The Role of Writing Prompts in a Statistical Knowledge for Teaching Course

Randall E. Groth
Salisbury University

Teachers of grades Pre-K-8 are charged with the responsibility of developing children's statistical thinking. Hence, strategies are needed to foster statistical knowledge for teaching (SKT). This report describes how writing prompts were used as an integral part of a semester-long undergraduate course focused on building SKT. Writing prompts were designed to help assess and develop the subject matter knowledge and pedagogical content knowledge of prospective teachers. The methods used to design the prompts are described. Responses to a sample prompt are provided to illustrate how the writing prompts served as tools for formative assessment. Pretests and posttests indicated that prospective teachers developed both SKT and knowledge of introductory college-level statistics during the course. It is suggested that teacher educators employ and refine the prompts in their own courses, as the method used for writing and assessing the prompts is applicable to a broad range of statistics and mathematics courses for teachers.

Key words: Statistical knowledge for teaching, Mathematical knowledge for teaching, SOLO Taxonomy, Writing prompts, Formative assessment

Current scholarship in teacher education reveals the complexity of the knowledge needed by teachers. Subject matter knowledge alone is not sufficient. The Learning Mathematics for Teaching (LMT) project characterized mathematical knowledge for teaching (MKT) as consisting of two primary elements: subject matter knowledge and pedagogical content knowledge (Hill, Ball, & Schilling, 2008). Pedagogical content knowledge helps teachers make subject matter comprehensible to students. It has been described as a "special amalgam of content and pedagogy that is uniquely the province of teachers, their own form of professional understanding" (Shulman, 1987, p. 8). Components of subject matter knowledge and pedagogical content knowledge hypothesized by the LMT project are shown in Figure 1.

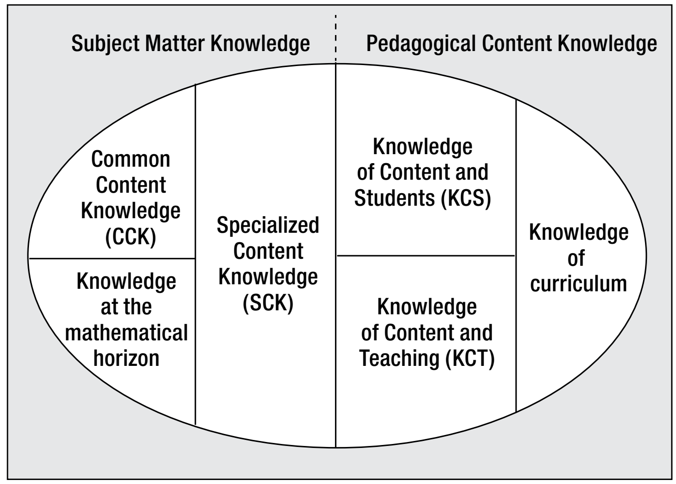

Figure 1. **Hypothesized components of subject matter knowledge and pedagogical content knowledge (Hill, Ball, & Schilling, 2008, p. 377).**

This report describes how I used writing prompts in a semester-long undergraduate course devoted to building prospective Pre-K-8 teachers' subject matter knowledge and pedagogical content knowledge for teaching statistics. In this article, I use the term "statistical knowledge for teaching" (SKT) rather than MKT to acknowledge statistics and mathematics as distinct disciplines (Groth, 2007; Moore, 1988). For example, many statistical activities, such as study design, survey question design, and measurement, have substantial nonmathematical components (Rossman, Chance, & Medina, 2006). Although the LMT model explicitly focuses on MKT, researchers have found it to be of use in describing SKT as well (Burgess, 2011). The degree of overlap between the two disciplines makes it feasible to ground the discussion of SKT in a theory of MKT (Groth, 2007), and the methods I describe for designing and assessing writing prompts are not restricted to use in statistics courses. Some specific examples of subject matter knowledge and pedagogical content knowledge for statistics will be discussed next.

Subject Matter Knowledge

Subject matter knowledge includes common content knowledge, specialized content knowledge, and knowledge at the mathematical horizon (Hill, Ball, & Schilling, 2008). *Common content knowledge* is that which is required in teaching as well as in other professions. Examples include knowing how to compute and interpret frequently used measures of center and spread, understanding the idea of random sampling, and recognizing variability as a central object of study in statistics (Groth, 2007).

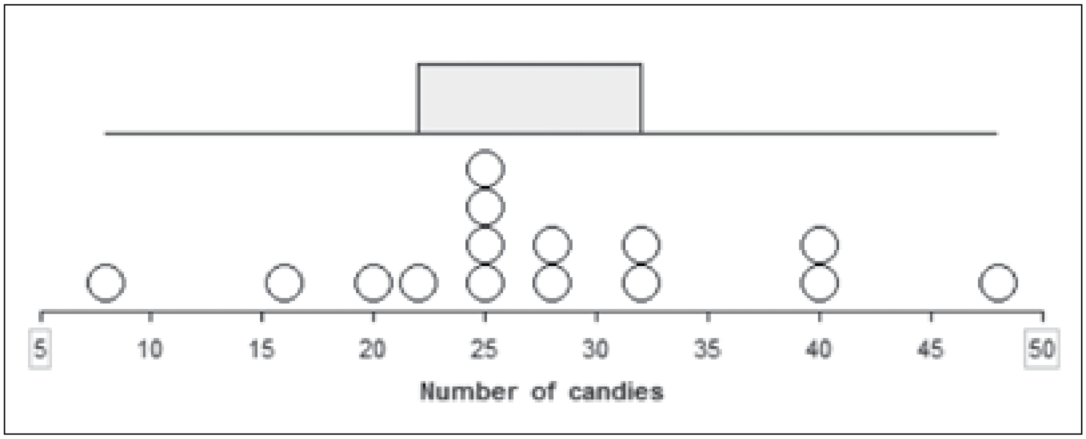

Figure 2. **Representing a data set for number of candies per student with a dot plot and hat plot.**

Specialized content knowledge is that which is unique to teaching. It allows teachers to select representations to make subject matter comprehensible. For instance, hat plots produced using dynamic statistics software (Konold & Miller, 2005; Figure 2) are not conventional data displays, but they provide a useful intermediate step between reading dot plots and box plots (Watson, 2008). Children often have difficulty interpreting box plots because they condense the data to display summary statistics rather than showing individual values (Bakker, Biehler, & Konold, 2005). Hat plots address this difficulty because they are generally displayed above plots showing individual values, as in Figure 2. By examining both representations simultaneously, students can begin to understand how individual values contribute to a more condensed display. The median is initially not included in a hat plot to avoid another intuitive difficulty: in a box plot, the median is usually closer to one of the quartiles than the other, even though the same number of data points resides within each quarter of a box plot (Watson, Fitzallen, Wilson, & Creed, 2008). Hat plots allow students to initially focus on the more intuitive idea of "modal clump" (e.g., the middle 50% of the data is highlighted in Figure 2), which connects to children's tendencies to partition data into low, middle, and high categories (Konold et al., 2002). Once students understand how a display can condense data and partition it into groups, adding the median to a hat plot showing the middle 50% of the data can complete the transition from dot plots to box plots. The hat plot representation, therefore, can be considered an element of specialized knowledge because it helps make box plots comprehensible, and it was invented to serve this purpose rather than to be widely used among those outside the teaching profession.

Horizon knowledge helps teachers understand how activities done during a given lesson foreshadow more advanced ideas to be studied later on. For example, elementary school students work on describing populations (e.g., students in their classroom) using descriptive statistics and graphs, but a transition to using samples to make inferences about larger populations must eventually be made. Teachers who have horizon knowledge of formal statistical inference can pose questions that prompt students to think about the extent to which data from one class may generalize to a larger population (e.g., all students in school or all students in the country). They can also look for opportunities to emphasize ideas that comprise the foundation for formal inference, such as sample size, randomness, sampling variability, and bias (Ben-Zvi, Gil, & Apel, 2007). As they do so, teachers can bring students progressively closer to techniques of formal inference by gradually formalizing their early statistical investigations.

Pedagogical Content Knowledge

Pedagogical content knowledge includes knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum. Each component marks out a type of knowledge needed specifically for tasks related to teaching.

Knowledge of content and students can be described as "content knowledge intertwined with knowledge of how students think about, know, or learn . . . content" (Hill, Ball, & Schilling, 2008, p. 375). This type of knowledge allows teachers to anticipate difficulties students will have in learning a subject and address them when planning and implementing lessons. Research provides a fair amount of information about developmental levels through which Pre-K–8 students are likely to pass as they learn statistics and probability. Jones et al. (2000) mapped levels of thinking one can expect from elementary school students in regard to describing, organizing, representing, and analyzing data. Mooney (2002) did the same for middle school students. One of the key insights from these studies is that students often use their own idiosyncratic strategies to handle data before progressing to conventional methods. Knowing about common student strategies can help teachers anticipate potential inroads and obstacles in developing students' statistical thinking.

Knowledge of content and teaching entails having a repertoire of content-specific strategies for teaching concepts. For instance, it is beneficial for teachers to know how to help students understand the arithmetic mean as a fair share and as a balance point. To portray the mean as a fair share, students can be given snap cubes to represent the numbers of a given object belonging to each person in a group. The cubes can then be piled together and redistributed so that everyone has the same amount (if the total number of cubes is not a multiple of group size, distributing fractional amounts of a cube can be discussed). The amount each person receives is the mean. To help students understand mean as a balance point, teachers can give students data on the number of pets each student in a class has. Sticky notes can be used to construct a dot plot of the data set, and students can move all of them to the mean value of the data set. To preserve the original mean, they can experiment with ways to rearrange the sticky notes so the balance point remains at the original mean. In doing so, they produce a number of data sets that all have the same mean (Franklin et al., 2007).

Knowledge of curriculum suggests knowing the structural characteristics of curricula. One part of developing curriculum knowledge can be coming to understand different philosophies underpinning Pre-K–8 curricular materials for teaching statistics. Several reform-oriented curriculum series funded by the National Science Foundation (NSF) are based on principles of inquiry-oriented instruction that may be unfamiliar to prospective teachers whose school experiences were more traditional in nature (Senk & Thompson, 2003). Lloyd and Behm (2005) found that when presented with reform-oriented curriculum materials, prospective teachers tended to gravitate toward traditional-looking elements of the texts rather than inquiry-oriented ones. Understanding the purpose and benefits of inquiry-oriented instruction enables teachers to implement innovative curricula with fidelity to the intentions of the curriculum designers.

The Statistical Knowledge for Teaching (SKT) Course

I used the Learning Mathematics for Teaching (LMT) conceptualizations of subject matter knowledge and pedagogical content knowledge as starting points in designing a one-semester SKT-focused course for prospective Pre-K–8 teachers. The use of the LMT framework prompted me to go beyond just common content knowledge goals for the course. Although common content knowledge is important, it is ideally developed in tandem with other types of subject matter knowledge and pedagogical content knowledge. An immediate implication of the LMT framework is that it is not adequate to ask prospective teachers simply to solve mathematics and statistics problems in such a course, even if those problems focus on conceptual understanding (Hiebert & Lefevre, 1986) and have high levels of cognitive demand (Smith & Stein, 1998). Therefore, I set out to design course experiences that required engagement with all aspects of the LMT framework.

The core teaching strategies used in the course and their connections to the LMT framework are shown in Figure 3. Inquiry-oriented statistics activities relevant to

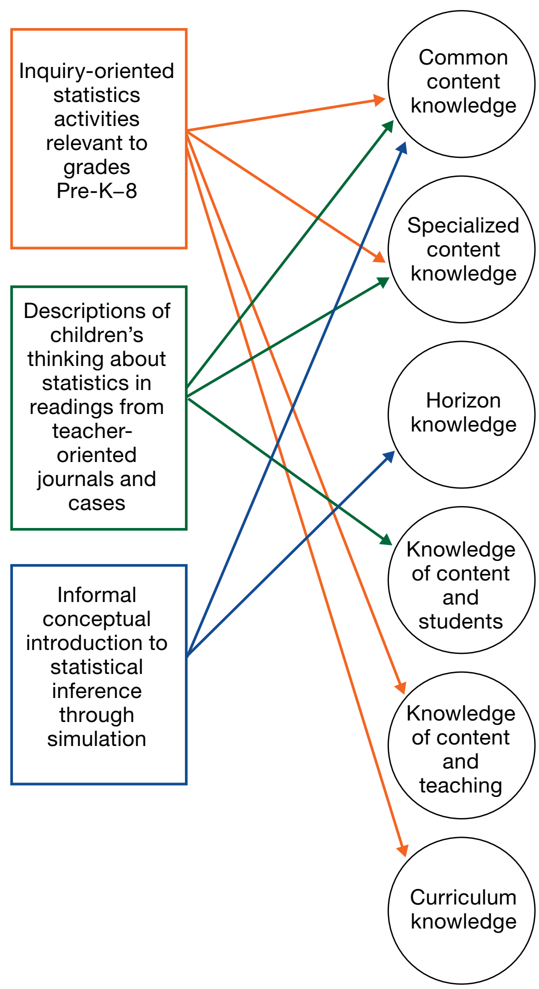

Figure 3. **Connections between core instructional strategies and the LMT framework.**

grades Pre-K–8 were selected from the required course textbook (Perkowski & Perkowski, 2007) and other sources (e.g., Burns, 2000; Rossman & Chance, 2008; Scheaffer, Gnanadesikan, Watkins, & Witmer, 1996). The required textbook integrated activities from NSF-funded curricula (Senk & Thompson, 2003). These activities were intended to build conceptual understanding of statistics (common knowledge) while also providing ideas for teaching specific content (knowledge of content and teaching) and an introduction to inquiry-oriented curricula (curriculum knowledge). To supplement and extend the inquiry-oriented activities done in class, I selected readings from *Teaching Children Mathematics* and *Mathematics Teaching in the Middle School*. These readings, as well as two classroom cases I selected from *Discovering Mathematical Ideas* (DMI) (Russell, Schifter, Bastable, Konold, & Higgins, 2002), helped build knowledge of content and students by providing descriptions of children's thinking about statistics. They also helped build specialized knowledge by introducing representations suitable for making content understandable to children (e.g., hat plots) and common knowledge by prompting readers to think conceptually about core statistics content (e.g., choosing between mean and median to describe data). Near the end of the semester, a unit introducing formal inference was included. The unit used materials described by Garfield and Ben-Zvi (2008) to provide an intuitive foundation on the meaning of sampling distributions, hypothesis testing, and confidence intervals through simulation. It was included in the course to foster horizon knowledge by providing a sense of statistical content studied beyond the Pre-K–8 curriculum and to build common content knowledge of inference ordinarily included in college-level statistics courses.

This report will focus in-depth on writing prompts used in conjunction with the course readings from *Teaching Children Mathematics* and *Mathematics Teaching in the Middle School*. The relationships among statistics content, the selected readings, and supporting course activities are shown in Table 1. Although the writing prompts were used in conjunction with most of the statistical topics included in the course, they were not used for all. Specifically, units on bivariate data and inference were not accompanied by articles and writing prompts. However, writing prompt sets did comprise more than 60% of the homework assignments for the course.

Writing Prompts in the SKT Course

I chose writing as a means to help prospective teachers analyze the teacher-oriented journal articles because it encourages learners to place organizational structures on their thinking (Vygotsky, 1987). As a self-reflective activity, writing supports learners' metacognition, enabling them to select and employ appropriate problem-solving strategies (Pugalee, 2004). Writing about a text can also help support generative reading of it. Generative reading involves applying background knowledge to interpretation of a text, thinking about relationships among ideas within a text and across texts, and identifying important concepts (Borasi, Siegel, Fonzi, & Smith, 1998).

Figure 4 provides an overview of the design and use of writing prompts in the SKT course. I wrote five prompts for each article. (The full set of writing prompts is available. See "Supplement: Assignments." Prompts addressed both subject matter knowledge and pedagogical content knowledge. Some of the questions in the prompts required literal reading, and others required generative reading. As I read responses to the prompts, I had both summative and formative assessment purposes in mind. I used a rubric to assign summative scores to each set of writing prompts, and I examined responses to selected prompts in more depth to gain insight about adjustments to the course to help advance prospective teachers' learning. It should be noted that although the process outlined in Figure 4 was developed in a statistics course for prospective elementary school teachers, it is not necessarily restricted to a single subject area or grade band. Details about the process are provided in the remainder of this section.

Each of the writing prompts was designed to address one or more of the six components of the LMT framework described earlier: common knowledge, specialized knowledge, horizon knowledge, knowledge of content and students, knowledge of content and teaching, and curriculum knowledge (Hill, Ball, & Schilling, 2008). Six types of questions described by Day and Park (2005) were used within the prompts to encourage active reading: literal comprehension, reorganization, inference, prediction, evaluation, and personal response. Examples of each type of prompt and their alignment with the LMT framework are provided in Table 2. Literal comprehension questions are those that can be answered directly from a portion of the text. The remaining five types of questions require generative reading. Reorganization questions require piecing together information from various parts of the text. Inference questions go beyond literal reading of the text to prompt students to draw on background knowledge and experiences while reading to formulate a response. Prediction questions involve extending a text by drawing on knowledge obtained from reading it. Evaluation questions prompt readers to express reasons for agreement or disagreement with a portion of the text. Personal response questions prompt readers to express their feelings about the text and its content. Some writing prompts contained more than one of the six types of comprehension questions, and some also were

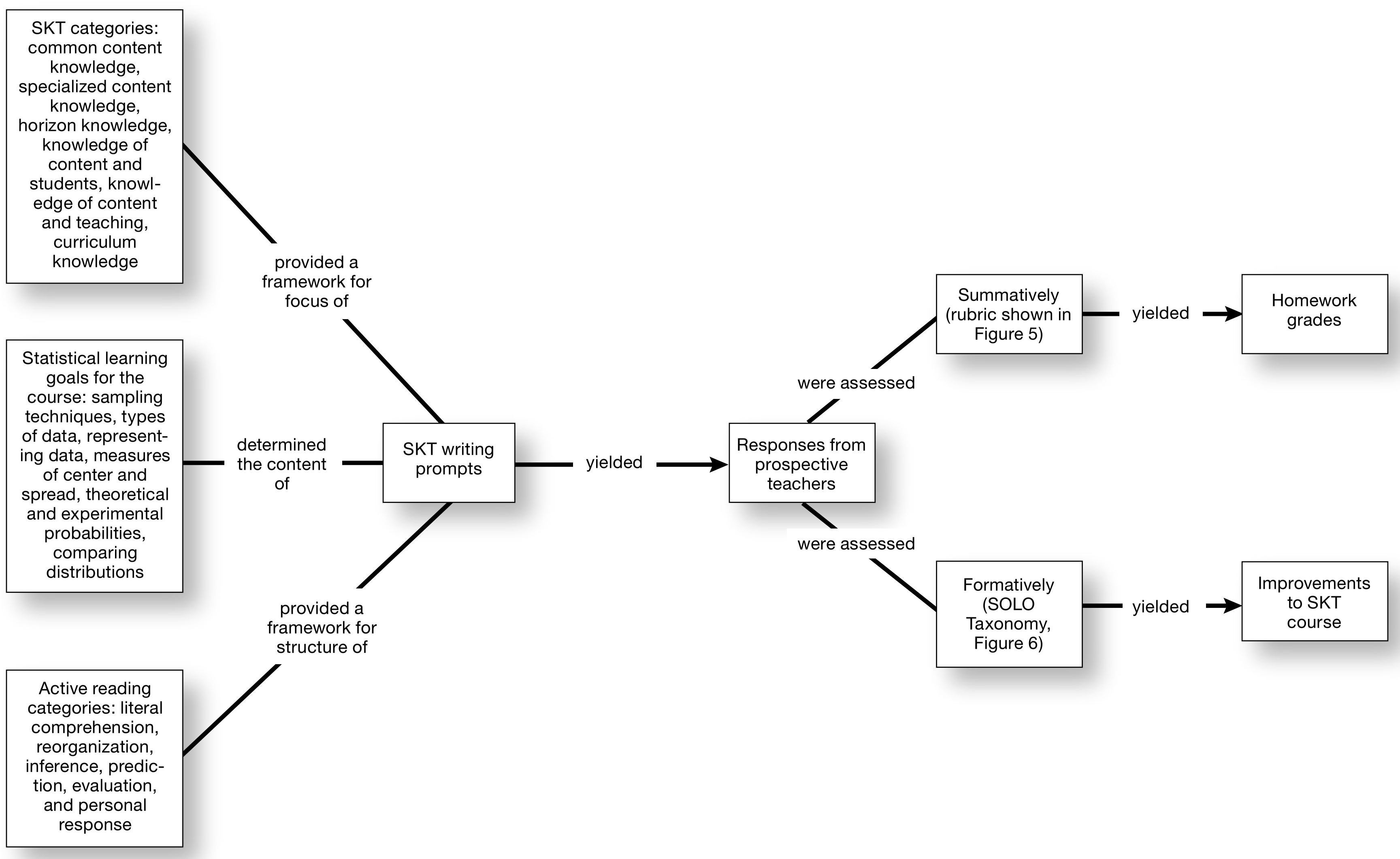

Figure 4. **Summary of design and use of writing prompts in SKT course.**

Table 1
Relationships Among Statistical Content for SKT Course, Readings, and Supporting Course Activities

Statistics content	Selected readings	Sample supporting course activities
Sampling techniques	"How do students think about statistical sampling before instruction?" (Jacobs, 1999) "Capture and recapture your students' interest in statistics" (Morita, 1999)	Completing "Random Rectangles" activity (Scheaffer, Gnanadesikan, Watkins, & Witmer, 1996) to establish the distinction between random and subjective sampling and to compare distributions of statistics produced by them.
Types of data	"Statistics in the elementary grades: Exploring distributions of data" (Franklin & Mewborn, 2008) "It's a fird! Can you compute a median of categorical data?" (Leavy, Friel, & Mamer, 2009)	Discussing classroom cases from *Developing Mathematical Ideas* series (Russell, Schifter, Bastable, Konold, & Higgins, 2002) to become familiar with how children tend to organize both quantitative and categorical data.
Representing data	"Reflecting on students' understanding of data" (McClain, 1999) "Students' interpretations of misleading graphs" (Harper, 2004) "The representational value of hats" (Watson, Fitzallen, Wilson, & Creed, 2008)	Analyzing hat plots and box plots with TinkerPlots (Konold & Miller, 2005) and Fathom (Finzer, 2002) to understand how a hat plot serves as a transitional representation between a dot plot and a box plot and to analyze how changes in a data set correspond to changes in accompanying box plot and hat plot graphical representations.
Measuring center and spread	"Mean and median: Are they really so easy?" (Zawojewski & Shaughnessy, 2000) "Statistics in the middle grades: Understanding center and spread" (Kader & Mamer, 2008)	Solving problems from *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2004) to understand the mean conceptually and to become familiar with the structure and goals of reform-oriented mathematics curricula.
Theoretical and experimental probabilities	"Predictions and probability" (McMillen, 2008) "Providing opportunities to learn probability concepts" (Tarr, 2002) "Enriching students' mathematical intuitions with probability games and tree diagrams" (Aspinwall & Shaw, 2000)	Completing "Numbers on a Line" activity (Burns, 2000) to compare theoretical and experimental probabilities for rolls of a pair of dice using a physical simulation. The physical simulation set the stage for computer-based simulations that took advantage of technology to run a large number of trials and analyze the resulting distributions.
Analyzing distributions	"Proportional reasoning: Lessons from research in data and chance" (Watson & Shaughnessy, 2004)	Describing shape, center, and spread of distributions of numbers of orange candies in a sample in an activity from *Workshop Statistics* (Rossman & Chance, 2008)

Table 2
Sample SKT Writing Prompts

Sample writing prompt	Article	Reading comprehension question types	Relevant SKT components
In your own words, explain how snap cubes can be used to determine the arithmetic mean of a set of quantitative data.	Franklin and Mewborn (2008)	Literal comprehension	Knowledge of content and teaching
How are hat plots similar to box-and-whisker plots? How are they different?	Watson, Fitzallen, Wilson, and Creed (2008)	Reorganization	Specialized content knowledge
On p. 438, the authors commented in regard to item 3, "This type of item assesses students' conceptual understanding of mean." Explain what the authors may mean by "conceptual understanding." How is it different from other types of understanding?	Zawojewski and Shaughnessy (2000)	Inference	Curriculum knowledge

Table 2—Continued

Sample writing prompt	Article	Reading comprehension question types	Relevant SKT components
Why did Eric, Paloma, and Kenji each have different estimates for the number of fish in the population of Lake Amanda? How much variability in student estimates do you think you would have if you had 25 students in your class? Why?	Morita (1999)	Reorganization, prediction	Common content knowledge
On p. 417, the author claimed, "They (the students) have formulated on their own this fundamental idea in statistical inference: larger samples tend to yield less sampling variability and therefore more accuracy." Do you agree with this claim? Why or why not? What evidence is provided in the article to support the claim?	Morita (1999)	Evaluation	Horizon knowledge, knowledge of content and students
In your own words, describe the different methods students used in combining their individual samples during the capture-recapture activity. Which method do you find the most appealing? Why?	Morita (1999)	Personal response	Knowledge of content and students

designed to assess more than one of the six components of SKT, as illustrated in Table 2.

The writing prompts served as summative assessments in that they were assigned homework grades. I graded the sets of writing prompts along five dimensions: inclusion of required information, clarity and organization, conciseness, depth of thought, and evidence of understanding. The rubric used to grade each set is shown in Figure 5. Prospective teachers were shown the rubric before completing the assignments. Along with giving them a sense of how the assignments would be graded, the rubric allowed me to provide feedback and assign grades in an efficient manner.

Grading with the rubric was only the first stage in my analysis of responses. I analyzed responses to prompts that elicited a wide range of thinking in more depth by using the Structure of the Observed Learning Outcome (SOLO) taxonomy (Biggs & Collis, 1982; Biggs & Tang, 2007), which has been gainfully employed in several studies of statistical thinking (e.g., Groth & Bergner, 2006; Jones et al., 2000; Mooney, 2002; Watson & Moritz, 2000). Figure 6 shows a diagram summarizing the SOLO levels of response observed for a prompt. It is based on a visual model devised by Biggs and Collis (1982). Figure 6 shows that *prestructural* responses draw on information not directly relevant to a task. *Unistructural* responses draw on a single relevant aspect, and *multistructural* responses draw on more than one aspect. *Relational*-level responses include connections among relevant aspects, and *extended abstract* responses include aspects beyond those required for a successful response. It should be noted that using the SOLO taxonomy does not limit one to identifying only five levels of response. Pegg and Davey (1998) theorized that the middle three SOLO levels form a repeating cycle that can be extended indefinitely. The depth of analysis provided by using SOLO was valuable for the purpose of formative assessment, as the levels of response to writing prompts indicated potentially fruitful adjustments to the course.

The Design and Assessment of a Sample SKT Writing Prompt

In order to illustrate the design and assessment of writing prompts in the SKT course, a sample prompt is described next. This extended example details the design of the prompt, a SOLO analysis of the responses, and use of the formative assessment information gained from the SOLO analysis to inform instruction.

Design of Prompt

A writing prompt to support the SKT learning goal of distinguishing between experimental and theoretical probability was assigned with an article that provided an overview of activities intended to facilitate the implementation of NCTM's (2000) data analysis and probability standards for grades Pre-K–8 (Tarr, 2002). One of the

	Levels of achievement		
Criteria	**Needs improvement**	**Meets expectations**	**Exceptional**
Inclusion of required information	**0 Points** Most components requested in the assignment description are missing.	**1 Point** Most components requested in the assignment description are present.	**2 Points** All components requested in the assignment description are present.
Clarity and organization	**0 Points** Problems with grammar, spelling, mechanics, or writing style obscure meaning.	**1 Point** Few problems with grammar, spelling, punctuation, mechanics, and writing style.	**2 Points** No problems with grammar, spelling, punctuation, mechanics, and writing style.
Conciseness	**0 Points** Writing is too brief to convey necessary points or the writing is long and rambling.	**1 Point** The main points requested in the assignment description are addressed with some degree of efficiency and eloquence.	**2 Points** The main points requested in the assignment description are addressed with a high degree of efficiency and eloquence.
Depth of thought	**0 Points** No unique insights related to the project components are provided.	**1 Point** The writer provides unique insight related to some of the components in the assignment description.	**2 Points** The writer provides unique insights related to all of the components in the assignment description.
Evidence of understanding	**0 Points** There is little evidence that the writer understood any of the main concepts related to the components in the assignment description.	**1 Point** Evidence that the writer understood the main concepts related to most components of the assignment description is provided.	**2 Points** Evidence that the writer understood the main concepts related to each component of the assignment description is provided.

Figure 5. **Rubric used to assign grades to sets of writing prompts.**

article recommendations was to use probability simulations to develop students' intuitions about random phenomena. Tarr provided the example of having children predict how many times a coin would come up "heads" when flipped a given number of times. After making a prediction, children were to perform coin flips and gather data. They were then asked to revisit their initial predictions in light of the data and revise their thinking as necessary.

To engage prospective teachers in reading Tarr's article generatively, one of the writing prompts I posed was, "Explain how simulations of random phenomena can help students develop correct intuitions about probability." In terms of the Day and Park (2005) reading comprehension question categories, this was an "inference" item because it prompted prospective teachers to draw upon examples presented in the article as well as similar activities they had experienced during class to formulate responses. In terms of the LMT framework, the prompt was intended to elicit knowledge of content and teaching because it assessed understanding of a content-specific teaching strategy. It was also intended to elicit knowledge of curriculum because probability simulation is not just a teaching strategy to be used for a single lesson, but rather recurs throughout the study of statistics.

SOLO Assessment of Prompt

Prospective teachers providing prestructural responses to

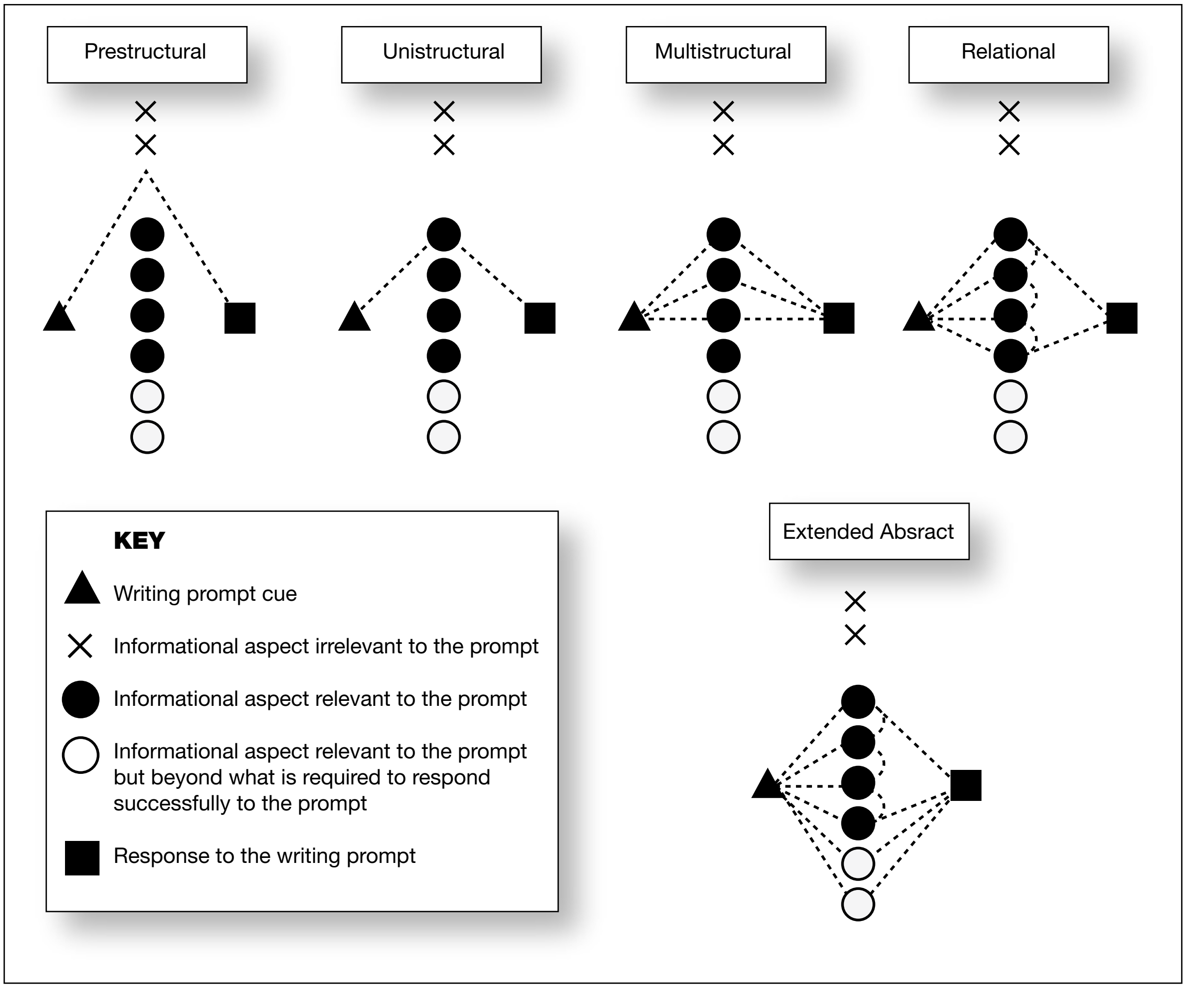

Figure 6. Diagrammatic representation for mapping SOLO levels exhibited in response to writing prompts.

the prompt exhibited no evidence of progress toward understanding the role of probability simulation in instruction. Steph, for example, wrote, "Random phenomena throws children off because it disproves what they have always thought was correct. It shows something that is extremely unlikely to happen and is hard to explain." Although portions of the response were true, it did nothing to explain how probability simulations may be helpful to students. It started to list potential difficulties in thinking students may have, but not how the recommended strategy may help remedy those difficulties.

Unistructural responses showed a degree of progress toward explaining how probability simulations may help students. However, the responses did not go beyond the single aspect of stating relevant terminology. In some cases, statistical terminology was included, but not explained, as in Sonya's response: "Simulations of random phenomena can help students develop correct intuitions about probability because it informally supports the idea of randomness and variability." Although all of the statistical terms in her response were relevant to responding to the writing prompt, she did not offer an explanation of *how* simulations would support children's understanding of randomness and variability. In other unistructural responses, pedagogical terminology was used, but not explained, as in Karen's response,

> **Using random situations to explain probability to students is a very successful way because it involves realistic situations that can be hands-on or they can relate to, to explain the material that is trying to be covered. When students are able to relate or do a project hands-on they are able to grasp the material better from my understanding.**

Karen used pedagogical terms like "hands-on" and "realistic situations" in lieu of giving content-specific insight about what probability simulations may contribute to children's thinking.

Multistructural responses included the aspect of relevant terminology, but also included the relevant aspect of what may happen statistically as students carry out simulations. Christine, for example, wrote, "Random phenomena can help students develop correct intuitions about probability because students can do trials to determine how often an event will happen. Doing trials can give them a range of different numbers, and then they can find the average." The response provided examples of activities that may occur during a probability simulation along with relevant terminology. The sample activities described, however, were not organized around a coherent theme (e.g., in Christine's response, it is not clear why students would want to "find the average"). Nonetheless, such responses suggested a greater amount of understanding of the nature of teaching strategies that incorporate probability simulation than unistructural responses.

Relational-level responses went beyond multistructural ones by describing how the elements of a probability simulation become useful pedagogically when the unifying theme of encouraging children's metacognition is placed at the forefront. Ken, for instance, wrote,

> **The simulations can help students because you can have them take an experiment and predict what they think will happen then run the simulation and see what the actual number would be. This allows the students to see what predictions they made actually were realistic and probable and what predictions were a little unreasonable. This process gives them a better understanding of probability.**

In relational-level responses, metacognition provided a means for linking the activities that occur during a probability simulation to their pedagogical purpose and value. Not only were the activities that occur during a simulation described, but the manner in which teachers can support children's reflection on their thinking was used to explain how simulation could be used as part of an overall teaching strategy.

Connecting the above SOLO analysis to the diagrammatic scheme in Figure 6, in the sample writing prompt, the cue (▲) was to explain how probability simulations can help develop children's thinking. Prestructural responses (■) simply stated that children have difficulty with random phenomena. Although true, this observation is irrelevant (×) to explaining how simulations might remedy the difficulties. Unistructural responses (■) touched on the relevant aspect of terminology (●), but did not mention aspects that would help illustrate its meaning. Multistructural responses (■) mentioned relevant aspects (●●●) of terminology and events that occur during a probability simulation. Relational responses (■) used the idea of metacognitive activity as an umbrella to explain how the various events that occur during a simulation can help develop children's thinking. This helped the responses progress beyond the multistructural level by explaining how relevant aspects (●●●) in multistructural responses complemented one another. No extended abstract responses (■) to the prompt were observed, but these might involve explaining how other pedagogical ideas, not specified explicitly in the writing prompt (○), might fit together with probability simulation to help form a coherent curricular approach to remedying children's difficulties with random phenomena. For instance, an extended abstract response might describe the advantages and disadvantages of online applets or dynamic statistics software for carrying out simulations.

Use of Formative Assessment Information from SOLO Analysis

The SOLO analysis for the sample writing prompt informed my approach to probability simulations with the class. To help more of the prospective teachers understand the importance of metacognition within the context of using probability simulations during instruction, I began to more consistently ask them to predict the results of probability simulations in class before running them. Once the simulations had been run, they were encouraged to compare the results to their original predictions and discuss reasons for discrepancies or agreement between the two. Additionally, to help them begin to reason about how technology can be used in conjunction with probability simulations, I introduced applets from the National Library of Virtual Manipulatives (http://nlvm.usu.edu/) and the freeware program Sampling Sim (http://www.tc.umn.edu/~delma001/stat_tools/). For example, when we began to study the behavior of sampling distributions, I asked the class to predict the shape, center, and spread for sampling distributions as the sample size varied. They then tested their predictions using Sampling Sim. As they did so, some began to predict that larger sample sizes lead to sampling distributions that are more tightly clustered around the population parameter. They then tested their conjectures with Sampling Sim, which allowed them to quickly simulate the gathering of various-sized random samples and then reconcile the results with their original predictions.

Effects on Prospective Teachers' Learning

Using writing prompts in place of solely subject matter-based homework problems was a substantive departure from conventional practices for undergraduate statistics courses. Although some of the prompts contained problems to develop subject matter knowledge, many were also designed to build elements of pedagogical content knowledge. Curious to examine the extent of prospective teachers' learning in a course where the primary homework tasks were writing prompts, I administered two assessments at the beginning and at the end of the course. The first was a statistics test developed by the LMT project (G. Phelps, personal communication, June 11, 2010). I used it to gain a sense of prospective teachers' SKT development during the course. This was an early draft of the test, and did not have equated forms, but it did align very closely with the course learning goals. The second assessment was the Comprehensive Assessment of Outcomes in a First Statistics Course (CAOS) test (delMas, Garfield, Ooms, & Chance, 2007). As the name implies, CAOS assesses the extent to which students develop conceptual understanding of ideas generally encountered in introductory college-level statistics courses.

Although writing prompts were assessed using the SOLO taxonomy, the LMT and CAOS examinations provided better assessments of learning gains from the beginning to the end of the course. The SOLO analyses provided valuable snapshots of prospective teachers' thinking at various points in time, but it was not feasible to track changes in SOLO levels across tasks because the sets of tasks all dealt with different statistical content. Hence, any changes in the level of response seem just as easily attributable to the difficulty of the content as they would be to general cognitive gains in SKT. SOLO analyses could, however, be used to track learning gains if similar sets of tasks were administered periodically throughout a course.

Results from the LMT and CAOS tests both indicated that prospective teachers made notable progress toward learning goals for the SKT course. On the LMT test, the mean difference between pretest and posttest scores was statistically significant ($N = 22$, $M = 0.64$, $SD = 0.52$), $t(21) = 5.79$, $p < .0001$, 95% CI [0.41, 0.87]. The mean change from pre- to posttest of 0.64 IRT units indicated that participants on average improved their SKT scores by 0.64 standard deviations between pretest and posttest administrations. On the CAOS pre-test, the mean percent correct was 36.54%, and on the posttest it was 51.54%. The mean difference between CAOS pretest and posttest scores was statistically significant ($N = 21$, $M = 15$, $SD = 12.01$), $t(20) = 5.72$, $p < .0001$, 95% CI [9.53, 20.47]. In comparison, the typical score on the CAOS pre-test for a national sample of students from undergraduate introductory statistics courses was 44.9%, and the typical posttest score was 54% (delMas, Garfield, Ooms, & Chance, 2007). Although the mean percent correct for the SKT class was slightly below 54%, the gain from pre-to-post was slightly greater.

The CAOS test results indicated that students in the SKT course left with approximately the same degree of conceptual understanding of statistics subject matter as students enrolled in conventional introductory college statistics courses. This was an important finding, since the SKT course had replaced a general education statistics course for the prospective teachers involved. Additionally, the LMT test results indicated that they gained statistical knowledge specifically required for teaching, which was not targeted in the general education course that used to be required. The observed gains in conceptual understanding of introductory college statistics and SKT helped justify continuing to steer prospective teachers into the SKT course. While it is not possible to attribute the observed learning gains directly to the writing prompts, the scores provide evidence that the writing prompts can play a prominent role in courses that build both subject matter knowledge and pedagogical content knowledge.

Conclusion

The ideas for designing and assessing writing prompts that have been discussed in this article can be used in a variety of content courses for teachers. Although statistical knowledge for teaching was the focus of this article, the ideas offered can be applied more broadly. Specifically, as teacher educators find articles that address elements of subject matter knowledge and pedagogical content knowledge, they can design prompts using the reading comprehension question types (Day & Park, 2005) that have been described and use the SOLO framework (Biggs & Collis, 1982; Biggs & Tang, 2007) to assess the levels of responses they elicit. The sample prompts described in this article, and included in the online supplement, provide examples of the types of items that may ultimately be designed and used. Readers are encouraged to experiment with the sample prompts and to design their own to elicit various aspects of SKT and MKT. Continuous design, trial, revision, and dissemination of prompts can contribute to a collective set of items to be used by those in the mathematics teacher education community for the purpose of supporting content courses for teachers. The development of tools to support such courses is particularly vital in light of calls to intertwine the development of subject matter knowledge and pedagogical content knowledge in the mathematical preparation of teachers (Kilpatrick, Swafford, & Findell, 2001; Conference Board of the Mathematical Sciences, in press).

I hope that this manuscript will contribute to teacher educators' discourse about SKT and MKT. Specifically, I hope it will spark discussions about the roles that writing prompts, generative reading, and the SOLO taxonomy can play in the process of developing and assessing prospective teachers' subject matter knowledge and pedagogical content knowledge. In addition to contributing to the practice of teacher education, such discussions can help refine theories of the components of SKT and MKT and how they may be assessed. The LMT framework and SOLO are useful tools to inform teacher educators' discussions, but they, like all models, can always be improved. The writing prompts discussed above and the method for producing them provide catalysts for further refinement of SKT and MKT theory and methods for assessment. Relevant questions for further discussion include: What other components of SKT and MKT might exist? How might the components interact with one another? What percentage of writing prompt responses fit well with one of the categories of the SOLO taxonomy? What other formative and summative assessment techniques might profitably be used in conjunction with SKT and MKT writing prompts? By examining such questions, we can develop increasingly effective approaches for fostering SKT and MKT and assessing their development.

References

Aspinwall, L., & Shaw, K. (2000). Enriching students' mathematical intuitions with probability games and tree diagrams. *Mathematics Teaching in the Middle School*, *6*, 214–220.

Bakker, A., Biehler, R., & Konold, C. (2005). Should young students learn about box plots? In G. Burrill & M. Camden (Eds.), *Curricular Development in Statistics Education*: *International Association for Statistical Education (IASE) Roundtable* (pp. 163–173). Voorburg, the Netherlands: International Statistical Institute.

Ben-Zvi, D., Gil, E., & Apel, N. (2007). What is hidden beyond the data? Helping young students to reason and argue about some wider universe. In D. Pratt & J. Ainley (Eds.), *Reasoning About Informal Inferential Statistical Reasoning: A Collection of Current Research Studies*. *Proceedings of the Fifth International Research Forum on Statistical Reasoning, Thinking, and Literacy* (SRTL-5), University of Warwick, UK.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning*: *The SOLO taxonomy*. New York: Academic.

Biggs, J., & Tang, C. (2007). *Teaching for quality learning at university* (3rd ed.). Buckingham, UK: SRHE and Open University Press.

Borasi, R., Siegel, M., Fonzi, J., & Smith, C. F. (1998). Using transactional reading strategies to support sense-making and discussion in mathematics classrooms: An exploratory study. *Journal for Research in Mathematics Education*, *29*(3), 275–305.

Burgess, T. A. (2011). Teacher knowledge of and for statistical investigations. In C. Batañero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education* (pp. 259–270). Dordrecht, the Netherlands: Springer.

Burns, M. (2000). *About teaching mathematics*: *A K–8 resource* (2nd ed.). Sausalito, CA: Math Solutions Publications.

Conference Board of the Mathematical Sciences. (in press). *The Mathematical Education of Teachers II*. Providence, RI: American Mathematical Society.

Day, R. R., & Park, J. (2005). Developing reading comprehension questions. *Reading in a Foreign Language*, *17*(1), 60–73.

delMas, R., Garfield, J., Ooms, A., & Chance, B. (2007). Assessing students' conceptual understanding after a first course in statistics. *Statistics Education Research Journal*, *6*(2), 28–53. Retrieved from http://www.stat.auckland.ac.nz/~iase/serj/SERJ6%282%29_delMas.pdf

Finzer, W. (2002). Fathom [software]. Emeryville, CA: Key Curriculum Press.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report*: *A pre-K–12 curriculum framework*. Alexandria, VA: American Statistical Association.

Franklin, C.A., & Mewborn, D.S. (2008). Statistics in the elementary grades: Exploring distributions of data. *Teaching Children Mathematics*, *15*, 10–16.

Garfield, J. B., & Ben-Zvi, D. (2008). *Developing students' statistical reasoning*: *Connecting research and teaching practice*. New York: Springer.

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. *Journal for Research in Mathematics Education*, *38*, 427–437.

Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning*, *8*, 37–63.

Harper, S. R. (2004). Students' interpretations of misleading graphs. *Mathematics Teaching in the Middle School*, *9*, 340–343.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1–28). Hillsdale, NJ: Erlbaum.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*, 372–400.

Jacobs, V. R. (1999). How do students think about statistical sampling before instruction? *Mathematics Teaching in the Middle School*, *5*, 240–246, 263.

Jones, G. A., Thornton, C. A., Langrall, C. W., Mooney, E. S., Perry, B., & Putt, I. J. (2000). A framework for characterizing children's statistical thinking. *Mathematical Thinking and Learning*, *2*, 269–307.

Kader, G., & Mamer, J. (2008). Statistics in the middle grades: Understanding center and spread. *Mathematics Teaching in the Middle School*, *14*, 38–43.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Konold, C., & Miller, C. D. (2005). TinkerPlots: Dynamic data exploration (Version 1.0) [Computer software]. Emeryville, CA: Key Curriculum Press.

Konold, C., Robinson, A., Khalil, K., Pollatsek, A., Well, A., Wing, R., & Mayr, S. (2002). Students' use of modal clumps to summarize data. In B. Phillips (Ed.), *Developing a Statistically Literate Society*: *Proceedings of the Sixth International Conference on Teaching Statistics, Cape Town, South Africa*. Voorburg, the Netherlands: International Statistical Institute. Retrieved January 26, 2012 from http://www.stat.auckland.ac.nz/~iase/publications/1/8b2_kono.pdf

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Philips, E. D. (2004). *Data about us*. New York: Pearson.

Leavy, A. M., Friel, S. N., & Mamer, J. D. (2009). It's a fird! Can you compute a median of categorical data? *Mathematics Teaching in the Middle School*, *14*, 344–351.

Lloyd, G. M., & Behm, S. L. (2005). Preservice elementary teachers' analysis of mathematics instructional materials. *Action in Teacher Education*, *26*(4), 48–62.

McClain, K. (1999). Reflecting on students' understanding of data. *Mathematics Teaching in the Middle School*, *4*, 374–380.

McMillen, S. (2008). Predictions and probability. *Teaching Children Mathematics*, *14*, 454–463.

Mooney, E. S. (2002). A framework for characterizing middle school students' statistical thinking. *Mathematical Thinking and Learning*, *4*, 23–63.

Moore, D. S. (1988). Should mathematicians teach statistics? *College Mathematics Journal*, *19*, 3–7.

Morita, J. G. (1999). Capture and recapture your students' interest in statistics. *Mathematics Teaching in the Middle School*, *4*, 412–418.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Pegg, J., & Davey, G. (1998). Interpreting student understanding of geometry: A synthesis of two models. In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 109–135). Mahwah, NJ: Lawrence Erlbaum Associates.

Perkowski, D. A., & Perkowski, M. (2007). *Data and probability connections*: *Mathematics for middle school teachers*. Upper Saddle River, NJ: Pearson.

Pugalee, D. K. (2004). A comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics*, *55*, 27–47.

Rossman, A., & Chance, B. (2008). *Workshop statistics*: *Discovery with data* (3rd ed.). New York: John Wiley & Sons. Emeryville, CA: Key College Publishing.

Rossman, A., Chance, B., & Medina, E. (2006). Some important considerations between statistics and mathematics and why teachers should care. In G.F. Burrill & P.C. Elliot (Eds.), *Thinking and reasoning with data and chance* (Sixty-eighth Annual Yearbook of the National Council of Teachers of Mathematics, pp. 323–333). Reston, VA: NCTM.

Russell, S. J., Schifter, D., Bastable, V., Konold, C., & Higgins, T. L. (2002). *Developing mathematical ideas*: *Working with data*. Parsippany, NJ: Dale Seymour Publications.

Scheaffer, R.L., Gnanadesikan, M., Watkins, A., & Witmer, J.A. (1996). *Activity-based statistics*. New York: Springer.

Senk, S. L., & Thompson, D. R. (Eds.) (2003). *Standards-based school mathematics curricula*: *What are they*? *What do students learn*? Mahwah, NJ: Erlbaum.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*, 1–22.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, *3*, 344–350.

Tarr, J. (2002). Providing opportunities to learn probability concepts. *Teaching Children Mathematics*, *8*, 482–487.

Vygotsky, L. S. (1987). Thinking and speech. In R.W. Rieber and A.S. Carton (Eds.), *The collected works of L. S. Vygotsky* (pp. 39–243). New York: Plenum Press.

Watson, J. M. (2008). Exploring beginning inference with novice grade 7 students. *Statistics Education Research Journal*, *7*(2), 59–82. Retrieved April 18, 2012 from http://www.stat.auckland.ac.nz/~iase/serj/SERJ7%282%29_Watson.pdf

Watson, J. M., Fitzallen, N. E., Wilson, K. G., & Creed, J. F. (2008). The representational value of hats. *Mathematics Teaching in the Middle School*, *14*, 4–10.

Watson, J. M., & Moritz, J. B. (2000). Developing concepts of sampling. *Journal for Research in Mathematics Education*, *31*, 44–70.

Watson, J. M., & Shaughnessy, J. M. (2004). Proportional reasoning: Lessons from research in data and chance. *Mathematics Teaching in the Middle School*, *10*, 104–109.

Zawojewski, J. S., & Shaughnessy, J. M. (2000). Mean and median: Are they really so easy? *Mathematics Teaching in the Middle School*, *5*, 436–440.

Author

Randall E. Groth, Teacher Education and Technology Center, Room 379R, 1101 Camden Avenue, Salisbury, MD 21801; regroth@salisbury.edu

SUPPLEMENT

The Role of Writing Prompts in a Statistical Knowledge for Teaching Course

Randall E. Groth

Assignment 1

Article to read: Jacobs, V. R. (1999). How do students think about statistical sampling before instruction? *Mathematics Teaching in the Middle School, 5,* 240–246, 263.

Questions:

1. Write three of your own original scenarios about sampling. The first should involve random sampling, the second should involve restricted sampling, and the third should involve self-selected sampling. Explain why each scenario fits each category.

2. On p. 244, the author stated, "Students seemed to focus on the possibility of extreme outcomes without realizing that the probability of their occurrence was low." What does this mean? Provide your own example of a situation where a student may exhibit this behavior.

3. What does "fairness" mean, in the statistical sense? What is a common student conception of "fairness" that differs from the statistical sense? Give an example of a situation a young student with a nonstatistical notion of fairness may consider to be unfair.

4. Why do some students believe that a survey is not useful if survey respondents do not all answer the same way?

5. In 200–250 words, describe a general strategy you would use for teaching young students about survey sampling and how you would assess their understanding. Then provide a rationale for your general strategy.

Assignment 2

Article to read: Franklin, C. A., & Mewborn, D. S. (2008). Statistics in the elementary grades: Exploring distributions of data. *Teaching Children Mathematics, 15,* 10–16.

Questions:

1. Explain the difference between categorical and quantitative data. Give your own example of a statistical question that young students could investigate involving categorical data. Also give your own example of a statistical question that young students could investigate involving quantitative data.

2. In the second column on p. 12, the authors provided a bulleted list of five extension questions for the shoe activity. Write two of your own extension questions, and explain why they should be added to the list.

3. On p. 13, the authors stated, "it is inappropriate to ask children to determine the mean of a set of categorical data." Why is it inappropriate? In your own words, explain how Snap Cubes® can be used to determine the arithmetic mean of a set of quantitative data.

4. On pp. 14–15, a bulleted list of questions is given that teachers can ask students when interpreting the results of the soccer investigation. Write two of your own questions to add to the list, and explain why they should be added.

Assignment 3

Article to read: Leavy, A. M., Friel, S. N., & Mamer, J. D. (2009). It's a fird! Can you compute a median of categorical data? *Mathematics Teaching in the Middle School, 14,* 344–351.

Questions:

1. Provide your own example of a data set for which the median cannot be determined. Then provide your own example of a data set for which the median can be determined. Explain your thinking.

2. How can the median of a data set be determined using a paper strip marked with square grids? Give two of your own examples of data sets that would help illustrate the approach for young students, and show how the model applies to your examples.

3. See problem 1.4 in Figure 3. Identify one question that cannot be answered by using the data from the graphs and tables the students created. Explain why the question cannot be answered and tell what additional information you would need to answer the question.

4. Describe two types of student errors that occur when working with nominal categorical data: computing a numerical value for the median and computing a categorical median. Illustrate how the two errors can occur using a data set of your own.

5. In 200–250 words, describe a general strategy you would use for teaching young students that you cannot find the median of categorical data and how you would assess their understanding. Then provide a rationale for your general strategy.

Assignment 4

Article to read: McClain, K. (1999). Reflecting on students' understanding of data. *Mathematics Teaching in the Middle School, 4,* 374–380.

Questions:

1. On p. 374, the author asked, "Do students first need to know how to construct various types of graphs before they can engage in an analysis of data, or can they learn how to construct various types of graphs by engaging in data analysis?" Write a response to the author's question. Explain how your response compares to the position taken by the author of the article.

2. On p. 375, the author stated, "My assessment of their (the students') performance would not be based solely on whether they made a histogram and made it correctly but would focus more on how they reasoned about organizing and representing the data." Do you agree with this decision? Why or why not?

3. On p. 377, the author stated, "I was not clear whether the students were making a modified histogram or simply grouping the data points into categories that they named with numeric intervals." What is the difference between the two activities?

4. Examine the student graphs shown in Figures 2, 3, and 4c. Discuss the strengths and weaknesses of each one.

5. On p. 380, the author stated, "As we deliberated, we decided to find situations in which the two data sets had very similar means even though the individual data points in one of the sets varied greatly." Invent two data sets that are very different but have similar means. Use a context for the data that would be engaging for young students (similar to the battery life example on p. 380).

Assignment 5

Article to read: Harper, S. R. (2004). Students' interpretations of misleading graphs. *Mathematics Teaching in the Middle School, 9,* 340–343.

Questions:

1. Respond to the NAEP test items shown in Figure 1 in your own words.

2. Respond to the NAEP test items shown in Figure 2 in your own words.

3. Respond to the NAEP test items shown in Figure 3 in your own words.

4. Invent a set of data that would be interesting for young students to analyze. Construct two correct graphs for the data. One of the graphs should be misleading. Explain why one graph is misleading and the other is not.

5. Drawing upon the sample student responses reported at the end of the article, describe three major types of difficulties students may have with interpreting misleading graphs.

Assignment 6

Article to read: Zawojewski, J. S., & Shaughnessy, J. M. (2000). Mean and median: Are they really so easy? *Mathematics Teaching in the Middle School, 5,* 436–440.

Questions:

1. Write a response to item 1 in Figure 1. Explain your reasoning completely.

2. Write a response to item 2 in Figure 1. Explain your reasoning completely.

3. Write a response to item 3 in Figure 1. Explain your reasoning completely.

4. On p. 438, the authors commented in regard to item 3, "This type of item assesses students' conceptual understanding of mean." Explain what the authors may mean by "conceptual understanding." How is it different from other types of understanding?

5. Explain why some students believe the mean is always a better indicator of typical value than the median. How might you convince these students that the median is more appropriate in some cases?

Assignment 7

Article to read: Kader, G., & Mamer, J. (2008). Statistics in the middle grades: Understanding center and spread. *Mathematics Teaching in the Middle School, 14,* 38–43.

Questions:

1. In your own words, and drawing upon the ideas in the article, explain why histograms and box plots are more challenging to use and interpret than line plots, dot plots, and picture graphs.
2. Construct two different sets of data that have the same mean. The data sets should have different numbers of values. Compute the SAD and MAD for each set of data. Show your work. Explain what the SAD and MAD tell you about the sets of data.
3. How is the MAD similar to the standard deviation? How is it different? How might understanding the MAD help students prepare to study the standard deviation?
4. Write your own responses to each of the questions shown in Table 1 on p. 41. Explain your reasoning.
5. Write your own responses to each of the questions shown in Table 2 on p. 42. Explain your reasoning.

Assignment 8

Article to read: Watson, J. M. (2008). The representational value of hats. *Mathematics Teaching in the Middle School, 14,* 4–10.

Questions:

1. Beyond generating hat plots, how can the software program TinkerPlots® help students learn statistics?
2. How are hat plots similar to box-and-whisker plots? How are they different?
3. Why is it desirable to have students work with hat plots before working with box-and-whisker plots?
4. How can hat plots help students make the transition from focusing on individual data values to focusing on group characteristics?
5. Invent a data set that would be interesting for young students to analyze. Construct a dot plot, a hat plot, and a box-and-whisker plot for the data. Describe the conclusions one can draw about the data from each representation.

Assignment 9

Article to read: McMillen, S. (2008). Predictions and probability. *Teaching Children Mathematics, 14,* 454–463.

Questions:

1. Explain the difference between experimental and theoretical probability in your own words.
2. Explain why experimental probabilities do not always match the theoretical probabilities.
3. Which cards in Figure 3 (p. 459) involve theoretical probability? Which cards in Figure 3 involve experimental probability? Justify your answers.
4. Explain how technology can be useful when teaching the distinction between theoretical and experimental probability.
5. Examine the worksheets at the end of the article for activities 1 and 2. Describe at least one modification you would make to the worksheets in order to help improve students' learning experience. Explain why you made the modification.

Assignment 10

Article to read: Tarr, J. (2002). Providing opportunities to learn probability concepts. *Teaching Children Mathematics, 8,* 482–487.

Questions:

1. What is the difference between estimating the relative likelihood of events and quantifying likelihood numerically? Which of the two should elementary school children do first? Why?
2. Write a problem or scenario you would share with young students to help them understand the idea that the sum of the probabilities of all sample space outcomes is 1 (or 100%). Explain how the problem or scenario would help them understand this idea.
3. Explain how simulations of random phenomena can help students develop correct intuitions about probability.
4. Describe an activity that could help students understand the idea, "that, for a given event, the experimental probability (through repeated trials) is more likely to approximate the theoretical (actual) probability as the number of trials increases" (p. 486).

5. Is the beanbag game described in the "Probability and Area" section of the article (p. 486 and Figure 6 on p. 487) fair? Justify your response. Is taking an equal number of turns an essential requirement for the game to be fair? Why or why not?

Assignment 11

Article to read: Aspinwall, L., & Shaw, K. (2000). Enriching students' mathematical intuitions with probability games and tree diagrams. *Mathematics Teaching in the Middle School, 6,* 214–220.

Questions:

1. Explain how the tree diagram in Figure 2 shows that "odd it out" is a fair game.
2. Describe how Bill, Clara, Denise, and Ahmed differed in their intuitions about activity 3 (in Figure 3) before doing the activity.
3. Explain why activity 6 (in Figure 8) is not a fair game.
4. Explain how tree diagrams can help students refine their intuitive ideas about probabilistic situations. Provide at least one specific example from the article to support your explanation.
5. Provide your own example of a probabilistic situation that can be analyzed by using tree diagrams. Explain how you would use the situation in a classroom setting to teach students.

Assignment 12

Article to read: Watson, J. M., & Shaughnessy, J. M. (2004). Proportional reasoning: Lessons from research in data and chance. *Mathematics Teaching in the Middle School, 10,* 104–109.

Questions:

1. Write your own responses to each of the four tasks shown in Figure 1 (and described on p. 105). Explain your reasoning.
2. Describe at least two different reasoning patterns students may make when comparing unequal-size groups. Discuss the strengths and weaknesses of each reasoning pattern.
3. Provide a response to each part of the task shown in Figure 2.
4. Describe three types of strategies you can expect students to use in answering the sampling task shown in Figure 2.
5. Add one of your own follow-up questions to the list in the first column of p. 109. Explain how your follow-up question would help enhance students' learning.

Assignment 13

Article to read: Morita, J. G. (1999). Capture and recapture your students' interest in statistics. *Mathematics Teaching in the Middle School, 4,* 412–418.

Questions:

1. In your own words, explain how the "capture-recapture" method of sampling works. Are samples produced using this method likely to provide reasonable estimates? Why or why not?
2. Why did Eric, Paloma, and Kenji each have different estimates for the number of fish in the population of Lake Amanda? How much variability in student estimates do you think you would have if you had 25 students in your class? Why?
3. In your own words, describe the different methods students used in combining their individual samples during the capture-recapture activity. Which method do you find the most appealing? Why?
4. Describe a method for helping students get a feel for sampling variability within the context of the sampling-resampling activity. Explain why the method is likely to help students understand the nature of sampling variability.
5. On p. 417, the author claimed, "They (the students) have formulated on their own this fundamental idea in statistical inference: larger samples tend to yield less sampling variability and therefore more accuracy." Do you agree with this claim? Why or why not? What evidence is provided in the article to support the claim?
6. At the end of the article, the author stated, "Now what or who else can we tag? The possibilities are endless." Write your own example of a situation where you could lead students to use the capture-recapture method to estimate the size of a population.