JOURNAL WRITING IN THE MATHEMATICS CLASSROOM: A BEGINNER'S APPROACH

Today's mathematics educators are advocating methods of assessment other than frequent tests, quizzes, and daily worksheets. In fact, the NCTM's *Curriculum and Evaluation Standards* states,"The assessment of students' ability to communicate mathematics should provide evidence that they can express mathematical ideas by speaking, writing, demonstrating and depicting them visually" (1989, 214). One alternative form of assessment that incorporates these standards is journal writing.

After reading about the potential benefits of journal writing, we each decided to try it in a high school mathematics class. Although we taught at different schools at the time, we undertook this project as a team so that we could share ideas and experiences with each other. Starting the writing process was difficult, but the end result was pleasantly surprising. This article is an account of our approach to journal writing in the mathematics classroom.

GETTING STARTED

Because this attempt was our first one at at using journal writing, we each selected only one class to participate. We had noticed that students' behavior and performance are generally better during the earlier part of the school day and that higher-level classes usually have more adequate foundations for communicating mathematically. For these reasons, Nancy chose her first-period informal geometry class and Brian chose his second-period advanced algebra II class. Knowing which classes would be involved helped us decide on other criteria to be used in the assessment process.

Our next concern was what the students would write. Because we wanted to be able to compare and contrast our findings across classes, we thought that having some structural similarities was important. We decided to use similar writing prompts, writing schedules, and assessment rubrics. We used two types of writing prompts—affective and mathematical (see **fig. 1**). We hoped that this arrangement would allow the students to communicate both their knowledge about mathematics and their feelings about the environment of the mathematics classroom.

Another concern that we addressed was the frequency of writing and when to write. At first, we decided to assign journal writing twice each week and collect the journal entries at the end of the week. To avoid the tedium that could be caused by writing about similar topics twice each week, we alternated the affective and mathematical topics on Tuesdays and Thursdays. We expected students to submit both journal entries to the teacher every Friday for evaluation, so the teachers would have sufficient time over the weekend for grading. We later decided to assign only one journal entry per week.

We also decided to present the writing topic on the chalkboard or on the overhead projector so that the students could begin their entries when they entered the classroom. Nancy's students were allowed five minutes to begin writing while she took roll and handled other classroom duties. Brian's students were given ten minutes to write. We told both classes that they would not have time to finish their entries in class. We expected them to complete their entries outside of class and hand in both entries on Friday at the beginning of the period. Because we were assessing individual thoughts and knowledge, we expected entries to reflect individual efforts. We did not allow students to collaborate with other students or to copy from books. However, we encouraged students to consult their textbooks and class notes for ideas.

Grading papers was a major concern while we were setting up the project. We were concerned about the length of time that we would need to read, evaluate, and post grades on fifty journal entries each week. We therefore decided to limit the length of the journal entries. We told students that one full page of explanation would be sufficient for

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Affective Prompts	Mathematical Prompts
1. Discuss (in paragraph form) three qualities of a good teacher and three qualities of a good student. Explain why these qualities are important.	1. Suppose that you are a math teacher and that it is your responsibility to do an SAT review session where the topic is radicals. Make up four sample problems, and explain to your audience how to work each one.
2. In your own words, describe or define what is meant by a "fair" test. Also, describe how you prepare for math tests and quizzes.	2. Explain the Pythagorean theorem in your own words. Discuss three types of problems that use the Pythagorean theorem, and explain how to solve each of them.
3. You have been assigned to teach this class. Describe how you would plan the course—grading procedures, checking homework, how often you would give a test or quiz, the format of the tests or quizzes, seating of the students, and so on.	3. Your best friend has enrolled in this class for next year. Your responsibility is to inform your friend about the prerequisites for doing radicals.
4. Explain how you organize your math notebook. How does your notebook help you?	4. Explain the isosceles triangle theorem. Use several examples.
5. Discuss whether or not you think journal writings have enhanced experiences in this math class. Which journal writing did you like most? Least? Why?	5. Discuss why you think the imaginary unit i is important. You may want to list some of its uses—solving equations, graphing complex numbers, simplifying radicals, and so on.
	6. Explain the angle-side relationship of triangles. Describe two types of questions that could be asked on a test.
	7. Describe the two methods that we have discussed for solving quadratic equations. Include an example of each.
	8. Explain to an absent classmate the distance formulas, with three examples of how to use them.

Fig. 1
Affective and mathematical writing prompts

a journal entry, but less would not be adequate. Initially, Brian gave one grade for both entries for a week, whereas Nancy graded each entry individually. Brian later adopted Nancy's method to allow each entry to stand on its own merit.

We next discussed what factors would be important in the grading process. From this discussion, we developed and employed the same grading rubric. The content of the entry accounted for half the grade, and effort counted for the other half. We evaluated mathematical entries on their mathematical content, whereas we examined affective entries for sufficient thought and expression. For the grade for effort, 10 percent of the grade was for neatness, 10 percent for vocabulary and grammar, 10 percent for the headings, and 20 percent for length. When we returned the entries to our students, we gave them a cover sheet detailing the points that they received. This cover sheet contained the grading rubric that also appears on the syllabus. See **fig. 2.**

When evaluating the mathematical topics, we corrected the students' mathematical content and vocabulary and made comments to help improve their thought processes, understanding, and expression. When we evaluated the affective topics, we supplied constructive criticism of the students' comments, acknowledged their difficulties and efforts, and encouraged them to do better. Because we generally graded quizzes and tests in red, we decided to use a different color, such as green, to grade the journals and write comments. The grades for all the entries were averaged to count as one test grade at the end of each marking period.

THE WRITING PROCESS

To introduce journal writing to our classes, we gave our students a syllabus, as shown in **figure 2.** This syllabus included the purpose for journals, materials that the students needed, the procedures and the expected format, and the grading criteria. To

PURPOSE: In my opinion, journal entries are an integral part of a mathematics class for three main reasons:

1) Journal entries tend to enhance a person's learning of a concept because he or she is taking an active part by conveying knowledge instead of just absorbing the material and working a set of problems.
2) The student receives individual feedback from the teacher in writing; the feedback may clear up any misunderstanding about the topic at hand.
3) The teacher is able to get an idea of a student's thought processes, as well as address any concerns that the student may have.

MATERIALS: Although I am not requiring that you purchase a second notebook for journal entries, I do strongly urge you to set up a separate section in your current math notebook so that you can access your journals easily—especially on due dates!

GRADING: You must include the following information for every journal entry: the topic (or title) and topic number, the current date, your name, and class period. In addition, you should write neatly and legibly in blue or black ink. Pencils are acceptable; however, pencils tend to rub off onto other pages. Although my main concern is your mathematics, good grammar and appropriate vocabulary are a must! Furthermore, each journal entry should be approximately one page in length. Points for journal entries are earned—not deducted, and to ensure a "fair" grading of journals, I am including the following grading rubric:

Criteria	Maximum Point Value
Name, date title, class period	10
Neatness	10
Vocabulary and grammar	10
Appropriate length (approx. 1 page)	20
Mathematical content	50

You should plan on one journal entry per week. These journal entries will be collected every week. No late journals will be accepted. You will find the day's topic on the chalkboard, and you should begin writing in your journal as soon as you enter the classroom.

Fig. 2
Syllabus for journal entries

begin the project, we used an affective topic: "Discuss three qualities of a good teacher and three qualities of a good student. Explain why these qualities are important." We believed that beginning the journal writing with an affective topic would make the students realize that their opinions do matter and would not be as intimidating as a mathematical topic might be for the first entry.

In the beginning, both classes complained about writing journal entries. Some students declared that writing should not be part of mathematics class, and others said that they did not know what to write. But the initial response was good. One student wrote, "I don't really enjoy doing journals, but I don't hate it either. I think the hardest thing about journals is the fact that some of our topics are hard to write a page about." Most students turned in legible, coherent journals that addressed the writing prompts and were appropriate in length and format. Some students forgot to hand in their journal entries the first Friday, but we decided to accept late work for that assignment. When we returned the first journal entry, we reminded our classes that we would no longer accept late journal entries.

The next topic was mathematical and therefore different for each class. The students complained more about writing in mathematics class. Some students said they did not know how to write what they wanted to say. Similar complaints occurred with each entry.

At the midpoint of this project, we examined students' grades and noted that one class had more zeros for work not submitted than the other class. We decided that the difference occurred because one teacher allowed only five minutes of writing time in class, whereas the other allotted ten minutes. Many students could finish their entries in ten minutes. We decided that ten minutes of in-class writing was better.

Thinking that comparing each other's grading practices would be useful, we traded five randomly selected journal entries. Brian tended to place more emphasis on the student-effort parts of the rubric, deducting more points for neatness, vocabulary, grammar, and length. Conversely, Nancy tended to be more critical of the mathematical content of the students' writing. Surprisingly, when we traded papers, the final grades were very close, with no more than one or two points difference on most entries. However, a twenty-point discrepancy existed between our grades on two papers. Nancy graded higher on one entry, and Brian graded higher on the other. We discovered that both entries were mathematical topics and that each of us graded the entry from our own class more harshly. After some pondering, we decided that the difference occurred because the teacher who taught the material knew which points to emphasize and was therefore looking for particular points in the journal entry. Accordingly, we thought that the grades given to our own students were fair.

At the beginning of this project, we were discouraged and overwhelmed. The students did not enjoy journal writing, and we spent much of our weekends grading them. Midway through the project, we decided to reduce the frequency of journal entries from twice a week to once a week. The students were as relieved to have a reduced workload as we were.

Toward the end of the project, we discussed whether to continue writing journal entries in our classes. We agreed that the entries gave us excellent feedback on our students' understanding of the curriculum taught, but we were not sure whether the students were seeing any benefits. Additional

drawbacks included grading time and student disinterest.

For our last journal entry of the grading period, we asked the students whether they found journal writing beneficial, and why. We were surprised to learn that most students in both of our classes wanted to continue with the journals. Their reasons included that the grades helped improve their average, although they did not do so for all students; that they learned to explain themselves better mathematically; and that our comments on the journals gave them immediate feedback on their understanding of the material. See **figure 3** for excerpts from students' final journal entries. We were pleased that our students benefited, and we decided to continue journal writing for the remainder of the school year.

By having to explain how to do a certain type of skill to someone who doesn't know helps you understand it better yourself.

I suppose I have benefited from journals somewhat, even though I don't want to admit it. It's helped me to describe methods I use to solve problems, and then when I get my journals back, I can see how well I understand the concepts we talked about, or if I need extra help.

In journal number three, we had to give a "game plan" for studying for tests. It really made me think about my own study habits. I wrote down a great plan, and I think that if I stick to it, then my test grades will improve.

Fig. 3
Samples from final entry

RECOMMENDATIONS AND ADVICE

After reflecting on our experiences, we think that the journal writing was a success for both our students and us. We persevered through the initial stages and compiled the following recommendations for others who would like to try journal writing.

- Choose the class that will write journal entries; attempting this project with one class is best for the first try.
  - ✓ Select a class with which you have good rapport.
  - ✓ Select a class that has good communication skills and a positive attitude toward mathematics.

  After you have gained experience with journal writing and are comfortable with your procedures and expectations, expanding your efforts to include more classes will be easier.
- Decide what you want the students to write.
  - ✓ Alternate affective and mathematical topics for variety.
  - ✓ Do not use journals as filler work. Mathematical entries provide an excellent student-centered source of closure.
- Decide when the students will write and for how long. Allow students approximately ten minutes to work on journals in class. Students dislike getting involved in an activity and then being asked to stop too early.
- Decide on the writing format that you expect the students to follow.
  - ✓ Assign students one entry per week. The students do not become overworked, and the teacher is able to keep up with the grading.
  - ✓ Create a grading rubric that details how each entry will be assessed.
  - ✓ Use a color other than red for grading.

AFTERTHOUGHTS

Beginning the journal-writing project seemed an overwhelming task at first, but it proved less daunting than we imagined. We believe that journals are a valuable form of assessment for both students and teachers. The mathematical entries furnish concrete feedback on a student's understanding of concepts taught. As Countryman notes, "Good journals give evidence of students using a wide variety of thinking skills, again providing a teacher with considerably more information about how students are approaching and using mathematical concepts than most formal assignments can offer" (1992, 32). The affective views encourage teachers to assess their teaching methods and classroom practices. Journals also give students opportunities to express themselves mathematically and to attain closure. Overall, we survived the initial struggles and firmly believe that journal writing is beneficial to the mathematics class. We are looking forward to continuing journals next year.

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STUDENT-RESEARCHED PROBLEM-SOLVING STRATEGIES

NCTM has called for teachers to create an environment in which students become independent problem solvers (1989; 1991). This instructional goal is not new but is one that continues to challenge all teachers of mathematics. I previously taught problem solving as an isolated lesson, often giving a lecture about problem-solving strategies. I described a list of strategies that students could use to solve problems: draw a picture, make an organized list, guess and check, and so on. These strategies represented the problem solving that we would practice, but they were no more meaningful to my students than memorizing steps to perform algebraic manipulations.

I eventually developed a project that enables my students to *experience* these strategies. They construct their own understandings of the problem-solving strategies instead of merely writing them in their notebooks. My students begin to research the problem-solving process itself, uncovering and defining strategies that they can subsequently use to solve problems, as well as exploring the impact that attitude has on problem solving.

Students construct their own understanding of problem-solving strategies

THE RESEARCH ASSIGNMENT

This assignment has been successfully used with middle school, high school, and college mathematics students. Early in the term, or at the beginning of a problem-solving unit, students receive a sheet of paper that includes several problems. The students use these problems to do research on problem solving. The teacher tells the students to find three individuals, one at a time, who will sit down and attempt to solve a problem for them. These individuals can be anyone—friends, parents, siblings (young children do not make good subjects), coaches, or anyone who will sit and work on a problem for them.

The students' first research decision is to select their subjects. I encourage them to ask people who are likely to have different problem-solving abilities, since the data are richer when students observe both expert and novice problem solvers. Observing someone who can easily find a solution is just as valuable as watching someone struggle.

A second research decision that students must make is to determine which problem to present to each subject. They may have each of their three subjects solve the same problem or different problems. The researchers must decide which approach will enable them to obtain the most interesting data. I collected the problems thoughtfully, to elicit a variety of problem-solving strategies. The number of students in the class is usually large enough that most of the problems are used, so students can observe all the traditional strategies. See **figure 1** for the problems that I have used—I encourage you to develop your own set.

As researchers, the students must take detailed field notes and record what they observe, listing the strategies and techniques that their subjects use to solve the problems. Determining the strategies and techniques can be tricky because many people do most of their thinking silently, in their heads. Consequently, the teacher must instruct the students to ask questions that prompt subjects to explain their thinking; supplying examples of prompts is helpful.

In addition, the students document the behaviors and emotions that their subjects exhibit while they solve the problem. Students can fold a piece of paper in half and write notes documenting the problem-solving strategies on one half and the behaviors that they observed on the other. Armed with their plans, the researchers begin to collect data. The field notes count as a homework assignment and are due several days later.

Students do not always understand the research assignment, so reviewing it at the next class meeting is helpful. In groups, students explain the assignment to one another. This procedure results in questions that the teacher can address to clarify the task. If confusion reigns, students can work in pairs to practice the research methods with each other. One student solves a problem while the other observes and takes notes. The field notes that stu-

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1. Fourteen clothespins are placed on a line at seven-foot intervals. What is the distance from the first clothespin to the last?

2. After hearing a gloomy Sunday sermon on the danger of smoking, Hugo promised himself to cut back two cigarettes a day, smoking two fewer on Monday than he had on Sunday, and so on. He kept his promise for a week, that is, through Saturday. During the whole week, he smoked sixty-three cigarettes. How many did he smoke on the day of the sermon?

3. If nine people are in a room and every person shakes hands exactly once with each of the other people, how many handshakes will occur?

4. I have already covered one-third the distance from Podunk to Boondocks, and after I walk one more mile, I'll be halfway there. How far is Boondocks from Podunk?

5. In the barnyard is an assortment of chickens and pigs. Counting heads, I get thirteen; counting legs, I get forty-six. How many pigs and chickens are in the barnyard?

6. Move only three disks from the first arrangement to produce the second.

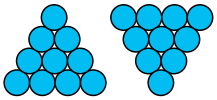

Fig. 1
Sample problems

My sister, [University] math freak. [Podunk/Boondocks problem, number 4] Drew 2 pts. Labeled P and B. Then divided the line segment into halves and also marked the first 3rd and called it (1/3)x. Called the distance from P to B is x. Then said (1/3)x plus 1 mile equals (1/2)x. Then solved for x. Answer, 6 miles.

—Geometry student

[Mom]
1. The first thing she did was read the [clothesline problem, number 1].
2. The second thing she did was make line with 14 x's on them, she counted the spaces and got 13 and multiplied that times 7.
Answer: 91 ft.

—Geometry student

[Disk problem, number 6] Draws the two diagrams thinks about it. Puts hand on head says "Oh my gosh." Runs fingers in her hair. Says "I don't know." "Can't you just turn it around?" Trys to move the circles around. Gives up. After a little then says okay and trys to look at the picture again. But doesn't get it.

—Intermediate-algebra student

[Friend]:
After assigning [smoking problem, number 2] to a friend their first response was to roll their eyes and ask why. After explaining they read the problem and continued to make a chart and throw random numbers into time spaces. They explained that the number went down by two daily and continued to equal 63 cigarettes. After establishing the correct chart through trial and error and educated guesses my friend came to his conclusion. His strategy was to dive directly into the problem and work it out by writing and drawing charts.

—Intermediate-algebra student

The second person I interviewed was my mom, she is also 44 yrs. old. and has a baclors degree. She did problem [disc problem, number 6] and used checkers to figure out how to move only 3 discs.

—Geometry student

Fig. 2
Examples of student work

dents turn in are often unclear and trivial, but the ensuing discussion, based on these notes, is the most powerful part of the activity. See **figure 2** for samples of students' field notes.

SHARING AND MAKING SENSE OF THE DATA IN CLASS

When the students return to class with their field notes, they share with their groups what they found. A whole-class discussion, facilitated by the teacher, follows. The teacher creates a master list of the collected data. Students share interesting problem-solving strategies that they observed, while the teacher writes them on the chalkboard using the students' words. In this way, the students own the problem-solving strategies; they give them names and define the terms for their peers. The list usually includes all the strategies described in the textbook, and more. These strategies usually include the following: use trial and error; use algebra; draw a picture; make a table; act it out; reread the problem; make a model; and be lucky, which is usually associated with guessing and not checking. Some tactics are not traditionally considered problem-solving strategies; but they represent the strategies to which the community of problem solvers in the class refers.

On another section of the chalkboard, the teacher generates a list of behaviors exhibited by subjects when they were solving problems. The

Students develop habits that serve them well as they continue their study of mathematics

teacher divides the chalkboard into two sections: one is labeled with a plus sign, and the other, with a minus sign. When students share the behaviors that they observed, they must decide in which column to record their contribution. That is, they must determine whether the behavior contributed positively to the problem solving that they observed or whether it was detrimental. Sometimes the researchers decide to place a behavior in both sections. This approach is reasonable, since people deal with problem solving on the basis of their own personalities and experiences. Typical items listed on the plus side include "smile," "talking to oneself," and "concentrate." On the minus side, "grimace," "tense," or "saying they are no good at math" frequently appear.

Next, the data are assembled for all the researchers and the teacher to consider. It is time to apply the research to the future activities of the class.

HOW THE RESEARCH IS USED

The teacher needs to review the findings with the students. Returning to the list of strategies, the teacher can make sure that everyone understands the problem-solving strategies and that the vocabulary used is clear. Students can be told that the teacher will respect strategies listed on the chalkboard when their work is evaluated, with the exception of "be lucky." When they turn in future assignments, their job is to prove that they were not merely lucky. They must show sufficient work to back up solutions and explain how they reached their answers. The strategies that they use can be any that appear on the chalkboard or new ones that arise during the year. This discussion of expectations can help students understand why they must show their work.

Another discussion should involve the behaviors listed on the other section of the chalkboard. Part of problem solving is being aware of one's own behaviors and dealing with the negative ones. For example, someone who is exhibiting negative behaviors might take a break or get help as a way of dealing with the negativity that is impeding his or her ability to solve a problem. In this way, students begin to see that problem solving is more than applying strategies; it is the way that they approach a problem, execute a plan, and look back on the answer and the process after they reach a solution.

One teacher who uses this activity creates a poster that shows the strategies listed by all her classes. The teacher mounts the poster on a wall and uses it throughout the year. If students become stuck when solving a problem, the teacher refers them to the poster for inspiration. The teacher writes the list of behaviors on another poster, to serve as a constant reminder that problem solving is more than using strategies. When they are working in groups, students look for negative behaviors in their peers, pointing them out and suggesting ways to help one another become better prepared to solve problems.

Another teacher uses the research problems in class. The students solve all the problems by working in groups. Then each group is assigned one problem to present to the class. They share their solutions and include the strategies that they used to determine their answers. The students therefore practice the vocabulary that they developed in their research. This process sets a precedent for problem solving that can be discussed and shared throughout the year.

After students have engaged in this problem-solving research and the subsequent sharing of data, they construct a vocabulary in which they use their *own* words to describe problem-solving strategies. This procedure increases the chance that problem-solving strategies will make sense and be useful to the students. Moreover, each student has an opportunity to think about her or his own presence of mind when solving problems, thereby developing metacognitive and reflective habits that serve them well as they continue their study of mathematics. Students who research problem solving are taking a first step toward becoming independent problem solvers.

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