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Using Everyday Knowledge of Decimals to Enhance Understanding

Kathryn C. Irwin, *University of Auckland, New Zealand*

The study investigated the role of students' everyday knowledge of decimals in supporting the development of their knowledge of decimals. Sixteen students, ages 11 and 12, from a lower economic area, were asked to work in pairs (one member of each pair a more able student and one a less able student) to solve problems that tapped common misconceptions about decimal fractions. Half the pairs worked on problems presented in familiar contexts and half worked on problems presented without context. A comparison of pretest and posttest results revealed that students who worked on contextual problems made significantly more progress in their knowledge of decimals than did those who worked on noncontextual problems. Dialogues between pairs of students during problem solving were analyzed with respect to the arguments used. Results from this analysis suggested that greater reciprocity existed in the pairs working on the contextualized problems, partly because, for those problems, the less able students more commonly took advantage of their everyday knowledge of decimals. It is postulated that the students who solved contextualized problems were able to build scientific understanding of decimals by reflecting on their everyday knowledge as it pertained to the meaning of decimal numbers and the results of decimal calculations.

Key Words: Collaborative learning; Decimals; Discourse analysis; Intermediate/middle grades; Piaget; Problem solving; Situated cognition; Vygotsky

In 1887 Howard wrote, "The system of decimal fractions is so eminently simple that when it is generally understood will entirely displace the clumsy system of common fractions" (Kerslake, 1991). Despite this optimism, there is considerable evidence from studies involving both school students and adults that the system of decimal fractions is neither eminently simple to learn nor generally understood (e.g., Brown, 1981; Thipkong & Davis, 1991). Mathematical concepts that are not intuitive, such as decimal fractions, fit within Vygotsky's definition of scientific concepts (1987). Decimal fractions are usually taught at school, but they need to be anchored in some way to students' existing knowledge.

In the study reported here, I focused on New Zealand students from a lower economic area who were known to have more difficulty with decimals than students from more affluent areas had. I investigated whether the understanding

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of decimals held by this group of students could be improved by anchoring their understanding to their existing knowledge by asking them to solve problems set in everyday contexts. The study's research questions involved the role of context, the role of peer collaboration, and the role of cognitive conflict in helping students understand decimals. Specifically, I sought to discover whether working in pairs made up of a more competent student and a less competent student increased the students' understanding of decimal fractions following joint exploration of problems—all of which were designed to encourage cognitive conflict and half of which gave problems in context. This design takes advantage of the strengths of peer tutoring (e.g., Limbrick, McNaughton, & Glynn, 1985).

BACKGROUND

This study arose out of the finding from a previous study (Britt et al., 1993) that students from lower economic areas had more difficulty than did students from more affluent areas in understanding decimal fractions. In that study, not only did the students attending a school in a lower income area have a poorer understanding at the start of the school year, but they also made less progress during the year. For example, the percentage of 13-year-old students from one school in a lower income area who understood the decimal concept of "hundredths" or more complex decimal relationships was 22% at the start of the year and 32% at the end of the year, whereas the comparable percentages from a similar cohort from a school in a middle-income area were 62% and 93%.

Textbooks are not routinely used in New Zealand elementary schools in the hope that teachers will tie mathematics to students' everyday experience. It may have been that the poorer performance of students from lower income areas was related to the fact that they were less likely to share everyday mathematical contexts with their teacher than were students from more affluent areas. In the study reported in this article, the students not only came from a lower economic area but also were from many different ethnic groups and cultures, thus making it impossible for their teachers to share the cultures of all. Therefore, their teachers would have been less able to help students integrate their everyday knowledge with school mathematics, or scientific knowledge of mathematics as defined by Vygotsky (1987). This lack of integration would be particularly true at what Hiebert (1985) calls Site 1, the meaning of the numbers that are to be calculated (e.g., what the 0s and the 1 stand for in 0.01), and at Site 3, the meaning of the answer to a calculation (e.g., that 0.87 + 0.95 will be about 2). Site 2, the calculation itself, can usually be done procedurally—for example, by "lining up the decimal points" —and does not necessarily require everyday knowledge.

Resnick, Bill, Lesgold, and Leer (1991) suggested that children from minority cultures are less likely than students from the dominant culture to spontaneously use the knowledge they have learned outside school when learning new concepts in school. However, Resnick et al.'s successful educational program and the successful program reported by Boaler (1998) have demonstrated the value of integration of everyday and school or scientific knowledge for students from lower economic areas. Other authors (e.g., Jones, 1991; Lubienski, 2000) have shown that students from lower economic areas may resist a pedagogy that is based on integration of school and everyday knowledge, and that resistance may be related to the fact that the specific problems or tasks that these students are asked to solve do not relate directly to their own experience. The study reported here was designed on the premise that the usefulness of everyday contexts depends on the appropriateness of such contexts for that particular group of students and that problems from textbooks may not be appropriate for students like those.

However, the integration of everyday knowledge and school mathematics can be problematic for reasons that have to do with the context of the problems that students are asked to solve. On the one hand, when the problems are too closely tied to students' lives, then factors other than mathematical ones can determine how problems are solved (see Lubienski, 2000). On the other hand, when the problems are too distant from their experience, students either fail to associate problems with mathematics that they know or they apply known mathematical skills without considering the appropriateness of the answer (e.g., Greer, 1987; Silver, Shapiro, & Deutsch, 1993).

Determining appropriate decimal problem contexts that were neither too close to nor too distant from the experiences of students from this part of New Zealand was an important consideration as I designed the study reported in this article. To do this, I interviewed 84 students from four schools in lower economic areas of Auckland, New Zealand, to find contexts for decimal fractions that were familiar to them and to explore their knowledge about decimals. Results of these interviews are reported in Irwin (1995a, 1995b, 1996). Among the main findings from these interviews were that 8-year-old children displayed a wide knowledge of the everyday use of decimals: they talked about seeing them in sports statistics, on hospital charts, in shops, on checks, in banks (including currency exchange), on calculators, in books, as a position on a racecourse (1.4 km), and on a 1.5-liter bottle of soft drink. However, children age 10 or older, who had been introduced to decimals in school, thought of a much narrower range of everyday contexts for decimals, primarily offering money or "math class" as contexts. These older children could write and read a number that included a decimal fraction that they had seen in school (e.g. 1.8). But, although they could divide an object or diagram into tenths and hundredths with reasonable accuracy, they could not give the names for these fractional portions.

Despite being able to divide a physical unit, many children did not think that the number 1 could be divided by 10 on a calculator or that anything came between 0 and 1. Many children dealt with decimal fractions in a manner that suggested that they did not see decimals as having a meaning that might relate to size or quantity. For example, one student (as reported in Irwin, 1995a) correctly added 1.5 and 0.1 to get 1.6. However, when asked which of these three numbers would be the largest, she was certain it was 0.1. She did not understand that 1.5 was larger than 1 and that when 0.1 was added to 1.5, the sum was even larger. If she had related meaning to these decimal fractions at Hiebert's (1985) Site 1, the meaning of 1.5 and 0.1 individually, and Site 3, the meaning of the sum, she would not have held this belief.

The interviews revealed these commonly held misconceptions about decimal fractions:

- Longer decimal fractions are necessarily larger.
- Longer decimal fractions are necessarily smaller.
- Putting a zero at the end of a decimal number makes it ten times as large.
- Decimals act as "a decorative dot" (Bell, Swan & Taylor 1981); when you do something to one side of the dot you also do it to the other side (e.g., 2.5 + 1 = 3.6)
- Decimal fractions are "below zero" or negative numbers.
- Place-value columns include "oneths" to the right of the decimal point.
- One hundredth is written 0.100.
- $^1/_4$ can be written either as 0.4 or as 0.25.

In summary, the results of these interviews demonstrated that many students had misconceptions about decimal fractions, and they had difficulty transforming their everyday knowledge of decimal fractions to school knowledge of decimal fractions. In school, many had not built their scientific knowledge on their everyday knowledge by attaching meaning to decimal fractions as described in Hiebert's Site 1 and Site 3.

METHOD

The purpose of this study was to investigate whether the understanding of decimals held by a group of students from a lower income area could be improved by asking them to solve problems set in everyday contexts. Research questions included the contribution that context made to increased understanding, the role of peer collaboration, the role of cognitive conflict, and the effect the interaction of these factors might have. The investigation involved two groups of students: one group solving decimal-fraction problems set in a variety of contexts and another group solving similar problems but without contexts.

Participants

The participants were 16 students from one class in an elementary school situated in a lower economic area of Auckland, New Zealand. It was one of the schools that had been involved in the interview studies done the previous year (Irwin, 1996), but the students were not the same. The class was a combined Year 7 and 8 class (ages 11 and 12) taught by a European (Caucasian) male from a middle-class background. Criteria for selection of students for the study were: (a) having had the majority of their schooling in New Zealand in the English language (Note: Many of the students were from Pacific Island nations and possibly spoke a different language at home); (b) having been at this school for at least one year; and (c) having parental permission to participate. Although 17 of 29 students met these criteria, one student was absent on the first day of the intervention and therefore was not included in the study.

The teacher was asked to rank the remaining 16 students by their general achievement in mathematics, not their understanding of decimal fractions. Students were then paired so that a higher ranked partner worked with a lower ranked one. The difference in ranking between students on the basis of teacher judgment was equal for each pair. Thus, the student ranked 1st was paired with the student ranked 9th, the student ranked 2nd was paired with the student ranked 10th, and so forth. This ranking did not necessarily represent equal-interval achievement, but it placed a relative expert with a relative nonexpert. Students were paired in this manner to encourage conversation about decimals and to maximize the chances of students learning from one another. I did not want students to assume that their partner shared either correct concepts or misconceptions about decimals. Information about the students (e.g., teacher's rankings, ages, year in school, ethnicity, and gender) appears in Table 1.

Table 1
Characteristics of Pairs of Students

Pair[a]	Rank	Ages[b]	Year in School	Ethnicity[c]	Gender
1 – Mereana & Ngata	1st & 9th	12 & 11	8 & 7	Samoan & Maori	f & m
2 – Geoff & Alan	2nd & 10th	12 & 11	8 & 7	European & Maori	m & m
3 – Heli & Amber	3rd & 11th	11 & 11	7 & 7	Maori/European & Samoan	f & f
4 – Glen & Bruce	4th & 12th	12 & 11	8 & 7	European & European	m & m
5 – Alex & Rob	5th & 13th	12 & 12	8 & 8	European & European	m & m
6 – Ruth & Colin	6th & 14th	12 & 11	8 & 7	European & Maori	f & m
7 – Lavinia & Viki	7th & 15th	13 & 12	8 & 8	Maori/European & Samoan	f & f
8 – Hine & Katene	8th & 16th	12 & 11	8 & 7	Maori & Tongan	f & m

[a] All student names are pseudonyms.
[b] Age based on student's self-reported birth date.
[c] Maori are the indigenous people of New Zealand.

Materials

I devised items for a pretest and posttest on decimal fractions and a set of decimal fraction problems for use in the intervention phase of the study. These materials are described next.

Pretest and posttest. The purpose of the pretest and posttest was to assess each student's understanding of decimal fractions before the intervention and to see if the intervention altered his or her understanding in any way. The test questions, which were selected from those used in previous research studies, appear in Figure 1.

There were three sections in the test, with the first section requiring students to order a set of numerals that were printed on separate pieces of paper and presented

1. Put these in order

0 0.01 0.10 [0.100] [0.11] 0.5 0.9 1 1.4 [1.425] 1.43 1.5 2

(Author Note: The numerals were written on pieces of paper, which were mixed up and placed in an envelope.)

2. Guess what the answer to these might be (estimate, don't work them out)

12.5 – 5.75
5.07 – 1.3
10 × 0.5
0.12 ÷ 10
5.07 + 1.3

3. How can you tell which is larger?

0.□ or 0.□□□□

[Give a rule for multiplying and dividing decimal fractions by 10. Use an example.]

Figure 1. Pretest and posttest questions. Items in square brackets were on the posttest but not on the pretest.

in a mixed order in an envelope. There were 10 numbers to be ordered on the pretest and 13 numbers on the posttest. This section of this test took advantage of misconceptions described by others or found in my previous interviews (Hiebert & Tonnessen, 1978; Irwin, 1996; Thipkong & Davis, 1991).

The second section involved numerical problems selected from Mason and Ruddock (1996) and was presented in a horizontal rather than a vertical format (e.g. 12.5–5.75). In the third section of the pretest, students were asked to predict the size of decimal fractions, and on the posttest, they were asked an additional question on multiplying and dividing a decimal fraction by 10. These items came from Resnick, Nesher, et al. (1989) and Brown (1981).

All items were selected because they had proved challenging for students of this age range. For example, the items selected from Mason and Ruddock (1996) had been passed by 50% or less of the 11-year-olds to whom those tests had been given. The items in the pretest and posttest dealt with abstract numerical and operational understanding and were intentionally chosen to be different from the problems in the intervention so as not to favor students receiving a particular intervention. Some additional numbers were added to the first section on the posttest and an additional question was added to the third section in order to avoid a ceiling effect; these additions appear in square brackets in Figure 1. Students were also interviewed individually after the posttest, particularly in regard to the first section of this test. The test was not timed, but in general the students completed it in about 10 minutes. No students wrote vertical algorithms that would indicate that they worked out the second section of the test procedurally.

Intervention problems. An intervention was devised in which pairs of students were to solve problems, on the premise that this problem-solving experience would increase their understanding of decimal fractions. The intervention problems, presented as verbal sentences, consisted of two types: contextualized and noncontextualized. Contexts were selected from those offered by students from this and similar schools in earlier interviews, although not by the same children as in this study (see Irwin, 1996). Contexts offered by students in these earlier studies included different sizes of soft-drink bottles, monetary exchange between countries, and other uses of metric measurement. The same numbers were used in the contextualized and noncontextualized problems as far as possible. There were three categories of decimal fraction problems: magnitude, addition and subtraction, and multiplication and division, mostly by multiples of 10. Within each of these categories a problem was written to address each of four misconceptions:

1. The misconception that led students to treat whole numbers and decimal fractions as distinct units separated by a " decorative dot."
2. Misconceptions that related the length of a number to its value.
3. Misconceptions about the way in which quantities were represented in decimal fractions.
4. Misconceptions about when a zero is important and when it can be omitted from a decimal fraction.

These four misconceptions were selected from those that had been identified in the previous studies (Irwin, 1996). Both sets of problems written for the third misconception addressed confusion about representation, but the contextualized problem dealt with money given to one, three or four decimal places, and noncontextualized problems dealt with translation from common fractions to decimal fractions. Thus, the problems themselves were obviously different, but both kinds of problems relate to the misconceptions concerning translation of different representations of the decimal fractions. Examples of of contextualized and noncontextualized problems and the misconceptions that they represent appear in Table 2.

All problems, whether contextualized or not, were designed to present students with a conflict between an answer that resulted from a misconception and a correct answer. This was done either by presenting written statements given by two hypothetical students, one holding a misconception and the other giving a correct answer, or through an expected conflict between a concept and the result given by a calculator. Students were required to decide which answer was correct and why. There were 12 problems for each pair to discuss, each of which was presented in two parts. The materials presented to students contained a series of statements, as the one in the following noncontextualized problem.

Teri said that 93 $^1/_4$ was written as 93.04 in decimals.
Why did she say that? Do you agree?

Peta said that 93 $^1/_4$ was written as 93.25 in decimals.
Why did she say that? Do you agree?

Who do you think is right?

Table 2
Sample Problems for the Contextualized and Noncontextualized Conditions and the Misconceptions

Category	Contextualized	Noncontextualized
Magnitude: Confusion of length and value	A soft drink like Coke comes in different sizes. One is 1.5 l and another is 355 ml. John says that 355 was bigger than 1.5 because that is a larger number. Amoura says that 1.5 l is more than 355 ml because that is a bigger bottle.	Alex said that .355 was more than .5 because 355 was more than 5. Jasmine said that .5 was more than .355 because 5 tenths was more than 355 thousandths.
Magnitude: Different representations of decimal fractions	The paper says that one New Zealand dollar = 0.9309 in Australian dollars. Susan said that would be 93.09 cents. Andrew said it would be 9309 dollars.	Teri said that 93 $^1/_4$ was written as 93.04 in decimals. Peta said that 93 $^1/_4$ was written as 93.25 in decimals.
Addition and subtraction: Confusion of length and value	How much do you think you will have left if you have a 1.5 liter bottle of drink and pour out enough to fill a 225 ml glass?	If you subtract 0.225 from 1.5 what will you get?
Addition and subtraction: Different representations of decimal fractions	If you go on a trip and you buy 1 liter of petrol @ 90.9¢, and a meal at McDonalds' at $4.95, how much will it cost?	If you add 90 $^9/_{10}$ and 4.95 what will your answer be?
Multiplication and division: Continuity of units across the decimal point	Louise is making elastics for skipping and is buying 2 meters 30 cm for each. She needs to make up 10 for the class. She says that she will need 10 times 2.30 meters and that would be 20 meters and 300 cms. Conrad says that 10 times 2 meters 30 centimeters would be 23 meters.	Louise thought that 2.30×10 would be 20.300. Conrad thought that 2.30×10 would be 23.
Multiplication and division: Different representations of decimal fractions	$1 New Zealand exchanges for 1.5989 Samoan tala. How much would you get for $10 New Zealand?	How much is $1\,^1/_2 \times 10$?

Given that the purpose of the intervention was to see if the use of familiar contexts could aid understanding, the two sets of problems should be viewed as serving two different intervention purposes: one in which students would bring their everyday knowledge to the problem and one in which students would solve purely mathematical problems, thus giving them extra practice with peer collaboration and potential cognitive conflict, but without familiar contexts.

Intervention materials. In the intervention sessions students were provided with a Casio SL-450 calculator, paper, and ballpoint pens. Students were given the contextualized problems and were also shown items that were referred to in their problems: for example, Sprite containers of 2 litres, 1.5 liters, 500 ml, and a six-pack of 355-ml cans; a glass that would hold approximately 255 ml of liquid; a measuring cup that showed measurements up to 250 ml; a clipping from the newspaper on monetary exchange rates; and a sheet from a bank showing the exchange rate between New Zealand dollars and the currencies of other countries. There was one sheet for each problem per pair and one calculator per pair, but students could take and use as much paper as they wanted.

Procedure

Pairs of students were divided into two parallel groups. Pairs 1, 4, 6, and 7 (see Table 1) were given the contextualized problems to solve, in which the numbers had referents common to these students' experience. Pairs 2, 3, 5, and 8 were given similar problems, but without referents. The mean rank of the students who were given the different types of problems was equal (8.5). The mean age of the group that was given contextualized problems was 12 years 1 month (problems without referents: 12 years 1.5 months), and the mean year level of the group that was given contextualized problems was 7.6 (problems without referents: 7.5). Thus, because the groups were similar with respect to mean rank, age, and grade level, the groups were assumed to be parallel for the purposes of this study. Both the week of intervention and time of day of the intervention were counterbalanced between the groups solving different types of problems.

Pairs in both conditions were asked to solve problems collaboratively. The discussion between classmates working together to solve a problem was a type of discourse that these students often experienced in class, and one in which they would naturally talk, think aloud, and explain their reasoning. Both the group solving problems in context and the group solving noncontextualized problems engaged in peer collaboration and potential cognitive conflict. This procedure that encouraged peer collaboration was based on Piagetian theory (1932/1965) and on the findings of several authors (e.g. Forman & Larreamendy-Joerns, 1995; Pontecorvo & Girardet, 1993), and the use of cognitive conflict was based on work by writers such as Piaget (1975/1985), and Swan (1990).

Students were withdrawn from class in pairs as described above and worked in a small room in the school. Following the pretest, which was taken individually, students worked in pairs on the intervention problems on three separate days. The duration of the intervention was especially chosen to be similar to the time a teacher would normally spend reviewing decimal fractions in class at this grade level. The posttest was given two months later. Decimals were not taught in school in this intervening period.

During the intervention, I acted as a clinical interviewer, making only predetermined types of statements such as reminders of what students were to do, requests for further explanation, and neutral statements that might facilitate collaboration (e.g., repeating a student's statement or suggesting that a pair come back to a problem after a break). My intention was to interfere as little as possible with normal collaboration (see Piaget 1932/1965) but to get an accurate record of students' thinking. All sessions were audio-taped, transcribed, and analyzed using Non-Numerical Unstructured Data Indexing Searching and Theorizing software (Qualitative Solutions and Research, 1994).

Concern that my intervention might have differed between the two groups was addressed by analyzing the proportion of statements that I addressed to each group. Overall the differences were small in that the students made most of the statements in any session. Of all the statements made, I made 16.2% to the students engaged in contextualized problems and 15.4% to the students working on problems without contexts. The slightly larger percentage of statements I made to the students working on the contextual problems can be traced to the need to remind students about the existence of particular intervention materials (e.g., soft drink bottles, sheet of exchange rates).

Analysis

All items on the pretests and posttests were scored according to a scoring key, and then the scores were checked by a colleague. Any disagreements in scoring were discussed to reach consensus. Students were given points in the first section if they placed numbers between a smaller and a larger number (1 point each for left and for right, including appropriate starting and ending of the row, with a maximum 20 points on the pretest and 26 on the posttest). They were given points in the second section for the number of decimal places that were correct (maximum 12 points). Their scores for the third section were based on the accuracy and appropriateness of the explanation given (maximum 5 points on the pretest and 10 on the posttest). The raw scores were converted to percentage-correct values for futher analysis.

I then analyzed the dialogues to provide evidence about why students changed their views when presented with contextualized problems. Additional analyses of the dialogues have been reported elsewhere (Irwin, 1997, 1998).

RESULTS

Pretest Versus Posttest Performance

Figure 2 shows the mean percent correct performance of the students on the pretest and on the posttest by ranking (higher and lower) and by group (contextual and noncontextual). The circles represent the mean performance of students who worked on the contextualized problems, and the squares represent the performance of those who worked on noncontextualized problems. The percentages depicted in Figure 2 are as follows: contextualized, lower ranked improved from pretest of 19% to posttest of 32%; contextualized, higher ranked improved from 34% to 48%; noncontextualized lower ranked improved from 21% to 27%; and the noncontextualized, higher ranked from 45% to 52%.

Analysis of variance confirms, as the figure suggests, that there was a significant difference in performance between the pretest and posttest, $F(1,12) = 14.76$, $p = .002$. In addition, there was a significant difference between the scores of students ranked higher (top half of the sample—rankings of 1st through 8th) and those ranked lower (bottom half—rankings of 9th through 16th), $F(1, 12) = 8.66$, $p = .01$. This finding is particularly important because it confirms the teacher's ranking of the students' mathematical ability.

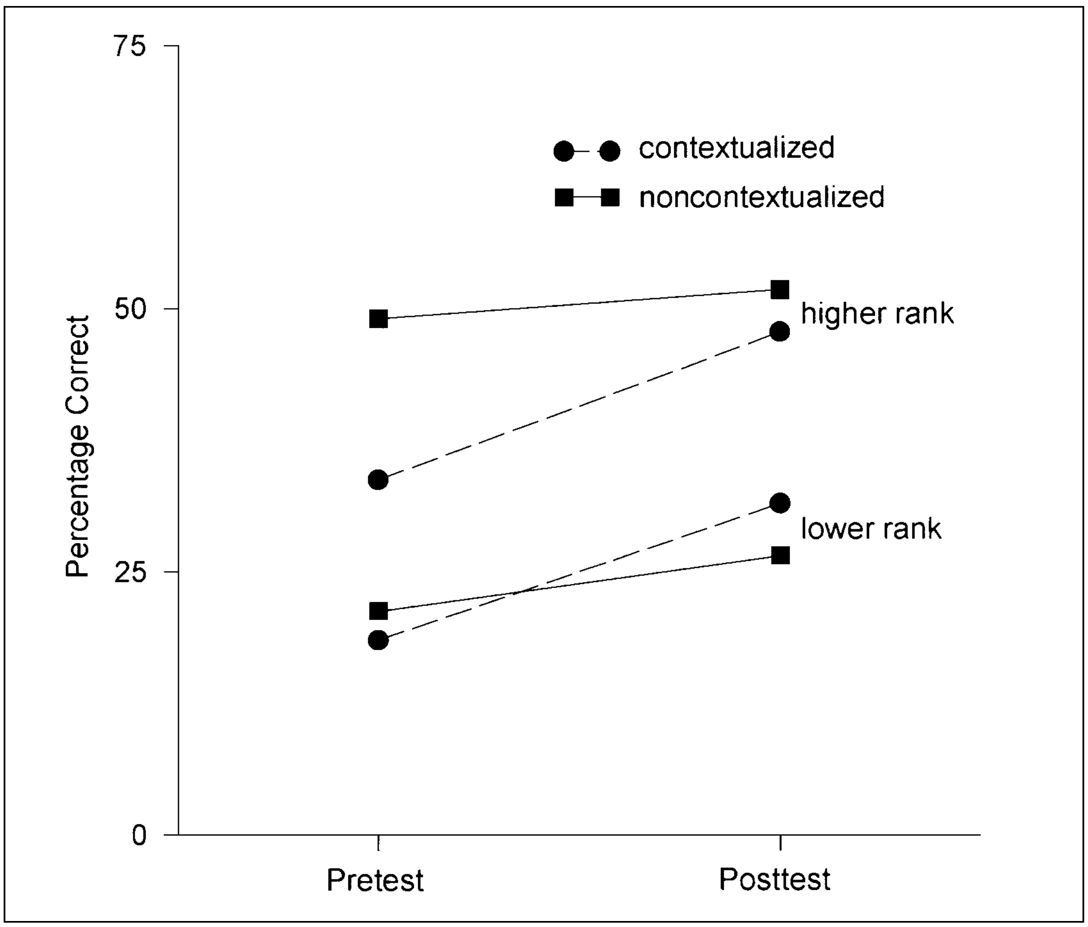

Figure 2. Mean percentage correct on the pretest and posttest of students working on contextualized and noncontextualized problems. The performance of students ranked higher or lower in mathematical ability by their teacher is shown separately.

Of most interest, however, was a significant interaction in improvement between the pretest and posttest and the type of problem that the students worked on, contextualized or noncontextualized, $F(1, 12) = 5.7$, $p = 0.03$. As Figure 2 illustrates, the nature of the interaction arises from the fact that students who worked on contextualized problems improved more than those who worked on problems without referents did. This greater improvement of students who worked on contextualized problems was independent of whether they were high ranked or low ranked. This independence was confirmed by the analysis of variance, which showed that the three-way interaction between type of problem studied, ranking, and performance was not significant, $F(1, 12) = 0.41$, $p = .53$.

Analysis of the Students' Dialogues

I analyzed the students' dialogues during the intervention to attempt to determine what might have enabled some students to improve their understanding of decimal fractions more than others. As reflected in the analysis of variance results, some students working on problems in context increased their scores as much as 28 percentage points, while students working on problems without contexts mostly changed very little.

My initial thought was that if partners contributed similar amounts of "talk" to the discussion, this could be a sign of productive peer collaboration in which partners were discussing and arguing as peers, thus enabling both partners to learn from each other. The total number of lines of typed dialogue for each student in a pair was converted to a ratio of the contribution of each partner, with these ratios ranging from 1:1 to 2:1. However, no relationship was found between these ratios and improvement in understanding of decimal fractions.

Another analysis involved identifying the contribution that each statement made to the discussion. From 24 dialogues, each up to six single-spaced pages in length, a sample of at least two dialogues from each pair was analyzed in an attempt to tease out why some students progressed and others did not. I created categories on the basis of the argument operations used by Azmitia (1988) and Pontecorvo and Giradet (1993) and the categories proposed by Wells (1994). The categories were: Answer, Explanation, Agreement, Disagreement, Challenge, Question, Consideration of other's view, Request for agreement, Comment, and Incomplete statement. In addition, modifying characteristics of these categories were noted, such as tentativeness, expansion, repetition, correctness, and to whom the remark was addressed.

The theory behind this analysis was that certain types of argument were more likely to advance understanding than were others and that a balance of arguments between partners was more likely to advance understanding than was a marked imbalance. Piaget (1932/1965) postulated that reciprocity depended on whether or not both members of a pair had some knowledge to contribute to a decision on a reasonable answer—a potential difficulty when a relative expert is paired with a relative non-expert. The balance between the type of statements proffered by partners would indicate if both partners offered answers and explanations and challenged one another or, alternatively, if only one of the partners made most of the statements that led to solution of the problem. An important condition for successful collaboration is that the knowledge offered must include some knowledge that is correct.

A difference in the nature of the dialogue was found between the pairs that progressed as a result of working on contextualized problems and the pairs who worked on noncontextualized problems but did not make much progress. The dialogue of students who worked on contextualized problems more often showed greater equity and reciprocity in the arguments than those who worked on problems without referents. In particular, the lower ranked partner in these pairs was often the one who used everyday knowledge to make sense of the problem at Hiebert's Sites 1 and 3, whereas higher ranked partners ignored context and manipulated numbers instead. In these pairs, both partners had contributions to make that were valued by his or her partner. Three examples of these dialogues and appropriate commentary are given next.

Example 1. The first example is from a pair who worked on noncontextualized problems. The dialogue is similar to those produced by the other pairs solving these problems who made little progress, in that the higher ranked partner gave most of the accurate answers and explanations and the lower ranked partner was relatively passive, agreeing with both correct and incorrect answers of the other student.

Heli and Amber were two girls of the same age; Heli was the higher ranked student. A representative portion of their dialogue is given. There was a marked imbalance in the types of statements made by each of the girls. In the full discussion for this problem, Heli gave 37 answers and explanations while Amber gave 6. Heli agreed 5 times and Amber agreed 20 times. Both partners asked each other questions, but Heli's questions were primarily polite in nature, such as "So, do we agree?" Amber asked for explanation—for example saying "Huh?"—but there was little genuine conversation. This pair lacked the common intellectual values and reciprocity that Piaget noted as essential for collaboration. In addition, Amber did not appear to work hard enough for the conservation of her ideas in the face of Heli's statements.

Problem (part A): Teri said that 93 $^{1}/_{4}$ was written as 93.04 in decimals. Why did she say that? Do you agree? What do you think is right?

	Dialogue Line	**Categories**
	… (following initial agreement that 93.04 [wrong answer] was correct)	
H:	Oh remember, remember, 'cus decimals leave, you know how this goes ones, tens, hundreds, this goes hundreds, tens, ones. (points to place-value columns for what would be hundreds, tens, ones, oneths, tenths, hundredths)	Explanation
A:	Oh yeah, and then …	Agreement Incomplete statement
H:	But you see that can—umm. Do we agree? 'Cus the four should be, there should be another zero between the zero and the four.	Request for agreement Wrong answer
A:	Yeah.	Agreement

Problem (part B): Peta said that 93 was written as 93.25 in decimals. Why did she say that? Do you agree? What do you think is right?

H: (23-second pause) Hold on. (Pause) Yeah, it [93.25] could be right.	Consideration of other's view Tentative agreement
A: (Starts to say something, interrupted)	Incomplete statement
H: See most, that is one quarter, 25 is one quarter …	Explanation
A: Yeah. (31 lines later)	Agreement
H: It's definitely that one [93.25]. 'Cus if you think about it, even if you got the 0 0 4 in there you couldn't divide it, oh you could divide it up by 4, but it would have been a zero one.	Explanation
A: Huh? (14 lines later)	Question
A: This one [93.04] could be right, though. (9-second pause)	Changes back to wrong answer
A: 'Cus that (0) is in the10s, isn't it? That's tenths, not oneths. [referring to column names]	Request for agreement Explanation

Note that Amber tries to correct Heli's misconception about place value, but Heli does not consider this.

H: It's tens, then ones, tens and ones [pointing to place-value columns]. (21 lines later)	Wrong explanation
H: It could be both. Could it be both? Or just one of them? (pause) [question to interviewer]	Tentative answer
I: That's for you to sort out.	
A: It could be the same one, right. (20-second pause)	Agreement

Amber appears ready to accept Heli's answer despite her earlier disagreement about place value.

H: Yeah, I reckon they're both right.	Answer
A: Yeah, and …	Agreement
H: Wait, wait! ... (49 lines and several calculations later)	Tentative disagreement
H: 93 and a quarter. Whereas that [93.04], 93 and a quarter as well. So it could be both. See 25 is a quarter. Twenty-five is one quarter of a hundred. … (8 lines later)	Explanation
A: Both of them?	Request for agreement

H: No, one of them. It's that one I reckon [93.25].	Disagreement Answer
A: (pause) Yeah that one.	Agreement
H: Because 25 is one quarter of a hundred.	Explanation
A: Is it?	Question to partner

Heli offered several incorrect explanations, which in the long run did not confuse her, but seemed to have thoroughly confused Amber. Heli appeared to be engaged more in a monologue that did not exclude her partner but rarely built on what she said. Amber appeared to have a better understanding of place value but lacked the confidence needed to persuade Heli that she was correct.

Example 2. This example is from the work of two students working on a contextualized problem. The lower ranked partner, Ngata, challenged the calculation of the higher ranked partner, Mereana, on the grounds of his everyday knowledge, and then Mereana subsequently challenged an answer of Ngata's on mathematical grounds. In the full discussion about this problem, their statements were well balanced, with relatively equal numbers of answers (Mereana–10, Ngata–12), number of questions (Mereana–4, Ngata–3), challenges (Mereana–4, Ngata–3), consideration of the other peer's views (Mereana–3, Ngata–1), and explanations (Mereana–7, Ngata–7). Ngata agreed or disagreed with his partner 11 times while Mereana agreed or disagreed 3 times.

Problem: If you go on a trip and you buy one liter of petrol @ 90.9¢ and a meal at Macdonald's at $4.95 how much will it cost?

Dialogue Line	**Categories**
M: Ninety-five dollars about, something cents ... four cents	Wrong answer
N: Or five dollars ... 'cus it's the whole thing.	Partial right answer
M: Yeah, I know, ninety-five dollars and something cents. Probably about ninety-five dollars and four cents.	Wrong answer, repeated
N: Where's the petrol?	Question to partner
M: That's how it costs all together, isn't it?	Question to partner
N: Ninety-five dollars? For a McDonald's and petrol?	Challenge
M: Yeah, (laugh) oh. ... (17 lines later)	Agreement (sees error)
M: Oh. See that's five dollars, five, six dollars.	Right answer, expanded
N: Six dollars.	Agreement
M: Six dollars point one.	Answer expanded
N: Five dollars, six dollars ... six dollars point eight, no point, no. Six dollars five ... uh, six dollars five cents point nine	Answer expanded
M: Six dollars five cents point nine?	Challenge
N: Yeah.	Agreement

M: Point nine? (Pause) Six dollars and five cents point nine.	Challenge again
N: Yeah	Agreement
M: That means six point five point nine.	Clarifies challenge
N: Oh man, nah.	Considers partner's view (sees error)

… (dialogue continues)

There were other instances of lower ranked peers contributing information from their everyday knowledge, whereas a higher ranked peer tended to manipulate numbers. One occurred when Ruth and Colin were discussing how much drink would be left in a 1.5 liter bottle after 225 ml were removed. Colin, the lower ranked partner, demonstrated how far down the bottle the drink would then come, calling it "one liter something," while his higher ranked partner, Ruth, suggested 115, presumably attempting mentally to subtract 15 from 225. Later when using the calculator, Ruth accepted the answer of –233.5. Another pair, Lavinia and Viki, were discussing whether there would be more soft drink in a 2-liter bottle or in a six-pack of 355-ml cans. The higher ranked partner, Lavinia, started to multiply without estimating, whereas the lower ranked partner, Viki, knew that a 2-liter bottle would serve six people, so that the answers would be about the same. A third pair, working on how many Samoan tala you would get for $NZ10, at an exchange rate of $NZ1 to $S1.5989, got an answer of 1.59890. When the lower ranked partner realized that this was less than $NZ10 he said, "No way, man"; he was not going to accept that exchange rate.

Example 3. There were also dialogues between pairs working on contextualized problems that showed relative equity in the types of contributions made to the dialogue but in which both partners used their everyday knowledge. The example given here shows both the higher and lower ranked students, Glen and Bruce, respectively, drawing on their everyday knowledge. They discussed the currency exchange problem given below. The students were given a newspaper clipping that included exchange rates for several countries, and these students used this information to give meaning to their decision. These two students did not fully understand currency exchange, for example for the Indonesian rupiah, but they discussed the problem as equals, with a similar number of answers (Glen–6, Bruce–4), a less equal offering of explanations (Glen–9, Bruce–4), and similar numbers of agreements (Glen–2, Bruce–3). The lower ranked student offering slightly more challenges, questions, and disagreements (Glen–1, Bruce–4). Their discussion included the segment that follows:

The paper says that one New Zealand dollar equals 0.9309 Australian dollars. Susan said that was 93 point 09 cents. Why do you think she said that? Do you agree?

Dialogue Line	**Categories**
G: Yeah it would be 93 point 09 dollars, Yeah (pause)	Wrong answer
B: No, it ... (13 second pause)	Disagreement
G: (unintelligible)	
B: Yeah, we're not really sure (pause)	Comment

G:	Maybe that's not cents, 'cus look, over here it says Indonesia 1516 [pointing to other exchange rates]. And then it can't be one thousand five hundred sixteen dollars, can it? (pause)	Explanation
B:	Umm, no, it has to be cents, 'cus I mean	Answer
G:	Unless that's very poor, something or cents	Explanation
B:	No, because if you took one dollar Indonesia, flip, you just take one dollar and you'd be a millionaire in Indonesia right away. (laugh)	Explanation
B:	No it would have to be cents, so she is probably most likely to be right	Answer
G:	Yeah	Agreement

… (dialogue continues)

DISCUSSION

An important finding from this study was that students who worked on the contextualized problems improved their competence with decimals more than did a comparable group of students who did not work on contextualized problems. This result (see Figure 2) is confirmed by the significant interaction between the type of problem worked on and an increased score on the posttest. Students' written responses to the final portion of the posttest ("How can you tell which is larger?") and interviews undertaken after the posttest showed that their increased competence was based on greater understanding. In the pairs who worked on contextualized problems, both the higher ranked partner and the lower ranked partner made progress. This result is similar to that found in peer tutoring in reading, where both the student tutor and the student tutored showed gains in their reading age (e.g., Limbrick, McNaughton, & Glynn, 1985).

The finding that students who worked on contextualized decimal problems showed significantly more competence two months later than did a similar group who worked on problems without context prompts the question of why this might have arisen. I offer an explanation which draws upon the work of two major theorists.

Acquiring Scientific Knowledge

The first explanation for the finding that working on contextualized problems increased retention of decimal fraction concepts derives from Vygotsky's discussion of scientific knowledge, as opposed to everyday knowledge, and from Wardekker's (1998) development of this theory. Wardekker proposed that scientific knowledge might be better termed "scholastic knowledge," as it refers to those aspects of our culture that are generally learned in school as opposed to those learned in daily life. These cultural or contextual aspects are not self-evident, but the result of development of a field over time. As stated at the beginning of this article, Howard's hope in 1887 that decimal fractions would soon be generally understood ignores the fact that the system is a multiplicative scientific concept that does not arise easily from everyday knowledge. Wardekker emphasizes the importance of reflection if students are to gain understanding so that this knowledge becomes knowledge-in-action and that this reflection usually happens through dialogue.

The misconceptions held by students in this study and others like them suggested that they had not reflected on the scientific concepts involved in decimal fractions in any way that resolved their misconceptions. Students who believed that "one hundredth" was written as 0.100 or that 1/4 could be written either as 0.4 or 0.25 had not reflected on the incompatibility of these notions with principles such as place value. On these problems, as on others, they did not necessarily believe a result shown on a calculator display but discussed the result at length before deciding on an answer. For all students in this investigation, discussing problems that involved some cognitive conflict gave them an opportunity to address such inconsistencies and reflect on their understanding. However, without reference to everyday knowledge, it was possible that this reflection could fail to advance their understanding. This was true for one pair working on noncontextualized problems who held the same misconceptions (see Ellis, 1995).

Wardekker, citing Applebee (1995), reported that knowledge-in-action was never intended to be knowledge-out-of-context, yet this may be what knowledge of decimal fractions is, especially for minority groups when the teacher does not share students' backgrounds or everyday knowledge. There are many contexts in which decimal fractions can be understood, including concrete models and the number line (Irwin, 2000). Problems presented in everyday settings provided the context that students needed for reflection on the scientific concept of decimal fractions. This reflection, for example, on the meaning of money given to more than two decimal places required students to reflect on how their existing concepts related to scientific knowledge. Such problems may have provided the reflection required for expanding their knowledge of decimal fractions.

Collaboration of Peers with Different Areas of Expertise

The second explanation for why students who worked on contextualized problems gained in their understanding of decimal fractions derives from Piaget's theory of peer collaboration. The question behind this explanation was why both higher and lower achieving students gained from peer collaboration, and in particular why both higher achieving and lower achieving students who worked on contextualized problems developed greater understanding. This question was addressed through an analysis of a sample of the dialogues as students reflected on correct and incorrect answers to problems. This question can be analyzed in terms of the criteria that Piaget discussed for peer collaboration from which both partners benefit.

Piaget (1932/1965) stated that learning from peers required that partners have a common scale of intellectual values that allowed them to understand both language and a system of ideas in the same way; that partners conserved their propositions so that they did not contradict themselves while searching for agreement on those propositions; and that there was reciprocity between partners so that the propositions of each were considered and treated interchangeably. When students consider each other's arguments, they are open to the perturbations of their own concepts that bring about learning, because these cause them to reflect on their own ideas (Piaget, 1975/1985).

There are several reasons why the partnerships in this investigation might have failed to meet Piaget's criteria. First, the fact that the partners had different achievement rankings (based on teacher judgement) might have made it unlikely that they would share a common mathematical language or system of ideas. Second, lower ranked partners would have less reason to hold to their original propositions in the face of the superior knowledge of their higher ranked partner, as was seen in Amber's conversation with Heli. Third, for similar reasons, reciprocity between the partners would not be likely, nor would it be likely that the propositions of each would be treated interchangeably. The sample dialogues analyzed in this article demonstrate that these expectations were borne out for the pair working on problems without referents. However, those working on contextualized problems did succeed in collaborating fruitfully. It is likely that this was because, for the contextualized problems, the lower ranked students not only drew on their everyday knowledge but also held to it, something that their higher ranked peers tended not to do until its value was demonstrated to them.

Analysis of the dialogues suggested that the collaborations of pairs working on contextualized problems were more like the collaborations of equals, as described by Piaget, than the collaborations of those working on noncontextualized problems were. This collaboration of equals occurred despite differences in overall mathematical understanding. A factor contributing to the reciprocity of those solving contextualized problems was the tendency of lower ranked students in a pair to draw on their everyday knowledge and thereby to challenge the incorrect procedures of the higher ranked students. Their contributions gained the respect of their higher ranked peers, who treated them as equals in the dialogue, as shown by the similarity of arguments offered by each. Hence, even though this investigation was not intended to be a study of the mechanism of collaboration, it nevertheless adds to an understanding of how collaboration might work.

In accordance with Hiebert's (1985) analysis, these students used their everyday knowledge to make sense of the numbers at Site 1 and Site 3. Analysis of the dialogues showed that numbers for calculation (Site 1), such as 90.9 cents, had meaning for them even though the notation was not one that they had dealt with in school. Similarly, the sum resulting from a calculation (Site 3) was checked against their everyday knowledge. Statements that epitomize this use of everyday knowledge at Site 3 were N's incredulous challenge "Ninety-five dollars for a McDonalds and petrol?" and B's "No way, man" when realizing that according to his calculation he would get only a little more than one Samoan tala for ten New Zealand dollars. Thus the students were compelled to think about the relationship of metric units and what the decimal point indicated instead of applying partially understood rules, such as "lining up the decimal points" or "adding a zero."

Integrating Vygotsky's and Wardekker's theory on the transition from everyday to scientific or scholastic knowledge with that of Piaget on the conditions in which peer collaboration leads to learning may lead to a fuller understanding of why this intervention was successful. Reciprocity in collaboration enabled both parties to reflect on the relationship of their existing everyday knowledge to their scientific knowledge and, thereby, to reconstruct and strengthen their scientific knowledge.

Contextual Problems and Students from Lower Economic Areas

The studies of Boaler (1998) and Lubienski (2000) have produced potentially contradictory findings on the role of context in learning mathematics by students from lower economic backgrounds. The findings from this study support those of Boaler rather than those of Lubienski. Two factors are important here. One is that these contexts were particularly chosen as familiar to this group of students, a situation that may also have been the case for the students in Boaler's study but was not so for the students in Lubienski's study. In addition, these problems were not used for initial learning of concepts but rather to help overcome misconceptions in a domain that the students had already learned, possibly in inappropriate contexts. The choice of appropriate contexts is essential if cognitive conflict is to occur.

Implications for Teaching

Decimal fractions are a particularly difficult domain for many students. Complete understanding requires multiplicative thinking, which is not natural but requires a reconceptualization of the relationship of numbers from that required in additive relationships. For reflection in classroom dialogues to be most effective, both teachers and students must be aware of the potential relationship between everyday and scientific knowledge. In particular, the teacher has the responsibility for mediating their discussions (Forman & Larreamendy-Joerns, 1998). In this intervention, my role as 'teacher' was in writing the problems so that they drew on the experience and misconceptions of students' from this particular area and ethnic mix.

It needs to be recognized that what amounts to everyday knowledge for one group may not be everyday knowledge for another group: cricket enthusiasts know cricket statistics and baseball enthusiasts know baseball statistics. Students will not draw on their everyday knowledge if it does not seem necessary to do so (e.g. Greer, 1987; Silver, Shapiro, & Deutsch, 1993). On the one hand, different decimal problems would be needed to take advantage of the everyday knowledge of different groups of students. In the area in which this study took place, most students would be unfamiliar with both cricket and baseball statistics. On the other hand, these students were familiar with monetary exchange rates because they or their parents traveled back and forth between countries or sent money overseas.

The interviews that preceded this intervention showed that information on the contexts that are familiar to students was not difficult to obtain. Once obtained, this information could be used to design problems suitable to help a particular group overcome misconceptions about decimals. These specially designed problems also have to have the appropriate level of difficulty for students so that conflict leads to successful scientific understanding of decimals.

Thus teachers of lower income or diverse classrooms need to be aware of students' everyday knowledge and any misconceptions developed on the way to achieving scientific knowledge of decimal fractions. They need to pose questions and mediate dialogue to promote reflection, interweaving this everyday knowledge with scientific knowledge. This study suggests one method, utilizing both conflict and collaboration, of helping students build scientific knowledge on everyday knowledge. Its findings could have important implications for teaching, especially among relatively disadvantaged groups.

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Author

Kathryn C. Irwin, School of Education, The University of Auckland, Private Bag 92019, Auckland, New Zealand; k.irwin@auckland.ac.nz