Posing Problems That Matter:

Investigating School Overcrowding

Angel and Naisha, two sixth-graders in Ms. Font's mathematics class, had the following conversation as they reflected on their experience with posing and investigating problems related to overcrowding at their school.

Angel. I think that every time you did the project [about overcrowding at the school], it makes you feel more *quizzitive*.
Naisha. What's quizzitive?
Angel. Curious. It's like—curious about different things, like it makes you want to go *deeper* into the project, and learn more stuff.… Like the legal width of the hallway, the different dimensions of each room. It makes you feel *mad* curious, because you want to know different things. And when you learn different things, that will help you with stuff, cause then you know how much space you got [in your school].

As teachers, we would like for all our students to be as "quizzitive" and as curious about mathematics as Angel. The problem posing she is discussing with Naisha was part of the study unit Overcrowding at Our School, in which students drew on important mathematical concepts such as measurement, fractions, and ratios to investigate their school space, compare it with that of other schools, and ultimately argue that their school was overcrowded.

Problem posing and problem solving have been core elements of reform initiatives for decades (National Council of Teachers of Mathematics 1989, 2000). Numerous researchers have argued for grounding problems in contexts relevant to students' experiences (e.g., Kahn and Civil 2001). What was unique about the problems posed by Angel, Naisha, and their classmates was that they were *authentic*—that is, they arose from an actual situation that was genuinely problematic: overcrowding at their school. The problems the students posed emerged from their daily experiences in navigating narrow, densely populated hallways, sitting in classrooms that felt "way too small," and observing a similar school that appeared to have more space. Students cared deeply about these issues, and whether or not they were solved made a difference to them personally. As Angel commented,

> It was what we wanted to learn about.… It was easier to do the math this way, instead of just learning it straight, like solving a problem, because we would actually like really get into it, and that made it easier.… Like the facts [about the school] they make you want to find out the answer. Like we *wanted* to know.

What happens when students pose and investigate *authentic* problems? What is the impact on their learning and engagement? And how do teachers negotiate a balance between helping students investigate real problems as they arise and ensuring

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that all students learn the mathematics they need to learn?

In this article, we describe what happened when we taught a five-week mathematics unit designed to foster posing and solving authentic problems. We tell the story of Angel's participation in this unit and use her story to highlight the value of this pedagogical approach. The merits include increased student engagement, enhanced opportunities for mathematical learning, and shifts in students' beliefs about mathematics (see **fig. 1**). We conclude with our reflections on the project and some suggestions for posing authentic problems in the elementary mathematics classroom.

Mathematics as a Tool to Investigate Crowding

Ms. Font teaches sixth-grade mathematics at "Francis Middle School," a diverse, urban public school in New York City. A small school with approximately 210 students, Francis Middle School is located in a predominantly working-class African American, Dominican, and Puerto Rican community. During the year of this study, the authors collaborated to design a series of project-based study units in which students used rigorous mathematics to investigate local and global issues. While each unit involved students in posing problems of interest to them, the overcrowding project in particular was driven by themes that emerged from the students' experiences (Freire 1993). To begin the study unit, Ms. Font asked the students to list issues about the school and local community that concerned them. The students' lists included topics such as neighborhood violence, gender-based salary discrepancies, decreases in school funding, and, repeatedly, the "space crisis" at their school (see **fig. 2**). Although any one of these topics might have sparked a rich mathematics unit, we selected the topic Overcrowding at our School for two important reasons. First, overcrowding was a salient issue for the students. Second, we knew that mathematically investigating the school space would provide an authentic context for several important mathematical concepts that Ms. Font needed to address, such as linear and area measurement and operations with fractions and ratios.

At the beginning of the study unit, the students claimed that Francis was "more crowded" than other schools and were eager to speak out in hope of increasing school space. Yet they were not sure how to discuss the crowding in terms that might convince school district administrators, and it was unclear to them how mathematics could enhance their argument. To help the students connect their concern about overcrowding with mathematics tools that would support their investigation, Ms. Font posed questions such as these: "How can we show the school district administrators how much space we have? What kind of information would we need to collect? What kinds of measurements? How might we prove that Francis is more crowded than Longmore?" ("Longmore Middle School" was located in the same building, one floor below Francis.) Students quickly realized that quantifying the school's space would be helpful. As one student noted, "We need to give them specifics … like [we need] to find the area!"

Ms. Font then prepared a series of lessons that addressed linear and area measurement, including how to find the area of a space having mixed-number dimensions, such as a hallway that measured 10 1/2 meters by 1 1/4 meters. [Note: The

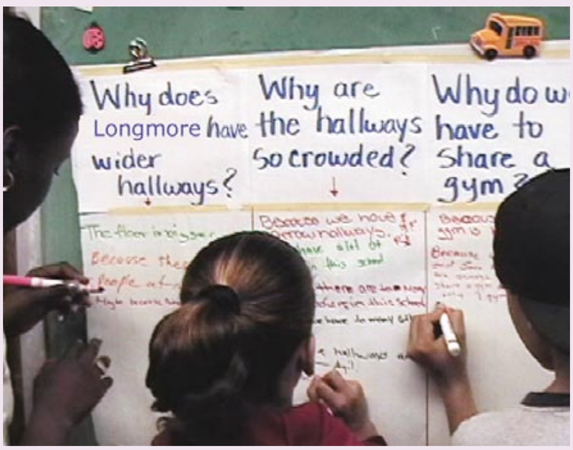
Figure 1

Merits of students posing and investigating authentic problems

1. Increased student interest and engagement in mathematics
2. Enhanced opportunities to develop mathematical understanding
3. Shifts in students' views of mathematics and its potentially transformative role in their lives

Figure 2

Students record their concerns about overcrowding at their school.



students initially measured length in meters and centimeters, but, considering the unit's mathematics goals—linear and area measurement, ratios, and operations with fractions and mixed numbers—Ms. Font guided them to record measurements expressed as the nearest fraction of a meter (e.g., 10 1/2 meters rather than 10 meters and 50 cm), thereby creating opportunities for the students to solve problems with fractions and mixed numbers.] After several days of measuring classrooms and hallways, calculating areas, and collecting information about the school district's space regulations, students formed small groups to investigate a particular aspect of the school space in greater depth. Ms. Font met with each group and helped the group members frame a problem that they could investigate mathematically. For example, one group, concerned about the schools' narrow hallways, posed the following problem: "How does our hallway space compare to the hallway space at Longmore? Are we really more crowded?" They generated relevant comparative data, such as area of the hallways and the number of students in each school, and constructed arguments based on their analysis. **Figure 3** displays students' analysis of the hallway space, and **figure 4** presents additional examples of the problems students investigated.

Figure 3

Lianna and Thomas's investigation of hallway space

Lianna and Thomas were concerned about overcrowded hallways, particularly between class periods, when all the students used the space at the same time. They first measured the length and width of the hallways in their school (Francis Middle School) and in a neighboring school (Longmore Middle School) and then calculated the total area of all hallways. Next, they calculated the amount of hallway space per student in the two schools and used these ratios to argue that the hallways in Francis were more crowded.

School	Total Area of All Hallways	Number of Students	Hallway Space per Student
Francis Middle School	199 1/2 square meters	213	0.95 square meters
Longmore Middle School	251 3/4 square meters	60	4.2 square meters

Figure 4

Space problems at Francis Middle School that the students investigated

- How does the total hallway area of Francis compare with the total hallway area of Longmore?
- What percentage of the school's hallways meets school district building codes (for minimum hallway width)?
- How much classroom space is there for each student at Francis?
- How does the classroom space per student at Francis compare with the classroom space per student at Longmore?
- What percentage of Francis's classrooms meets district building codes (for minimum classroom area)?

Angel's Topic: Overcrowding in the Girls' Restroom

Angel was a tall, rather quiet African American student in Ms. Font's class who spoke candidly about her preference for other subjects, such as language arts, over mathematics. She frequently made comments such as, "Math is all right, I guess, but I'm not really good at it," and her class participation was minimal. She often failed to complete assignments and was not very "quizzitive" about mathematics. However, when the class began to investigate overcrowding at the school, Angel's level of engagement increased noticeably.

Angel was extremely concerned about the school's bathrooms. She was bothered that all the female students in the school shared one small facility with only three working stalls and that the "wait time" during "peak use periods" made using the restroom almost impossible. Having to navigate among ten or twelve people in a tight space caused her significant frustration. Given the opportunity to pose her own problem about the school, Angel responded: "We want to know, why are the girls' bathrooms so small? And how does the size of our bathroom compare to the girls' bathroom at Longmore?" She assumed a leadership role in her group and with great enthusiasm gathered relevant data such as the number of female students in the school, the restroom's dimensions, and the number of stalls. Her group collected similar data from Longmore in order to compare the girls' restrooms across schools.

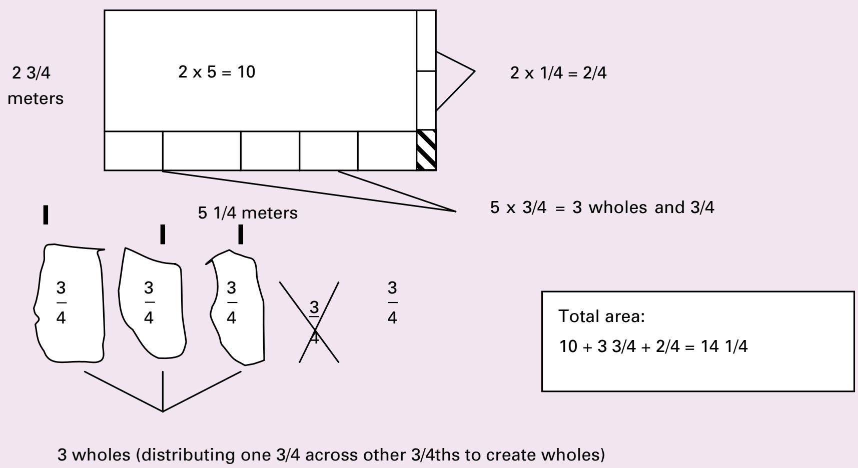
As Angel posed an authentic problem that mattered to her, her desire to understand and change the situation motivated her engagement in mathematics. This was not the first time that Angel's class had participated in rich problem solving. However, for Angel the opportunity to pose problems related to issues she cared about set this experience apart from the rest. In her words,

> Look, it's like you are learning about things you *be* [*sic*] in every day, and it's a part of your

Figure 5

Angel's strategy for calculating the area of the girls' restroom

Angel partitioned a drawing of the girls' restroom into smaller rectangular spaces to calculate the area. She began by dividing the space into areas of whole and partial square meters (i.e., ten whole square meters, five 3/4 square meters, and two 1/4 square meters). To find the area of the five 3/4 square meters, she broke one 3/4 into individual fourths, and then combined these fourths with the remaining 3/4s to create wholes. Her calculation of the area was an estimate; she did not include the area of the small rectangular space in the lower right corner of the diagram, which measured 3/4 meter by 1/4 meter. The students in Ms. Font's class decided that they could accurately estimate the area of rooms and hallways without including the area of this small space. The decision to omit this space was made, in part, because multiplying two fractions with conceptual understanding was beyond the ability of some of the students.



> life.… Because you know something more, it's like adding to your knowledge. Because you can remember that. So when we did that project about the space, it was something you could keep with you, it is like information that could involve *you*.

Angel's story is not unique. We found that posing authentic problems increased many students' interest and engagement in mathematics. They assumed ownership over the problems they posed and acted as agents in their own learning. We argue that this increased engagement, especially for students such as Angel, who tend to remain on the margins of classroom activity, is an important merit of this kind of problem-posing pedagogy.

Enhanced opportunities for developing mathematical understanding

The students in Angel's group continued their investigation by constructing a floor plan of the girls' restroom and calculating the area. To figure the area, Angel partitioned the 5 1/4-by-2 3/4-meter space into smaller rectangular areas so that she could deal with whole and partial square meters separately (see **fig. 5** for a diagram and full description of Angel's strategy). This partitioning strategy was invented by one of Angel's classmates as a way to deal with rooms whose dimensions included fractions of a meter. The strategy was made public in a whole-group discussion and then appropriated by many students.

As **figure 5** demonstrates, Angel had a developing understanding of area, including how to calculate the area of a rectangular space with mixed-number dimensions. Also evident is her ability to multiply a whole number by a fraction (i.e., 5 × 3/4) and combine fractional parts to create wholes. These concepts, important for all sixth graders, were concepts that Ms. Font aimed to address in this study unit. In addition, for Angel they were new mathematical understandings she developed and applied over the course of the project. Given her history as a low-achieving mathematics student, this understanding is significant.

Other teacher-created problems may have addressed the same mathematical goals (i.e., area and fractions) but may not have engaged Angel. The authentic problems Angel posed were mathematically rich *and* engaging, thus motivating her to participate in problem solving and, in turn, resulting in learning. We argue that such enhanced opportunities for constructing mathematical understanding are a second merit of this problem-posing pedagogy.

Shifts in students' view of mathematics and its usefulness

Posing problems that matter resulted in new mathematical understandings for Angel and also changed how she viewed the discipline. Before this study unit, Angel spoke of mathematics as "numbers" and "operations" and topics she never understood, such as "that division thing." She knew that mathematics was "important for the future" but could not describe how it related to life outside the classroom. Investigating authentic problems pushed Angel to broaden her conception of mathematics, particularly how it might be useful in her life. She commented:

> With math … it's like you have more defense. You know the length and the width, and you know—let's say you go and have an argument with somebody, and you say the hallway or the bathroom is small, and they say, "What do you mean it's small?" and you don't even know how big it is. And it's like, this room, you know the length and the width and the area, and then it's like you have more defense right there, because you know more stuff that they didn't even know about.

Angel was not the only student who experienced a shift in her views about mathematics. In general, students came to view mathematics as a tool that could help them investigate important personal and social issues, explore issues of equity and fairness, and argue and prove their point of view. Students often struggle to identify personal reasons why they should learn mathematics (Martin 2000; Noddings 1993), and yet understanding the utility of mathematics in one's life is a core component of the Equity Principle as set forth in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000, p. 12). Thus these shifts in Angel's and her peers' understandings about mathematics are significant.

Balancing authenticity and mathematical goals

The stories of Angel and her classmates are compelling and highlight the merits of posing authentic problems in the mathematics classroom. Yet we recognize that this kind of teaching is challenging and complex. In this section, we describe a particular challenge we encountered as we implemented the study unit, one that we believe other teachers may face and one that is inherent in instruction driven by multiple goals: How can teachers ensure that the problems students pose are "real" and authentic and at the same time address particular mathematical concepts?

We recognize that teachers often have to address certain content, and Ms. Font, as noted, did have clear mathematical goals in mind for this unit—linear and area measurement, ratios, and operations with fractions and mixed numbers. But also important was the fact that the students engaged in mathematics that was personally and socially relevant. In most cases, the problems the students posed about overcrowding led to mathematical problems consistent with the study unit's goals. The students measured hallways, calculated areas, and learned to use space-per-person ratios as tools to evaluate crowding. But occasionally the students posed problems about the school space that did not easily lend themselves to mathematical investigations.

L. J. and Joel, two students passionate about basketball, were concerned about the safety hazard created by floor-to-ceiling poles in the school gym. Ms. Font's role was to help the students see how their interests—proving to the school district that their gym was not a safe place to play basketball—intersected with the mathematical content they needed to study. This is not an easy challenge, and we acknowledge that mathematics may not always be the best discipline to address the problems the students pose. In this case, Ms. Font was able to negotiate this intersection. She helped L. J. and Joel expand their study so that it included an analysis of how the gym space was shared among multiple schools and how the presence of poles in the gym reduced the area of the court where students could play basketball. We believe it is important for problem solving to be "real" and for students to pose questions they genuinely care about. At the same time, we recognize teachers' responsibility to address particular mathematics concepts. Pursuing these multiple goals is bound to create moments of tension, and we found the following suggestions helpful in addressing these challenges.

Suggestions for Problem Posing in the Elementary Mathematics Classroom

1. Start small, with an instructional unit lasting several weeks. Although the mathematics unit we

Figure 6

Students practicing the speech they presented to a school advisory team

have described spanned five weeks, curriculum mandates may make it difficult for teachers to set aside the standard text for extended periods of time. Teachers can integrate posing authentic problems by developing short units of study that draw on grade-level mathematics objectives to investigate important local issues. A study of the school cafeteria, including menu items, their cost, student preferences, and nutritional value, could address mathematics concepts related to collecting, organizing, and analyzing data, while designing a space for a school function, such as a carnival or a field day, might ask students to apply measurement concepts and spatial reasoning. We acknowledge that not all important mathematics concepts will "emerge" from problems that students pose, and we do not envision investigating authentic problems as a replacement for the curriculum. But we see authentic problems as a way of enhancing and, in cases such as Angel's, transforming students' experiences in learning mathematics.

2. *Look for the mathematics potential in local issues*. Teachers can begin by paying attention to situations in the school or community that have the potential to spark mathematically rich problem solving. As we listened to Ms. Font's students complain about overcrowding at the school, we realized that analyzing the school space could lead to a variety of mathematics problems involving measurement, fractions, and ratios. Teachers can work together to brainstorm possible topics, and the particular mathematics concepts that each topic might involve (for additional examples of "real world" issues that other teachers have drawn on to teach mathematics, see De Orilla a Orilla 2005; Gutstein 2001; and Gutstein and Peterson 2005).

3. *Be flexible*. We quickly discovered that the ability to pose authentic problems develops naturally. On almost a daily basis, we found ourselves adjusting lesson plans and gathering information and materials related to the particular problems the students posed. For instance, the students initially used metric units for all measurements. Several weeks into the project, before sharing the results of their investigation with school district administrators, the district staff requested that all data be converted to standard units because, as one student explained, "The district only speaks feet." Ms. Font quickly planned a lesson that addressed how to convert units of measurement. The flexibility to adjust daily instruction as needed was essential to the success of this project.

4. *Incorporate mini-lessons.* Because the students investigated complex "real" situations, anticipating all the mathematical skills and understandings they would need was challenging. We found that mini-lessons spaced throughout the study unit were an effective tool for introducing new mathematical concepts on an "as needed" basis. Mini-lessons covered topics such as multiplying fractions and mixed numbers, converting between metric and standard units, and calculating space-per-person ratios. Although mini-lessons typically involved the whole class, Ms. Font sometimes presented lessons to small groups of students.

5. *Allow for student choice and ownership.* Although the entire class investigated the same general problem—overcrowding—we found it beneficial for students to pose their own problem about a particular aspect of the issue that was of interest to them. This element of choice helped all students assume ownership of the project, thus, as we found, increasing their engagement and their opportunities for learning mathematics.

The Impact of the Students' Investigations

Toward the end of the study unit, the students shared the results of their investigation with others. They compiled analyses of various aspects of the school space—hallways, classrooms, and restrooms—and presented their findings at a school advisory team meeting and later to school district administrators (see **fig. 6**). The students argued:

> The board of education has a building code that the classrooms have to be at least 750 square feet for 30 children. As you can see on the [floor

plan], only 3 classrooms are big enough.… The board of education has another building code that says the hallways must be 5 feet and 8 inches wide.... There is only one hallway that is 5 feet 8 inches.… So as a school we think we should have less students or more space.

What became of the students' investigations? Did the data they presented effect change?

At the end of the school year, it was unclear whether the school district would increase Francis Middle School's allocated space or limit its projected enrollment. But over the summer, the district decided to reduce the incoming class by approximately 30 students, thus allowing the school to retain its current size instead of increasing from 210 to 240 students, as initially planned. Although this result may seem inconsequential, the school district's action prevented an already crowded school from becoming even more overcrowded, an action that the students welcomed as one small success. As one student, Jhana, commented,

Yes [we made a difference], because first of all, we found out something for ourselves and we actually proved a point. We actually, like *we* made the difference.… We learned what we learned, and we told people.… And math made our argument make more sense. You couldn't do it without the math.

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