Research Commentary

Amidst Multiple Theories of Learning in Mathematics Education

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Currently, there are more theories of learning in use in mathematics education research than ever before (Lerman & Tsatsaroni, 2004). Although this is a positive sign for the field, it also has brought with it a set of challenges. In this article, I identify some of these challenges and consider how mathematics education researchers might think about, and work with, the multiple theories available. I present alternatives to views of the competition or supersession of theories and indicate the kinds of discussions that will support effective theory use in mathematics education research. I describe the potential for mathematics education researchers to make informed, justified choices of a theory or theories to address particular research agendas.

Key words: Epistemology; Learning theories; Research issues

Theories of learning orient much of the research in mathematics education. Each theory brings with it basic assumptions about the nature of learning, the phenomena of interest, constructs that can be used in the conception and implementation of research, and the types of explanations that can be generated. Inclusion of an explicit description of the theory or theories employed is an accepted practice in research reports. Currently, there are more theories of learning in use in mathematics education research than ever before (Lerman & Tsatsaroni, 2004). This is a positive sign for the field. However, with multiple theories has come a set of issues that require ongoing discussion (Silver & Herbst, 2007), including difficulties in communication (Sriraman & Kaiser, 2006) and accumulation of knowledge across theoretical perspectives, a lack of acceptance of work done from particular perspectives,[1] challenges in bringing to bear appropriate theory for specific research problems, and lack of transparency in the process of selecting theoretical and conceptual frameworks. In this article, I endeavor to contribute to, and provoke discussion of, issues related to the use of theories of learning. Toward this end, I propose some particular ways of thinking about theory use and present a synthesis

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[1] I have observed such lack of acceptance not in published articles but in discussions at conferences and in reviews of articles and grant proposals.

of some important ideas articulated by others. I also challenge some ideas pertaining to theory use that persist in mathematics education.[2] The article is by no means exhaustive of issues related to theory use.

My focus, in this article, is on major background theories (e.g., constructivism, enactivism, sociocultural theory) adopted by researchers and not on more specific theoretical constructs meant to explain particular phenomena in mathematics education (e.g., quantitative reasoning, Thompson, 1994) or on conceptual frameworks that are particular to individual projects. The rationale for the choice of this focus is threefold. First, a discussion that attempts to consider all theory in mathematics education is complicated by the lack of consensus of what constitutes a theory. Second, the diversity of what might be considered in this category (theory) makes it difficult to make statements about theory use that are not in conflict for some elements of the category. The focus on major background theories of learning affords a more focused discussion, a discussion that I argue is timely. Third, a focus on theories of learning allows me to make arguments that build on my research experience. Indeed, some of the points made in this article are applicable to theory use for theories other than theories of learning.

Ernest (2006), in reviewing four background theories of learning, "simple" constructivism, radical constructivism, enactivism, and social constructivism, alternately referred to them as "philosophies of learning" (p. 8) and noted that some might argue that they are not theories in the sense that they can be falsified (Ernest, 2006). What would it mean to falsify constructivism or enactivism? This issue is discussed indirectly in the section, Challenging the Notion That Each New Theory Supersedes Its Antecedent.

In this article, I attempt to respond to current tendencies and discourse with respect to the use of theories of learning in mathematics education research. I have not conducted formal research on theory use and current discourse, nor does any such research seem to have been done.[3] Thus, I have based the article on my experience as an active member of the research community; as a contributor to, and consumer of, the research literature; as an author and reviewer of grant proposals; and as a member of several journal editorial boards.

THE PERSPECTIVE OF COMPETING THEORIES

Researchers in mathematics education have many different ways of relating to multiple theories of learning. In some cases, researchers view diverse theories as

[2] In some cases, these ideas may exist in the field more implicitly than explicitly, although influencing the research enterprise nonetheless.

[3] Lerman and Tsatsaroni (2004) conducted an analysis of articles published in two journals. However, their analysis does not extend to the issues that I raise that are relevant to current discourse and use of theory in the field. Silver and Kilpatrick (1994) conducted a set of interviews on mathematics education research. Although their informants made comments about theory in mathematics education research, the report discusses their individual ideas and not the types of trends in theory usage or discourse in the research community discussed here.

competing theories. Goldin (2003) commented on one result of this view, "Unfortunately, in emphasizing its own central idea, each of these [behaviorism, radical constructivism, social constructivism, and mind-based mathematics] has insisted on *excluding* and *delegitimizing* other phenomena and other constructs" (p. 196, cited in Törner & Sriraman, 2006, p. 4). Cobb (2007) similarly observed, "Proponents of various perspectives frequently advocate their viewpoint with what can only be described as ideological fervor, generating more heat than light in the process" (p. 3). In this article, I argue that advocacy of a theory from an ideological standpoint is not what is needed to advance mathematics education research.

In the mathematics education research community, one frequently hears researchers identified as constructivists or social theorists. What does the "ist" suggest? It might suggest expertise, as in the labels *psychologist* or *physicist*. However, it also may suggest the notion of doctrinal adherence, as in Methodist or Baptist. That is, the label suggests adherence to one doctrine and not others. It identifies not only the constructs that one uses but also the constructs that one may not use. I see this as a symptom of an unproductive divide that sometimes is a part of discourse in our field. In the following sections, I offer perspectives that might contribute to effective use of theories of learning and suggest conversations that might contribute to progress in the field.

CHALLENGING THE NOTION THAT EACH NEW THEORY SUPERSEDES ITS ANTECEDENT

Contributing to some researchers' advocacy of a single theoretical perspective is the notion that when a new theory takes hold in the field, it supersedes preceding theories. Cobb (2007) challenged the depiction of "the history of a field as an ordered progression that typically culminates with the perspective to which the writer subscribes" (p. 31). Thus, some would say that constructivism superseded behaviorism and that socioculturalism superseded constructivism.[4] The assumption is that constructivism is a better theory than behaviorism and socioculturalism is a better theory than constructivism. However, the unanswered question is, "Better for what?"

The notion of superseding theories is unhelpful because it has the effect of "delegitimizing" prior theories. Consider the following analogies. A theory that the Earth is round rather than flat has become well established. Nonetheless, as I navigate around the big city to which I recently moved, I do so using a model of the world as flat. Would I be better off thinking about the great circle route to the West Village or to Brooklyn? Also, I am well aware of Einstein's theory of relativity. Yet, in my day-to-day experiences, I rely on Newtonian or even Aristotelian physics, hardly

[4] I am omitting many important theories and variations on those theories for efficiency in making the argument.

giving a thought to special relativity. Is this misguided or unenlightened? Tyree (2003) pointed out, "Newtonian physics is still good at predicting everyday events on earth. It is even good enough for NASA to plan trajectories of spacecraft in our solar system" (p. 3).

A theory that has proved useful over a significant period of time does not typically lose its usefulness when a new theory is adopted or developed. Although I chose to use examples from the physical sciences, the same argument could be made in the social sciences. For example, in psychotherapy, personality theories have not superseded psychoanalysis, and systems theory has not superseded personality theories. Each of these psychotherapeutic orientations provides a different way to conceptualize psychotherapy, and therefore each brings with it different affordances, constraints, and domains of applicability. Sfard (1998) argues that different theories of learning offer "differing perspectives rather than competing opinions" (p. 11) and offers additional examples from the physical and social sciences.

The advent of the idea that the Earth is round, the development of the theory of relativity, and the adaptation to mathematics education of constructivism and socioculturalism afforded a kind of work that previously existing theories were not well suited to doing. However, the pre-existing theories continue to do important work. Lerman (2006), who has been a leader in the adaptation of sociocultural theory to mathematics education, emphasized this idea by demonstrating his respect for work emanating from other theories. He observed that one way that social sciences grow is,

> . . . by the insertion of new theoretical discourses alongside existing ones. Constructivism grows, and its adherents continue to produce novel and important work; models and modeling may be new to the field, but already there are novel and important findings emerging from that orientation. (p. 9)

Lerman's description of how social sciences grow stands in stark opposition to the notion of superseding theories. Whereas sociocultural theory has afforded new research possibilities and new understandings, constructivism continues to generate useful research that cannot be done from a sociocultural perspective. Behaviorism still has its utility when the goal is influencing behavior directly.

When a new theory is proposed or imported from another discipline, it should be viewed critically. Does the new theory afford a kind of work that cannot be done, or done as well, with existing theories? Are there empirical warrants for the theory? However, when considering theories that have proved useful over decades, attempts to delegitimize work from these theories seem counterproductive. A more useful discourse would concern the affordances, limitations, and domains of applicability of these theories.

When discussing the use of multiple theories, the notion of incommensurability of theories is raised. *Incommensurability* is a term attributed to Kuhn (1962). Kuhn, who continued to work on the issue of incommensurability after completing *The Structure of Scientific Revolutions*, explained that for incommensurable theories, the terms of the theories are nonoverlapping and not intertranslatable (Bird, 2004).

However, this does not imply a supersession of theories and does not rule out the possibility of coordinating the results of work from different theories.

Cole and Wertsch (1996), who are major contributors to sociocultural theory, asserted, "There is little doubt in our view that there is still much to be learned from both Piaget and Vygotsky, and in many cases the strengths of one theorist complement the weakness of the other" (p. 254). Bruner (1996) made a multifaceted argument that the theories are *incommensurate*. However, he too celebrated the contributions of each, "Just as depth perception requires a disparity between two views of a scene, so in the human sciences the same may be true: depth demands disparity" (p. 19). Thus, theories can be judged to be complementary and incommensurate: complementary with respect to their affordances and incommensurate with respect to the obstacles to combining the theories. Sfard (1998) claimed that theories can be "incommensurable rather than incompatible, and because '[i]ncommensurability entails irreducibility [of vocabularies], but not incompatibility' (Rorty, 1979, p. 388), this means a possibility of their peaceful coexistence" (p. 11).

Mathematics education benefits from the contributions of different programs of research that are based on (different) incommensurate theories. In some cases, these incommensurate theories can be used productively within one program of research (e.g., Cobb & Yackel, 1996).

BACKGROUND THEORIES VERSUS WORLDVIEWS

A more slippery issue is a subtle confounding of background theories and worldviews. *Worldview* refers to the sum total of one's beliefs, understandings, and deeply held commitments. It is only partially conscious and is related to one's personal identity. Even though one's worldview can evolve over time, the current state of one's worldview has an impact on one's actions, perceptions, thoughts, and emotions. Whereas one's worldview is fairly well set at any particular point in time, I argue that theory use should be a matter of ongoing choice based on factors related to one's research rather than one's personal identity (see the section, Theory Use as a Matter of Choice).

The conflation of background theory and worldview may be the result of the overlap that exists between personal beliefs about knowledge and available theories. For example, one can believe that notions of right and wrong are socially and individually constituted. Or one might believe that there is a major social influence on knowledge development. In contrast, it is important to see background theories not as a set of beliefs but as elaborated systems of thought that orient identification of problems, observation of situations, and analysis of data. Therefore, for research, there is an advantage in attempting to make a distinction between one's worldview and theories of learning.

However, this issue is more complicated than the distinctions I have just made. In some cases, acculturation into a research program that is guided by a particular theoretical perspective can lead to changes in one's worldview. This is not, in

itself, a problem. I emphasize the distinction between worldview and theoretical perspective out of concern that education of such researchers be aimed at deep understanding of other theoretical perspectives and developing the capability to use them, if their use becomes indicated.[5] Communication, coordination of research results, and accumulation of knowledge across theoretical perspectives are enhanced by education of this type.

IMAGES OF THE FUNCTION OF THEORIES

Consistent with the arguments that I have begun to present, I offer two images for theory use. The two images are similar but, as analogies, emphasize somewhat different characteristics.

Theories as Tools

Each theory that has a history of being useful offers advantages in doing certain types of work. There are no universal tools in any field. Every tool offers the possibility of doing certain kinds of work well (when used optimally) and being less helpful for other kinds of work. Of course, new uses for a tool may be developed, but some tools will never be the tools of choice for particular types of work. The advent of new tools might restrict the domain of common usage of an older tool. This is not the same as superseding it. The advent of the power nailer has not eliminated the use of hammers by those engaging in home repair and construction, only restricted their use. Part of good tool use is knowing about a tool's affordances and limitations. Niss (2007) argued that, "No single imported theory encompasses all of mathematics, mathematics learning and teaching, the relations between all kinds of individuals, groups, classrooms, institutions, communities, and societies with mathematics, nor all significant contexts and dimensions therein and thereof" (p. 10).

Theories as Lenses

Theories make explicit a set of assumptions underlying particular research and delineate the kinds of questions that are asked and the types of phenomena that are researched (Cobb, 2007; Putnam, 1987). Theories can be thought of as lenses. When one looks at a situation through a particular theoretical lens, some phenomena are prominent, whereas others are not (e.g., cultural practices from a sociocultural perspective, prior knowledge from a cognitive[6] perspective).[7]

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[5] This requires more than a course on learning theories in which two or three classes are spent reading and discussing seminal works related to different theories.

[6] *Cognitive* is used to denote theories that focus primarily on mental processes and to contrast them with theories that focus primarily on social processes.

[7] Worldviews can also be thought of as lenses through which we see our experience. However, theories afford a choice of lens.

"Like the lens of the video camera, each theory directs our attention towards particular settings, situations, roles, relationships, actions and objects, it simultaneously discards or ignores other features of the classroom" (Clarke, 2007, Slide #30).

The focus on particular phenomena (or relationships) characteristic of a theoretical perspective does not deny the existence or the importance of other phenomena. The mathematics research community needs a varied set of theoretical lenses to study the complexity of mathematics learning and teaching. With respect to theories of learning, Kieren (2000) described different theories as "separate 'truths,' providing different lenses through which to attain a more complete reciprocal embodied view of mathematics education" (p. 228).

Cobb (2007) pointed out the advantage of not only having multiple theoretical perspectives in mathematics education but also of ongoing attempts to compare and contrast them:

> As Hacking (2000) observed, while the specific questions posed and the ways of addressing them are visible to researchers working within a given research tradition, the constraints on what is thinkable and possible are typically invisible. . . . [Comparing theoretical perspectives] provides a means both of deepening our understanding of the research traditions in which we work, and of enabling us to de-center and develop a basis for communication with colleagues whose work is grounded in different research traditions. (p. 7)

Hacking's point is a key one. Because we are often unaware of the constraints of the perspectives that we take, we can fail to recognize the need for other perspectives and the importance of questions asked from those perspectives.

THE ADVANTAGES OF MULTIPLE THEORIES IN MATHEMATICS EDUCATION

Up to now, I have argued that theories do not supersede each other and have implied that theories of learning in mathematics education are not in conflict. Different theories allow us to work on a different set of problems or work on the same problem differently. In this section, I discuss two other advantages of doing research from different theoretical perspectives.

Important Work Is Done at Different Levels of Organization

Research at one level of organization (e.g., individual, small group, classroom, school, school system) does not negate the need for research at all the other levels. One can use the analogy of research on the human body, which can be done at the molecular (e.g., DNA), cellular, tissue, organ, system, or whole-body level. Work looking at the human body in physical and social contexts is also important. Similarly, psychotherapists can work at the individual, relationship, or system level. It would be wrong to value the macro level because it is more encompassing or "realistic" and devalue the more micro levels that can provide key knowledge not

accessible when working at macro levels. Contributions from all levels of organization are needed in mathematics education as they are in biology and psychotherapy. These contributions might come from different projects, or the coordination of analyses at different levels within a project. To work at different levels, different theories are often needed.[8]

When More Theories Are Available, a Broader Set of Explanations May Be Offered

One of the advantages of background theories is that they focus observations and provide a set of explanatory constructs. In many cases, a single theory is adequate and appropriate for the research undertaken. In other cases, the use of a single theory can unnecessarily limit the observations that are made and the types of explanations that can be generated. Whether looking at a task-based interview, a collaborative group, a classroom, or a school, there may be limitations in assuming a single theoretical perspective, that is, explaining all observed phenomena through that perspective. The availability of multiple explanatory theories and the use of multiple layers of analysis can, depending on the research, provide a richer set of constructs for accounting for observed phenomena. "Parallel analyses informed by complementary theoretical frameworks offers a form of safeguard against the possibility that use of a single analytical framework might render the study insensitive to potentially salient considerations and significantly reduce its explanatory potential" (Clarke, 2007, Slide #41).

In my own research, I find that I cannot make sense of an interview or a classroom situation without the availability of at least four lenses—constructivist, social, sociocultural, and affective. Each lens affords a different view of the same situation. To develop a compelling analysis, I need to be able to explain activity in terms of what one knows, what one feels, the norms of the situation, the affordances and limitations of the tools used, and the impact of cultural practices and values, among others. Often one type of analysis is foregrounded while others are backgrounded. However, I need to be ready at any time to invoke one of the backgrounded perspectives and/or conduct a separate analysis using one or more of the previously backgrounded perspectives. If I give a cognitive explanation to an excerpt of a task-based interview for which there is a more compelling affective explanation, or if I give a social explanation for a student's contribution to a classroom discussion for which there is a more compelling cognitive explanation, the effectiveness and the credibility of my account is diminished.

Confrey (1995), argues for a thoughtful adaptation of sociocultural and constructivist theories based on the problems to be addressed: "The question is not what is the relationship between the two theories, but what relationship does one want to construct between the perspectives, given the problem one seeks to study" (p. 202).

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[8] Ernest (2006, p. 6) argued, "learning theories do not imply particular research approaches." However, this point is not in conflict with the observation that different theories have different affordances and tend to emphasize different phenomena.

Cobb, Yackel, and Wood (1993) took the lead in making use of more than one theoretical perspective in a single mathematics education research program. Cobb and Yackel (1996) refer to their coordination of constructivist and social analyses as the *emergent perspective* (Cobb & Yackel, 1996). It derived from the researchers' perception that constructivism alone was unable to explain adequately phenomena inherent in mathematics classrooms. Other researchers have followed suit; for example, Goodchild (1998) coordinated analyses based on three theoretical perspectives in his studies of classrooms.

In a special issue of *The Journal of the Learning Sciences* (Sfard & McClain, 2002), "Analyzing Tools: Perspectives on the Role of Designed Artifacts in Mathematics Learning," six researchers examined the same classroom data to explicate the role of artifacts identified in the data. Each perspective contributed to understanding the data under study. The common feature of the six researchers' theoretical perspectives was the "centrality of cultural and social factors" (Sfard & McClain, 2002, p. 157). The purpose of this collective endeavor was to "present and contrast the differing approaches while trying to show that they are complementary rather than mutually exclusive" (Sfard & McClain, 2002, p. 157). The limitation on the range of theoretical frameworks represented seems to have been determined by a perspective on artifacts underlying the project, a view in which artifacts have a central rather than a secondary role. One could imagine a modification of the project in which researchers were charged with accounting for student learning in relation to the computer minitool used (the artifact on which the researchers focused primarily). In such a project, how might cognitive (e.g., constructivist) perspectives have broadened the explanations of the complex learning processes involved?

THEORY USE AS A MATTER OF CHOICE

The issue of theory use as a matter of choice is a complex one. I previously discussed the difference between worldviews and background theories of learning. I pointed out that a theory can be selected as appropriate for a particular kind of work and made the analogy with selecting a tool or lens.

In some cases, the identification of a problem or a research question is rooted in a particular theoretical perspective. For example, the researcher who asks how students' ethnic backgrounds might explain their perceptions of the power relationships in a particular style of teaching, and the researcher who asks about the conceptions of students who approach a mathematics problem in a particular way, are posing questions from different theoretical perspectives. It may be difficult to argue that these researchers made a choice about which theoretical perspective to use. The researchers may have identified the problem or question as a result of working or thinking within a particular perspective.

On the other hand, there are problems that do not derive from a particular theoretical perspective. How can we improve the teaching of ratio? What are the effects on learning of a recent initiative to reduce class size? Silver and Herbst (2007)

referred to these as "problems and needs of educational practice" (p. 41), that is, the initial articulation of the problem might not be rooted in a particular background theory of learning. In these cases, if we accept the notion of a theory as a tool or a lens, there are choices to be made about which tool (lens) or set of tools (lenses) would be most useful in addressing the problem.[9] Of course, the ability to make good choices is based on the researchers' knowledge of and ability to use various perspectives. The ability of researchers to make good choices would be enhanced by richer discourse in the field as to the relative affordances and limitations of particular theories and of the ways to make use of more than one theory (in those cases in which it is indicated) to increase the analytic and explanatory power of the work.

There also are cases in which a research question is generated based on a particular theory, and later, during data analysis, the researchers determine that the theory cannot be used to do all of what is needed. In such cases, choices are made to make use of other theoretical tools.

In cases in which theory use involves a choice by the researchers, the following two points become relevant.

1. *It can be important to distinguish between what one is looking at and what one is looking with.*

Lerman (2006) argued that, "Too often theories are taken to be unproblematically applied to a research study" (p. 12). One consideration in the application of a theory to a research study is the following. In theory use, what one looks *with* (e.g., social perspective, cognitive perspective) and what one looks *at* (individual, classroom lesson, school district) may be confounded. Lack of consideration of this distinction may result in a decision to choose a theory because of what one is looking at, rather than the perspective that might be most useful to look with. An example of this is an assumption that research focused on small groups or classrooms requires a social perspective, because these situations involve social interaction, or that analysis of an individual interview requires a constructivist perspective. However, as I mentioned previously, what one can examine and explain in an observation of, for example, a classroom lesson is different depending on the theoretical perspective adopted by the researcher. It is generally worth asking, "What lens or lenses would be appropriate for examination of this situation, given the types of claims that we endeavor to make or the range of explanations that we are attempting to offer?"

I illustrate this point with contrasting examples. Cobb (1999) conducted a whole-class teaching experiment focused on the learning of statistics. For his purpose, which involved characterizing the mathematical learning of a class over the course of a 10-week period, he used a social construct, *mathematical practices*, to identify landmarks in that learning. The construct refers to observations by an observer of mathematical ways of acting that have become accepted in the classroom and no

[9] For a thorough discussion of the roles of theory in mathematics education research, see Silver and Herbst (2007).

longer require justification. In contrast, my colleagues and I conducted whole-class teaching experiments (e.g., Simon & Blume, 1994) in which we endeavored to understand the students' learning in the context of particular mathematical tasks and discussions. We were interested in understanding classroom interactions in terms of the diversity of students' mathematical ideas. For this we needed a finer-grained analytic framework. For us, a whole set of issues of interest occurred prior to the establishment of a new mathematical practice. As a result, we focused on the mathematical conceptions underlying particular mathematical discussions.

This example demonstrates the use of a cognitive lens when looking at a group situation—a mathematics classroom—and empirical evidence from previous cognitive studies. However, one could ask whether such an adaptation of a cognitive lens is warranted.[10] Cognitive lenses have evolved as means to characterize the mental structures of the individual. How can we use such tools when the data derive from a classroom discussion? On the other hand, in what ways do we limit our analyses, if we are not able to examine how the cognitive resources being used afford and limit the conversation? This is an important problem and deserves significant discussion that is beyond the scope of this article. However, the point to be made here is larger than the issue of the use of cognitive lenses for analysis of large-group data. My intention is to encourage the disengagement (to the extent that it is possible) of what one is looking at from the theoretical perspective through which one is looking. The example also serves to point out the ongoing work that is needed to adapt theoretical tools for new uses and to coordinate the use of theoretical tools to accomplish particular research goals.

To make clear that this discussion of *looking at* and *looking with* is not specific to classroom research, consider an example from a different type of research, an investigation of the academic success of mathematics students from a Latino community. Researchers are most likely to use a sociocultural perspective to investigate the problem. However, might the research not also benefit from consideration of the norms and practices of the classrooms in which these students are involved and the understandings, and weaknesses in understanding, of the students, vis-à-vis the instruction? How to coordinate these lenses in ways that advance both empirical research and theory development will require ongoing efforts and discussion in the field.

2. *To the extent that theoretical perspectives are a matter of choice, justification of that choice can be an important part of research reports.*

My perusal of articles in *JRME* over the last 5 years (consistent with my experience with the literature more broadly) reveals very few articles in which the authors provide justification for the theory of learning used in the study. There likely are several reasons for this. In some cases, a justification may not be needed. For

[10] My own research (e.g., Simon & Blume, 1994) has done so at a level of research practice. We have attempted to use the perspective in a reasonable and reasoned way; yet, our research reports lacked any real articulation of general principles involved in such an adaptation.

example (as discussed previously) sometimes the question and study originate from within one theoretical perspective. However, in many studies, in which a justification of theory choice would be appropriate, the researchers only declare the theory that they are using. Ideally, the researchers would justify not only the choice of theory but also the aspects of the theory being emphasized for the particular study. Discussion of such rationale is important within a research program, as the theoretical framework is established, and in reporting results of the research program. Two benefits accrue from greater emphasis on justifying theory use. First, if one accepts the notion that a research report is a warranted argument, the justification of theory use contributes to the overall coherence of that argument. Second, such justifications (and the critiques they might engender) can advance discussions in the research community of theory use, adaptation, and coordination.

CONCLUSION

In this article, I have tried to advocate a view of theory use that is pragmatic, ("There is nothing as practical as a good theory," Lewin, 1951, p. 169), informed, rational, and inclusive. Mathematics education research is an applied science like medical research or psychotherapeutic research. Mathematics education researchers cannot afford to engage as philosophers in debating established theories. Rather, we need to (when appropriate) engineer pragmatic coordination of analyses done from different theoretical perspectives. Although some research is generated within a particular theoretical perspective, larger problems within the field of mathematics education—problems that are not grounded in a particular theoretical orientation (e.g., how might we reduce the achievement gap)—require that we find ways to bring together research done from different theoretical perspectives and generate research programs that make use of multiple perspectives.

I envision future researchers who deeply understand many of the theories available, are aware of the affordances and limitations of each, and use these theories strategically as they attack their research problems. Further, these researchers think about how to coordinate and accumulate research-based knowledge across perspectives. This vision requires serious, ongoing discussion comparing and contrasting theories and necessitates theoretical work on coordinating analyses from different perspectives. It sets a high bar for the education of doctoral students and active researchers. Lerman (2006) used the construct of apprenticeship to account for one's acculturation into the mathematics education discourse. The question I raise is whether the apprenticeship is into a particular theoretical perspective (constructivists beget constructivists and likewise for socioculturalists), or whether the apprenticeship is into a community characterized by the knowledge and abilities that I have just described. I conclude with a quote from Goldin (2003), who was writing about the schism between mathematicians and scientists on one side and mathematics educators and science educators on the other. I believe the idea is just as applicable within mathematics education.

> It is time to abandon, knowledgeably and thoughtfully, the dismissive fads and fashions—the "isms"—in favor of a unifying, non-ideological, scientific and eclectic approach to research, an approach that allows for the *consilience* of knowledge across the disciplines [in our case, across theoretical perspectives]. (p. 176)

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