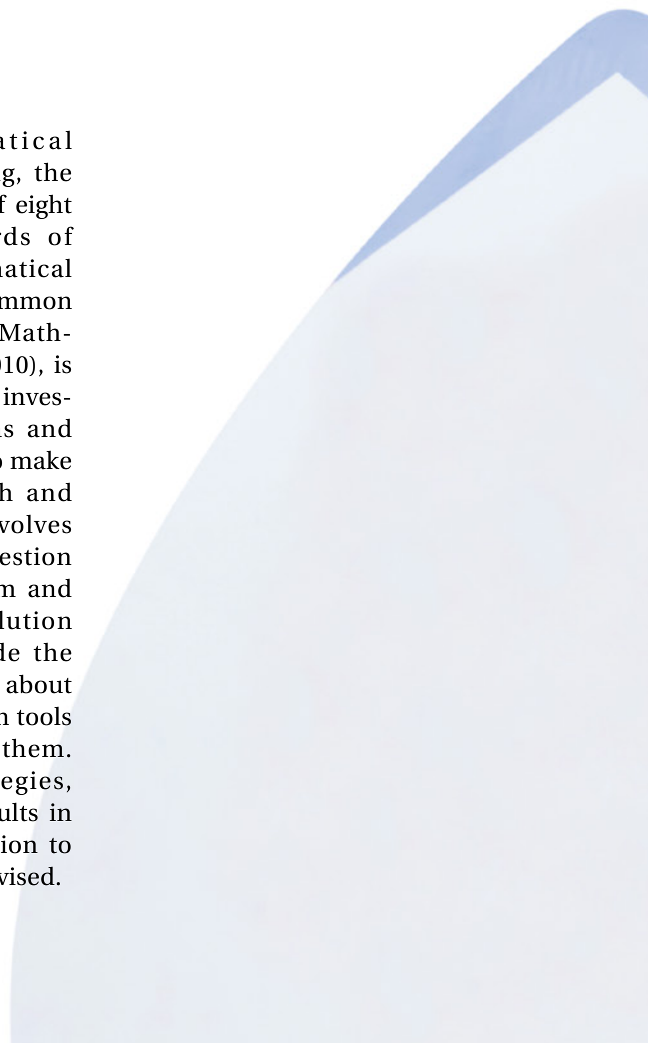
and Tracy Aytes

Examine second-grade students' investigative processes, thinking, and revisions in this lesson using **fish crackers**.

Mathematical modeling, the fourth of eight Standards of Mathematical Practice (SMP 4) in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), is a process in which students investigate important questions and use mathematics as a tool to make sense of a situation (Lesh and Doerr 2003). Modeling involves asking a mathematical question about a real-world problem and mapping out possible solution strategies. Students decide the assumptions they will make about the problem as well as which tools and methods might help them. As they implement strategies, students interpret their results in terms of the original question to see if strategies should be revised.



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SHARING

Although the process of mathematical modeling is often reserved for secondary and college students, researchers (Carlson et al. 2016) have recently proposed that engaging in the process of modeling is equally important for elementary school students. Mathematical modeling, at all grade levels, is of importance in mathematics education for several reasons. First, mathematical modeling builds proficiency. In describing modeling specifically, CCSSM states,

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. (CCSSI 2010, p. 7)

Problem posing is equally as important as problem solving.

The process of modeling involves making assumptions and approximations to reason about a situation, identifying relevant information, using tools to interpret data, and analyzing solutions through the lens of the real world (CCSSI 2010). Thinking critically, solving complex, open-ended problems, and being creative have all been highlighted as twenty-first-century skills that students must develop to become productive members of society (Partnership for 21st Century Skills 2008). Modeling addresses many of these skills through a mathematical lens.

In addition to building proficiency, mathematical modeling can empower students in multiple ways. By investigating problems of interest, students see mathematics as relevant and worthwhile (Greer, Vershchaffel, and Mukhopadhyay 2007). Allowing students to be creative in how they use mathematics to solve a problem (Lehrer and Schauble 2007) fosters ownership. Finally, when students present multiple solution strategies, mathematical modeling allows for the opportunity to engage in discussions and pose further questions or revisit the task through a different lens (Carlson et al. 2016). In this article, we elucidate a mathematical modeling task in a second-grade classroom. Through the Counting and Fair-Sharing task, we illustrate aspects of mathematical modeling that are beneficial to elementary school students in building mathematical proficiency.

Open-endedness and problem posing

The first step in the modeling process is translating a real-world situation to a mathematical problem or posing a potential question to investigate (Hansen and Hana 2015). Because mathematical modeling in the elementary school setting is relatively new, few resources document modeling tasks for young learners. We suggest looking for problems that interest students and arise organically in the classroom because they hold meaning for students. In determining if a situation is appropriate, teachers may want to consider the following pedagogical questions drawn from Carlson et al. 2016:

- What mathematical content tools have these children developed?
- What mathematical process tools could these children access and use as they engage in mathematical modeling?
- What settings are interesting and accessible to all students?
- What might students do (mathematically) as they engage in the modeling process?
- What mathematical understanding and insights might emerge as students engage in the modeling process?

A key feature of the modeling process is to present students with a situation and allow them to pose questions. Problem posing is equally as important as problem solving (Cai et al. 2015) and has been shown to help students become better problem solvers (Perez 1985). When using mathematics in their daily lives, students will not always be presented with clean-cut problems to solve. Instead, when faced with a dilemma, they must be able to use reasoning to see if and how mathematics can be applied. Furthermore, for teachers, problem posing can act as a window into students' thinking. It reveals what they might wonder about and what they find interesting. Teachers can use students' questions as an entry point into a mathematical task.

GARRETT AITKEN/THINKSTOCK

Fair sharing of goldfish

In some elementary school classrooms, students receive snacks during the school day. From students' perspectives, distributing, counting, and sharing snacks fairly is both an important and interesting situation. We began our modeling task by having students consider sharing a container of goldfish crackers. With the container at the front of the classroom, we asked the second graders what they wondered about the container. We rephrased some of what we heard, such as these student questions:

- How many goldfish crackers are there?
- How many of each color goldfish are there?
- Should we share the whole box, or are there too many to eat?
- How many goldfish will each of us receive for a snack?
- What will we do with the leftovers? (***Note:*** Students counted and handled goldfish crackers from one box, but they ate from a second box.)

When selecting a mathematical question, considering the content and process tools that students currently have and those that could be developed is important (Carlson et al. 2016). We thought any of the counting questions were accessible because students could add and subtract within 1000 (CCSS Math Content 2.NBT.B.7) and could add up to four two-digit numbers (2.NBT.B.6). They had also learned about rectangular arrays as an organizational structure to help them skip-count (2.OA.C.4, 2.NBT.A.2). We considered questions involving sharing to be accessible; they could also foster future concepts surrounding multiplication and division.

After a whole-class discussion, students said that the two most important questions to them were (1) How many goldfish crackers are there? and (2) How many goldfish will each student receive? Part of the process of modeling is to describe constraints and assumptions about the situation before moving forward as a class. We asked students to describe what was important to them and what assumptions they

→ **reflect and discuss**

"Elementary Modeling: Connecting Counting with Sharing"

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions, related to "Elementary Modeling: Connecting Counting with Sharing," by Megan H. Wickstrom and Tracy Aytes, are suggested prompts to aid you in reflecting on the article and on how the authors' ideas might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

In the Common Core State Standards for Mathematics (CCSSM), mathematical modeling is identified as one of the Standards of Mathematical Practice (SMP) or "an expertise that mathematics educators at all levels should seek to develop in their students" (CCSSI 2010, p. 7).

1. What types of expertise did mathematical modeling help to foster in this activity?
2. Why is it important to allow students to pose their own mathematical questions?

In their article, Wickstrom and Aytes identified sharing crackers as a classroom scenario that would lend itself to problem posing and mathematical modeling.

3. List some scenarios in your classroom that might lend themselves to mathematical modeling.
4. What questions might students pose surrounding the scenarios you identified?
5. Why is problem posing a valuable practice for students to engage in?
6. How might you incorporate the practice of problem posing in your classroom?

When building a solution, mathematical modeling allows for creativity in thinking and the opportunity to make connections.

7. In what ways were the second-grade students creative in using mathematics?
8. How did students make connections?

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These second-grade students used an array to help them count.

could make when considering the problem. Students described several assumptions, such as, “It needs to be fair” and “We all should get the same.” As a class, we discussed the idea of fair sharing. One student explained fair sharing by stating, “If you and I share a candy bar, we should both get half. One of us shouldn’t get more.” The class agreed that all students should receive the same number of goldfish crackers.

One student asked, “What about the different colors?” Classmates agreed that color did not matter because all the crackers taste the same. Some students stated, “We can’t eat any (while counting).” Students explained that eating some would be unfair because it would alter the total number of crackers. We decided to challenge students to see if they could reason about both important questions at the same time. We posed the following questions:

- Can we count the total number of crackers and fair share at the same time?
- How would we do it?

Creativity and choice in building solutions

The next phase of the modeling process is to decide on the mathematics we might use to build a solution. Students are given the freedom to consider how they want to make sense of the problem. We gave the second graders several minutes to consider how to solve the problem and then had students share out loud. As students shared, two ideas emerged.

A member of one group of students stated, “We could have a helper pass them out and count together.” Everyone in this group wanted the teacher to assign one or two helper students to pass out the crackers to the whole class, one student at a time. They envisioned the helpers visiting each student as the class counted each cracker the helper distributed, one at a time.

A member of a different group stated, “We all want to help. Can we count and share at our tables?” These students proposed using a mug to give each group some crackers and letting tablemates count and fair share together. When we asked the class how we would know how many total crackers we had, one group suggested, “We could add [crackers from] all the tables together.” After both ideas were shared and explained, students discussed which approach they liked better and why. Pondering the first idea, students reasoned, “It might take a long time” and “We can’t all help.” Students chose the second idea so that they could all be helpers.

Student strategies

After groups received a mug full of fish crackers, they were allowed to count and fair share, at their table, in any way that made sense to them. We observed that students approached the task in different ways: Those at two of the tables fair shared first and then counted the total. At one table, each student grabbed a fish at the same time as the group counted chorally, “One, two, three, four, . . . ” until no more fish were left. In another group, a student stated, “I will be the leader and pass out the fish.” The leader distributed fish crackers to students one at a time.

As students shared the crackers, we asked them how they would determine how many they had as a group. Once the pile of crackers was exhausted, students wrote on their whiteboards how many they had and added the numbers together, sometimes using doubling strategies.

Students at the other two table groups counted all the fish first and then fair shared. Some children grabbed a handful and counted one at a time; others used arrays to help them organize and skip-count. Students often counted smaller portions—such as by color—and then combined the totals.

Evaluating and validating solutions

The last step in the modeling process, and often the most challenging, is determining whether the solution works to answer the mathematical question or whether revisions must be made. We found that determining how they might find the total number of goldfish was somewhat easy for students: "We can put our number on the board and add them together."

They decided that each table group could provide their total number, and as a class, we could add them to find the overall total. Students added two sets of numbers at a time and discussed the sum as a class to make sure they had added correctly. When all the fish crackers were added, we had 452.

Although distributing the fish crackers to each table was a good model to help count them efficiently, it was not a good model for fair sharing. Students in one group complained, "They (the other groups) have more fish than us."

Although they were shared fairly at the tables, the fish were not shared equally across the classroom. Students in group 1 (each group had four students) had four more fish each than students had in group 2 (see **table 1**).

At this point, we decided it would be appropriate for students to consider how they might remedy the situation so that everyone received the same amount without redistributing all the crackers. We asked, "How could we readjust without having to start over?" We gave groups several minutes to consider what they might do next, and then we discussed their ideas.

One student stated, "We could give fish to another group," so we decided to experiment. Members of group 1 (which had twenty-eight fish per student) decided they would each give one fish to a student in group 2 (each of whom had twenty-four fish) to see what would happen. They concluded, "Our number went down one, and their number went up one." They reported that students in group 1 now had twenty-seven fish each and students in group 2 had twenty-five fish each. They decided they could take another turn so that both groups would have twenty-six fish crackers per student. Groups 3 and 4 shared in a similar fashion until both groups had thirty fish per student.

Looking around, students stated, "We still do not all have the same." They realized that it was still unfair, so they brainstormed how to remedy the situation.

Members of group 1 stated, "(Group) 3 and (group) 4 have more than (group) 1 and (group) 2." Several students noticed that if students in group 3 and group 4 gave two of their fish to members of group 1 and group 2, then all students would have twenty-eight fish. Four fish

TABLE 1

Distributing fish crackers to each table was a good model to help students count them efficiently, but it was not a good model for fair sharing. Students shared the crackers fairly at their respective tables but not across the classroom.

A fair-sharing model

Group number	Fish per student	Leftover crackers
1	28	2
2	24	1
3	27	1
4	33	0

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Students often counted smaller portions and combined them. For example, these children sorted the goldfish crackers by color before they counted them.

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These second graders each grabbed a fish at the same time and counted chorally until no fish remained.

remained, so we asked students what we should do. A student suggested, “Four is not enough to share, so those fish will go to the two teachers.”

At this point, we decided to end the modeling experience because students not only had answered the question but also had modified their strategy and developed concepts surrounding giving, taking, and equality.

Conclusion

The Counting and Fair-Sharing modeling task highlights several features of the process of mathematical modeling that are important for elementary school students. First, math can come from simple, accessible, everyday experiences. Rather than having students solve problems, let them practice asking relevant, meaningful questions. By allowing our students to pose questions, we could see what they were naturally interested in (quantity and sharing), and we were able to gear our mathematical questions toward topics that were accessible to second-grade students.

Second, the modeling task allowed students to make connections between mathematical topics. Students were able to apply what they knew about addition and arrays (2.NBT.B.7, 2.NBT.B.6) to reason about the situation. With regard to the practice of modeling, CCSSM states,

> Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. (CCSSI 2010, p. 7)

Our students were comfortable articulating assumptions and reframing a classroom task through a mathematical lens. In addition, the teachers were interested to see how students used their creativity to build solutions in their own ways. Allowing creativity and choice sparked rich discussions as well as ownership of the mathematical strategies created because students saw their work as important.

Last, we found the process of mathematical modeling to be an ideal context to promote productive struggle. Mathematical modeling involves allowing students to—

> routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (CCSSI 2010, p. 7)

The motivation of solving the problem as well as being creative and making choices allowed for an environment in which students were comfortable grappling with mathematics and readjusting their methods when the initial solution was not ideal. In describing the process, a second-grade student stated, “That was really hard but really fun.” The practice of modeling fosters an environment where students are challenged to apply their mathematical content knowledge to investigate situations that are important to them.

Mathematical modeling is an important and accessible process for elementary school students because it allows them to use mathematics to engage with the world and consider if and when to use it to help them reason about a situation. It fosters productive struggle and twenty-first-century skills that will aid them throughout their lifetime.

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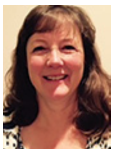

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