Searching for the Most Amazing Thing Over the Last 20 Years: A Review of *Research on Technology and the Teaching and Learning of Mathematics, Volume 1 and Volume 2*

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Back in 1988, Tom Snyder (of Tom Snyder Productions, one of the most famous early software publishing companies) and Jane Palmer wrote a prophetic book called *In Search of the Most Amazing Thing: Children, Education, and Computers.* Their thesis was twofold: First, they pointed out that technology, which was just beginning to be introduced in grade schools, was so compelling that educators were "… more interested in so-called computer literacy than the real thing, *literacy*" (p. 2). Snyder and Palmer called for stakeholders to determine what their educational priorities were, and then to figure out what technology could do to support them. Second, they emphasized the view that teachers are indispensible components in the teaching and learning process, and that no computer will ever take their place. After 20 years, we believe that Snyder and Palmer would be gratified to read Heid and Blume's newly published two-volume set that contains a thorough anthology of how educators have defined priorities for the teaching and learning of mathematics and the pivotal roles that both the teacher and the technology play within that process. In our view, the editors have attained their goal of assembling a comprehensive digest that "… will enable the creation and implementation of curricula that capitalize on technology and will help teachers orchestrate the use of technological tools in school mathematics classrooms" (vol. 2, p. viii).

The literature is divided into two volumes. Volume 1 contains nine chapters that describe ways in which mathematics learning is affected by the use of technology across a variety of content areas. Volume 2 contains ten case studies describing the process by which successful technology-intensive curricula and technological tools were developed and seven additional chapters containing perspective pieces describing issues that arise as experts experiment more broadly at the intersection of technology, pedagogy, and mathematics education. This review addresses some of the most salient issues that appeared to emerge throughout many of the chapters.

ISSUE 1. IMPLEMENTING TECHNOLOGY IN THE SOCIAL SETTING OF THE CLASSROOM

One key point emphasized throughout many of the chapters is that technology alone is not a "silver bullet" that can overcome all the difficulties inherent in teaching and learning mathematics. As Heid and Blume state, ". . . it is the confluence of technological environment, teachers, learners, curriculum, and mathematical activity that sets the stage for changes in the teaching and learning of mathematics in the context of technology" (vol. 1, p. 420). This idea may well stand as the greatest takeaway comment from the chapters because each, in its own way, addresses one or more of these components.

One chapter that focuses most specifically on this confluence is Zbiek and Hollebrands' (vol. 1, chap. 7). They describe research regarding the incorporation of technology into classroom practice. The authors use the metaphor of a teacher professional continuum to describe the various teacher roles that researchers have identified as teachers become more comfortable with technology. Their description of the synergistic relationship between teachers' experience with technology and their content knowledge and practice characterizes Mishra and Koehler's (2006) notion of "technological, pedagogical, content knowledge" (TPACK) for mathematics educators. The major emphasis of the chapter is not for teachers to acquire any particular set of skills or pick one particular role; instead, teachers need to be more aware of how to integrate technology into the classroom discourse so that technology-based conjectures and arguments become normative. Suggestions included asking more open-ended questions that demand technological explanations and encouraging students to become the ultimate arbiters of whether those explanations are clear and accurate.

Several other chapters describe how the interplay among technology, content, practice, discourse, and activity plays out when teaching specific topics. In the context of teaching geometry, Laborde and Laborde (vol. 2, chap. 2) and Goldenberg, Scher, and Feurzeig (vol. 2, chap. 3) describe the teacher's critical role in encouraging students to make and discuss conjectures as they use various features in dynamic geometry environments (DGEs) such as dragging, scripting, and measuring. The "math activity" then becomes using deductive arguments to justify and further explain their findings and conjectures. Clements, Sarama, Yelland, and Glass (vol. 1, chap. 3), Sarama and Clements (vol. 2, chap. 5), and Battista (vol. 2, chap. 6) all describe analogs of this technology-enhanced discourse at the elementary level. Their main emphasis was to describe the ways in which the teacher can create curricula and support social climates so that students' geometric explanations reach increasingly higher van Hiele levels. Sarama and Clements describe *learning trajectories* that "consist of rich descriptions of children's thinking and learning in a specific mathematical domain and a conjectured instructional route for that learning" (p. 114). These learning trajectories were used as guidelines to design software and activities that guided children through seven levels of increasingly challenging tasks. Battista also describes how the implementation of Shape Makers is successful when the teacher and students enter into cooperative dialogue, so that the teacher can support the students' transitions from static thinking to considering the interrelationships between spatial structurings and geometric properties.

Dunham and Hennessy (vol. 1, chap. 8) describe the confluence of teacher, technology, and curriculum in terms of equity. They point out that although the most creative users of technology in school classrooms tend to be teachers with technological expertise who take part in related professional development, past research has shown that children from suburban schools were more likely than their urban and rural counterparts to have teachers with such a profile. One interesting corollary to this observation emerged as well: The teachers of disadvantaged students tended to believe that their students needed to focus primarily on drill-and-practice problems and were best served by staying away from even handheld devices such as calculators. Dunham and Hennessy argue that this view served as a roadblock by citing a study on "detracking" that showed a great deal of success came from having these same students engage in higher order thinking skills through the proper use of technology. The findings from equity studies concur with those from content-based research: Professional development is vital for supporting the convergence of technology, dialogue, and mathematical activities in any classroom.

ISSUE 2. THEORETICAL FRAMEWORKS FOR RESEARCHING TECHNOLOGY-ENHANCED LEARNING

The editors described the critical role of research by stating, "We were interested in making visible the extent to which research related to the creation of technological tools and curricula went far beyond the often-minimal market research associated with commercial products" (vol. 2, p. 462). Many of the authors highlight this goal by making explicit references not only to their research methods and results but also to the epistemological ideas that drove their design and research processes. We found it interesting to note that several of the chapters assume a sociocultural perspective to examine the use of technological tools as a process of semiotic mediation, whereas others take a more Piagetian approach to both design and research.

In their review of DGE software, Hollebrands, Laborde, and Sträßer (vol. 1, chap. 6) assume a Vygotskian perspective to describe the ways in which researchers view computers and teachers as mediators of activity. Learning from this perspective is seen as occurring when students internalize their activity, which, in turn, supports further mental actions. The advantage of this view is that it provides designers with a focus on how to create features that would support students' activities such as dragging, scripting, and creating loci to support the internalization processes. Heid and Blume (vol. 1, chap. 2) also assume a sociocultural perspective to frame their observations in terms of activity theory. This approach enables them to examine how technological tools mediate students' activities and the consequent cognitive activities in which students engage. Their main thesis is that activity with technology can ". . . change the nature of opportunities for the mathematical activities of conceptualizing, representing, generalizing, symbolic work, and modeling as well as for student roles" (vol. 1, p. 59).

We found it interesting to note that although authors such as Hollebrands, Laborde, and Sträßer assume a Vygotskian perspective to research the role of DGEs in classrooms, they state that development efforts (such as the one involved in the creation of Cabri) are often inspired by a "Piagetian approach" that focuses on the use of microworlds as sources for perturbations. This approach was echoed by Battista who takes a constructivist perspective on mathematics learning and teaching by encouraging students to "invent, test, and refine their own ideas rather than unquestioningly follow procedures given to them by others" (p. 136). Olive and Lobato also assume a Piagetian perspective on design by describing how one project called Tools for Interactive Mathematical Activity (TIMA) was designed to support students' efforts to "disembed," or partition, fraction bars of various sizes into smaller sections, disassemble them, and then re-create the original bar and other bars from these smaller parts. In so doing, the students are building on their whole-number knowledge to construct new rational number understanding.

ISSUE 3. DESIGN FRAMEWORKS FOR TECHNOLOGY-ENHANCED CURRICULA

We found Doerr and Pratt's (vol. 1, chap. 7) distinction between expressive and exploratory modeling activities to be a useful framework for categorizing many of the projects described in the two volumes. Expressive activities generally involve having students use open-ended approaches to model a problem solution for later class discussion. This distinction characterizes many of the content-based chapters in volume 1 such as the use of The Geometer's Sketchpad and computer algebra systems (CAS). In contrast, many of the case studies in volume 2 can be considered exploratory modeling activities. These projects involve having students explore targeted relationships among mathematical objects that are built into smaller, more controlled environments. Advantages of this approach include a very short learning curve and a shared context on which an entire class can reflect.

As Doerr and Pratt note, one benefit of expressive modeling activities is that they evoke a rich diversity of approaches that can be debated in whole-class discussions. In their chapter, Confrey and Maloney (vol. 2, chap. 8) illustrate this point by describing how students using Function Probe chose different models to express algebraic relationships by exploiting different representations of the same function. Similarly, Schorr, and Kaput (vol. 2, chap. 9) offer fascinating transcripts to demonstrate how students used different SimCalc graphs to express ideas of rate using distance and velocity.

Modeling tasks enacted in DGEs are often expressive. According to Goldenberg, Scher, and Feurzeig (vol. 2, chap. 3), who offer a fascinating look into the parallels between the development trajectories of Cabri and Sketchpad, pedagogy was not a driving factor in the initial development of either program; instead, both projects were based on their designers' "mathematical and aesthetic inclinations, which led them to produce two software environments that users agree are elegant, functional, and valuable both in schools and in mathematical and mathematics education research" (p. 79). In terms of Doerr and Pratt's distinction, one could conclude that DGEs provide tools for expressive modeling in Euclidean geometry but do not direct a user to explore one specific concept or relationship.

In contrast to these examples, many of the case studies in volume 2 describe tasks that could be considered exploratory in nature because they target specific relations among mathematical topics. In particular, Battista's (vol. 2, chap. 6) Shape Makers microworld is an add-on to Sketchpad designed to support students' explorations of the hierarchical relationships among different polygons. Students do not create the shapes, they drag prototype "makers" and infer the relations based on their explorations. Olive and Lobato's (vol. 1, chap. 1) description of an add-on to SimCalc can also be considered exploratory in nature. This microworld targeted students' explorations of rate by limiting their activities in the SimCalc environment to entering one character's distance and time in order to support the construction of rate as an intensive ratio between the two variables. Kieran and Saldanha (vol. 2, chap. 15) illustrate the power of asking students to use CAS to explore the factoring of $x^n - 1$. Their chapter illustrates the power of using technology to support students' explorations to make conjectures and formalize proofs of topics that would otherwise be out of the conceptual reach of many students.

ISSUE 4. FOCUSING ON RESULTS: WHAT WORKS?

The two volumes contain relatively few chapters describing studies that compare the "effectiveness" of computer-enhanced curricula with that of traditional methods. Tall, Smith, and Piez (vol. 1, chap. 5) explain the paucity of comparative research in the field in general by noting that it is difficult (and not scientifically valid) to compare two instructional approaches if they do not share the same instructional foci. The heart of the impasse is that most traditional curricula emphasize procedural knowledge as the path for conceptual development, whereas technology-infused curricula often focus on developing conceptual understanding by exploring multiple representations and by off-loading procedural work. Tall, Smith, and Piez also note that the results of such comparison studies are not productive, because they tend to report that the computer use did little to improve procedural skills but the measures used could not reveal the benefits of computer use, such as the insights that students gain when faced with the challenge of solving novel problems.

Another reason for the small number of scientifically based research reports in these volumes is that the idea of combining what is known among cognitive scientists, educators, and mathematics education researchers is relatively new. Like all new fields of research, the majority of groundwork must be conducted through iterative processes of research and design. Dugdale's (vol. 2, chap. 1) discussion of the evolution of Green Globs provides a telling example of the importance of cyclic design. She states that when she first imagined designing Green Globs, she envisioned students working individually at their own computers to create various functions that would hit as many targets ("globs") as possible. However, her classroom-based observations revealed that students tended to be natural conversants who enjoyed sharing various "glob" strategies. This led her research-and-development team to think of novel ways to expand students' involvement in a mathematical community. One of the most successful innovations was the idea of leveraging a computer network to store and display work in a shared space. When students were able to view other students' successes, the evolution of functions generated by struggling students was impressive. Students appeared to be developing a deeper understanding of functions and how various parameters affect the graph. These insights would not have surfaced from a strictly controlled, randomized trial design.

One chapter that reports on several comparative studies is Ritter, Haverty, Koedinger, Hadley, and Corbett's (vol. 2, chap. 7) analysis of the Cognitive Tutor. The theory behind this software is that a cognitive tutor can come to know individual students on the basis of the steps that they take during practice exercises. Results from three scientifically based research designs (one within-teacher design, one matched-pairs study, and one study comparing various subgroups of students including English language learners) indicate that students who used the Cognitive Tutor scored significantly higher on standardized tests than those in control groups. These results highlight that the inverse of Tall, Smith, and Piez's theory holds: If the overall instructional goal of the software does align with non-computer-enhanced instruction, then comparative research can be conducted.

In their summary, the editors of the series address the overall findings from research to this point by stating that the answer to the question of "what works" refers to the main message of the book: Technology provides the potential for large gains in classroom settings if and only if the teacher implements the software or CAS in a way that is consistent with the social norms of the classroom and the teacher's epistemology and is in close proximity to the stated goals for each course in which it is used. At this point, very few comparative studies are able to take all these variables into consideration.

ISSUE 5. CALLS FOR FUTURE RESEARCH

In their chapter on research, policy, and technology use, Ferrini-Mundy and Breaux (vol. 2, chap. 17) describe their view on the importance of scientifically based research because of its critical effect on technological decisions.

> In general, policy about technology use is quite varied and seems to be based largely on local opinions, values, and perspectives about whether technology can support learning effectively. It is perhaps in this area that the concerted efforts of researchers stand a chance of having the most impact. (p. 435)

The authors review multiple state and national frameworks to argue that future policy decisions and research funding will rely on paradigms of "scientifically based research" (SBR). In other words, the era of funding programs that report on results not demonstrated (RND) is over. It is now critically important that technology-related education research includes systematic, empirical methods. The data collected must be measureable and establish a path for causality.

This mandate highlights a critical question: Can researchers address the pressing calls for future research using SBR methods? For example, it is difficult to consider how the following calls for future research could be made within an SBR design:

- Hollebrands, Laborde, and Straber (vol. 1, chap. 4) call for more "research devoted to longer-term teaching with regular use of DGS" (p. 191),
- Doerr and Pratt (vol. 1, chap. 7) call for "classroom-based research on modeling" (p. 280),
- Heid and Blume (vol. 1, chap. 2) call for research on ". . . how students move between, connect, and reason from multiple representations" (p. 98).

In their conclusion to volume 2, the editors address this conundrum by stating that the field needs both types of research. On the one hand, we need to continue to pursue the design-research cycles that have enabled the development of provocative educational technologies. On the other hand, we also need to move forward with SBR to guide policy and larger-scale adoptions.

SUMMARY AND FINAL COMMENTS

In summary, *Research on Technology and the Teaching and Learning of Mathematics* provides a broad perspective on the history and current state of the field. All the authors are well known and speak eloquently about the insights gained from their design and research cycles. We now look forward to a sequel in which experts share their visions for future-oriented research and development. Perhaps we will be reading analyses focusing on embodied cognition, the widespread use of the Internet as a social and practice-based medium for schools, or the potential for newer mathematical modeling programs that might serve as the DGE and CAS of tomorrow. Regardless, this compilation supports the hypothesis that working at the intersection of mathematics, pedagogy, and technology will produce significantly better educational results than each discipline would achieve on its own. As Frans Johannson, author of *The Medici Effect*, put it, "In every arena, whether in the sciences or the humanities, business or politics, there is a growing need to connect and combine concepts from disparate fields. That is how we will find new opportunities, surmount new challenges, and gain new insights. That is the way we will create our future. The future lies at the intersection, and if you wish to help create it, find your way there" (p. 187).

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