MY FAVORITE lesson

Judith Macks

The Dog Pen Problem

My favorite lesson is the Dog Pen problem:

> Suppose that you had 64 meters of fencing with which to build a rectangular pen for your large dog.
>
> - What are the dimensions of some different pens that you can make if you use all the fencing?
> - What dimensions are best to allow the most space for the dog to run?
> - What dimensions will allow the most play area? What dimensions allow the least play area?

This problem is quite flexible, with variations that can address different learning objectives. Further, the discussions before, during, and after work on the problem can focus on almost every one of the Common Core Standards for Mathematical Practice. The problem requires little background knowledge, is based on a familiar situation, and has many entry points.

Although the problem asks only for "some different pens," questioning during work time usually results in lists of at least all whole-number possibilities (see **fig. 1**). Students generally arrive at solutions by using some combination of strategies: drawing a picture, creating a table, guessing and testing, following a pattern, or using variables. Discussion during and after the student work session encourages good use of mathematics vocabulary, which helps improve student understanding of area and perimeter. Further, by the end of the discussion, students know that rectangles with the same perimeter may not be congruent or have the same area; thus, their understanding of how perimeter, area, and congruence relate to each other has increased. They also see that there can be more than one way to solve a problem successfully and that problems can have more than one accurate answer. But the learning does not have to stop there.

Width	Length	Area
1 m	31 m	31 m^2
2 m	30 m	60 m^2
3 m	29 m	87 m^2
4 m	28 m	112 m^2
5 m	27 m	135 m^2
⋮	⋮	⋮
14 m	18 m	252 m^2
15 m	17 m	255 m^2
16 m	16 m	256 m^2
17 m	15 m	255 m^2
18 m	14 m	252 m^2
⋮	⋮	⋮

Fig. 1 This list includes some of the whole-number solutions to the Dog Pen problem.

Questions about patterns can lead to rich discussions that help develop students' expertise in looking for structure. Although students may not readily notice any pattern in the sequence of areas of pens with integral sides, questioning usually helps some notice that differences in areas form a sequence of consecutive odd numbers.

To explore still more variations on this lesson, consider adding questions such as these:

- Which rectangular pen has the largest area? Does your result hold for any length of fence?
- What about pens that are not rectangles? Do any of them have a larger area?
- If x is either the length or the width and $f(x)$ is the area, what does the graph of $f(x)$ look like?

I am still discovering new questions to ask about this problem. After reading Steven Siegel's "The Ratio of Perimeter to Area" (Reader Reflections, *MT* May 2010, vol. 103, no. 9, p. 632), I look forward to asking students to use the Dog Pen problem to investigate Siegel's concept of *density*: "the ratio of perimeter to the area of the region it bounds."

I have also used a variation of the lesson to focus on pedagogy when working with preservice and in-service teachers. Asking teachers first to find the area and perimeter of a square and a nonsquare rectangle and then to compare these tasks with the Dog Pen problem demonstrates clearly the difference between an exercise and a true problem.

JUDITH MACKS, jmacks@towson.edu, is a lecturer in the mathematics department at Towson University in Maryland. She is interested in improving the learning and teaching of mathematics by working with preservice and in-service mathematics teachers.

The Back Page provides a forum for readers to share a favorite lesson. Lessons to be considered for publication should be submitted to mt.msubmit.net. Lessons should not exceed 600 words and are subject to abridgment.

*Edited by **Jennifer Wexler,** wexlerj@newtrier.k12.il.us, New Trier High School, Winnetka, IL*

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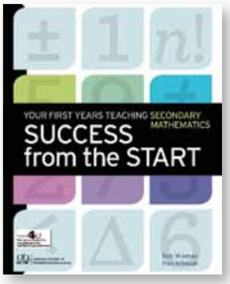

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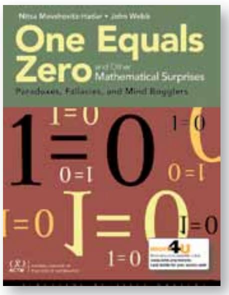

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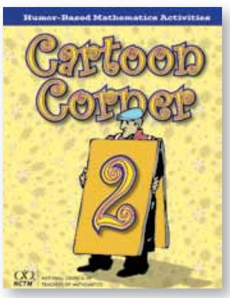

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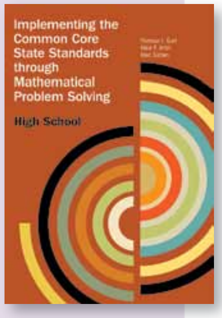

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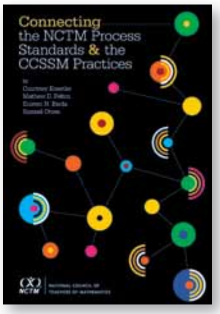

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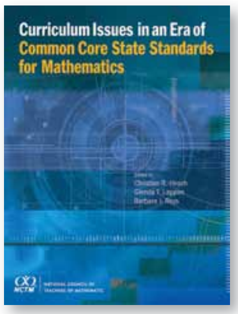

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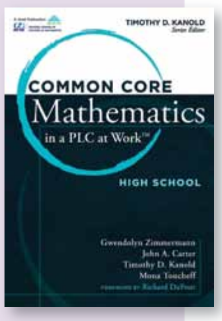

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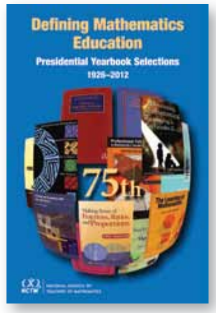

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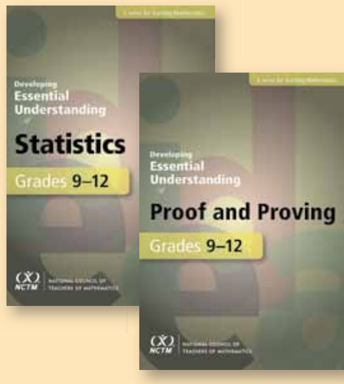

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