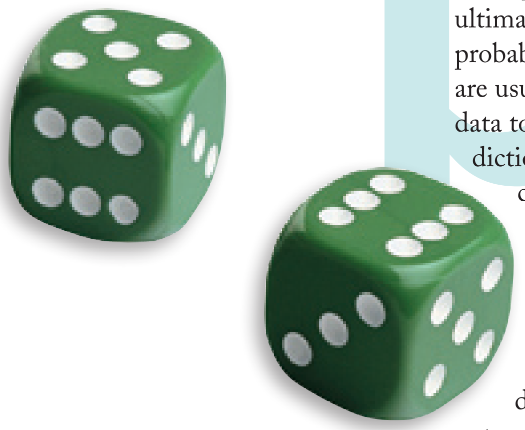
Darin Beigie

Probability Experiments with Shared Spreadsheets



Edited by **Gwen Johnson**, gjohnson@coedu.usf.edu, Secondary Education, University of South Florida and **James Dogbey**, jdogbey@mail.usf.edu, Secondary Education, University of South Florida. This department's classroom-ready activities may be reproduced by teachers. Teachers are encouraged to submit manuscripts in a format similar to this department based on successful activities from their own classroom. Of particular interest are activities focusing on NCTM's Content and Process Standards and Curriculum Focal Points as well as problems with a historical foundation. Send submissions by accessing **mtms.msubmit.net**.

Probability experiments illustrate how apparently random events are ultimately governed by the laws of probability. A large number of trials are usually necessary for experimental data to converge to a theoretical prediction. Having students electronically combine their data into a shared spreadsheet provides an efficient and powerful way to collectively analyze a large amount of information. In this activity, seventh-grade students use a shared spreadsheet to collaboratively investigate what happens when they roll a pair of dice. They compare the case of two 6-sided dice with the case of a 4-sided die rolled with an 8-sided die.

ROLLING 6-SIDED DICE

Students should have a basic understanding of counting outcomes using lists, charts, or tree diagrams. Students will start with a pair of 6-sided dice, one red and one white. Emphasize at the outset that rolling a 3 with the red die and a 5 with the white die is a different outcome from rolling a 3 with the white die and a 5 with the red die.

Students work in small groups and complete the chart on **activity sheet 1** to determine the theoretical probability of rolling each sum using dice. Some students will need to be convinced that there is only one outcome when the numbers are the same (e.g., a 2 on each die). One common mistake that students make is continuing the chart with impossible combinations, such as 7 + 3 = 10. Another common mistake is for the list to fail to add to 36 outcomes. Since students are just beginning to be understand why there are $6 \times 6 = 36$ outcomes, they have varying proficiency with being able to pull themselves out of the list-making mode and see the abstraction (even after using a tree diagram).

The students then draw a bar graph showing the theoretical probability. They generally learn quickly that 7 is the most likely sum because it has the most possible combinations of numbers on the two dice, whereas 2 and 12 are the least likely sums because they each have only one way to reach the sum. Each group then rolls their dice 100 times and tallies their results to compare with their theoretical results. As they roll the dice, they experience the sense of randomness that accompanies any game of chance.

Each student draws a bar graph of the experimental results from their 100 rolls (see **fig. 1**), and compares their result with the theoretical prediction. Only as the number of rolls increases do patterns begin to emerge in the tally. Although 100 rolls produce enough data to see a pattern, noticeable discrepancies generally remain from the theoretical and experimental graphs. Students have decidedly mixed reactions about whether their experimental graph and theoretical graph match, as shown in these comments:

- The most surprising difference was that in the experimental graph the probability of rolling an 8 was much higher than that of a 7.
- The most prominent similarity was that both were roughly shaped in a triangle.
- The probability [theoretical] graph is steadily going up to a peak and down. The experimental graph goes up and down unpredictably.
- I think that if we rolled the dice enough times it would begin to look roughly like the probability [theoretical] graph.

The students generally see similarities between their theoretical and experimental work. To some, the differences are striking, and the experiment does not seem to exhibit much of a pattern.

A shared spreadsheet provides an ideal opportunity for students to combine their data and thus dramatically increase the number of rolls. For homework, students from three classes are given a week to enter their data on a shared Excel spreadsheet on our school's shared math folder (see **fig. 2**).

The percentages are linked to a bar graph that shows the combined experimental results. The theoretical graph is inserted below the experimental graph for comparison. Once all the data have been gathered on the spreadsheet, students analyze the results to see the strong resemblance between the theoretical and experimental results. The effect of combining the data is clear, and students generally show a good rudimentary understanding. In explanation, Jack writes:

Fig. 1 Students roll two 6-sided dice 100 times and record the sum on each roll. Four student teams' experimental probability results, which barely resemble the theoretical probability, are shown.

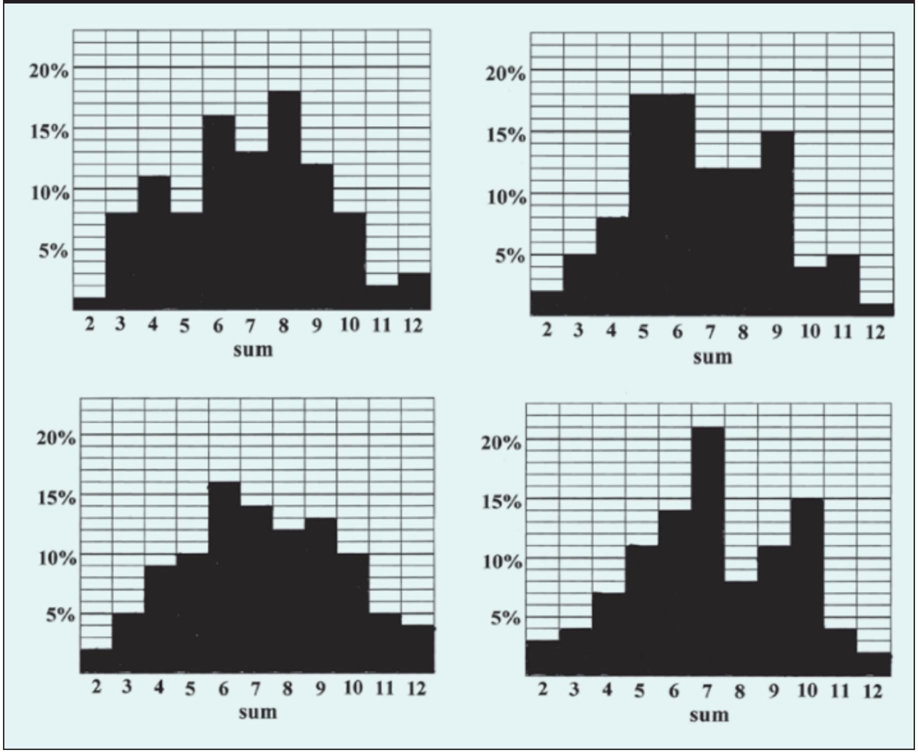
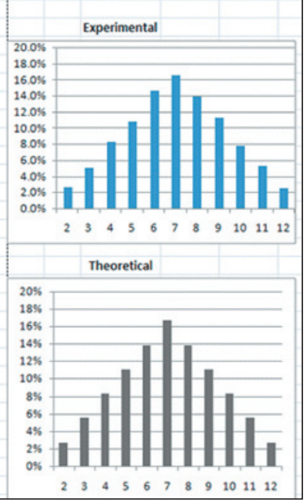


Fig. 2 Using a spreadsheet program, all student experiments can be combined to see that with more trials, experimental probability mirrors theoretical probability.

Sums of Two Rolled Dice

	2	3	4	5	6	7	8	9	10	11	12	Total
Teacher	5	3	7	8	16	16	12	13	8	6	6	100
Brit & Alex	1	4	15	11	17	17	10	11	9	4	1	100
Eric & Ben	4	8	8	7	14	11	22	13	5	6	2	100
Michael R. & Sam	2	4	10	15	12	16	17	11	7	4	2	100
Milan & Kavin	3	8	5	9	13	16	15	12	10	6	3	100
Alexis & Katie	3	5	12	14	10	17	15	10	7	5	2	100
Julia & Michael V.	2	5	9	10	16	14	12	13	10	5	4	100
Andrew & Daniel	1	1	12	9	19	15	15	14	9	5	0	100
Alex & Shana	3	6	6	9	16	17	14	9	8	10	2	100
Noah & Sinclair	4	4	8	5	12	19	16	10	8	11	3	100
Tara & Sanah	3	5	8	8	16	20	15	7	8	6	4	100
Patrick & Nick	3	6	6	9	10	16	16	12	8	7	7	100
Kay & Logan	2	5	8	18	18	12	12	15	4	5	1	100
Aidan & Taleen	1	8	11	8	16	13	18	12	8	2	3	100
Anthony & Jack	5	7	8	11	16	13	18	8	9	3	2	100
Caroline & Angela	3	3	11	12	13	15	9	16	8	7	3	100
Liza & Emily	4	8	5	11	15	20	17	11	3	5	1	100
Mathis & Mara	2	3	5	20	15	21	15	6	6	4	3	100
Shelby & Monica	2	9	6	10	10	21	17	10	8	3	4	100
Juan & Alex	1	3	8	14	21	20	8	14	7	4	0	100
Marten, Angela, Liam	3	4	9	16	12	20	8	14	4	5	5	100
Justin & Margeaux	3	4	7	11	14	21	8	11	15	4	2	100
Ray & Alessandra	5	3	12	15	13	17	8	10	11	4	2	100
Jade	2	8	5	2	20	13	20	11	9	8	2	100
Average	2.8%	5.2%	8.4%	10.9%	14.8%	16.7%	14.0%	11.4%	7.9%	5.4%	2.7%	100%



The Shared Spreadsheet

Setting up the spreadsheet for students is fairly straightforward.

- Enter student names beforehand in column A, along with teacher data as an example in row 5 (see **fig. 2**).
- Set up column M to use the sum formula to add the number of rolls entered by each group (e.g., SUM(B5:L5) for row 5).
- Use the last row (row 29, in this case) to apply the average formula to obtain the percentage of rolls for each sum (e.g., AVERAGE(B5:B28)/100 for column B).

As students enter their data, the formulas and linked graph are automatically updated. Although we use a shared spreadsheet that is available on our school network, other options are available that allow students to edit a shared spreadsheet, including online access with Google Docs at docs.google.com and Zoho Docs at zoho.com. Although online sharing of student data is the most streamlined option, teachers can also simply enter all the student data and have each group verify that their data are entered correctly.

One set of data is not enough to explain the probability of rolling dice. When all the outcomes are recorded, they balance each other out, creating a smoother graph. In the end, the experimental graph was nearly identical to the theoretical graph.

Although the correspondence between the theoretical and experimental is striking, it is helpful to acknowledge that some differences will persist after several hundred rolls are combined. After students analyze the spreadsheet, they are shown a double-bar graph for side-by-side comparison of theory and experiment, to highlight subtle differences (see **fig. 3**). We discuss how, as the number of rolls increases further, these differences get even smaller.

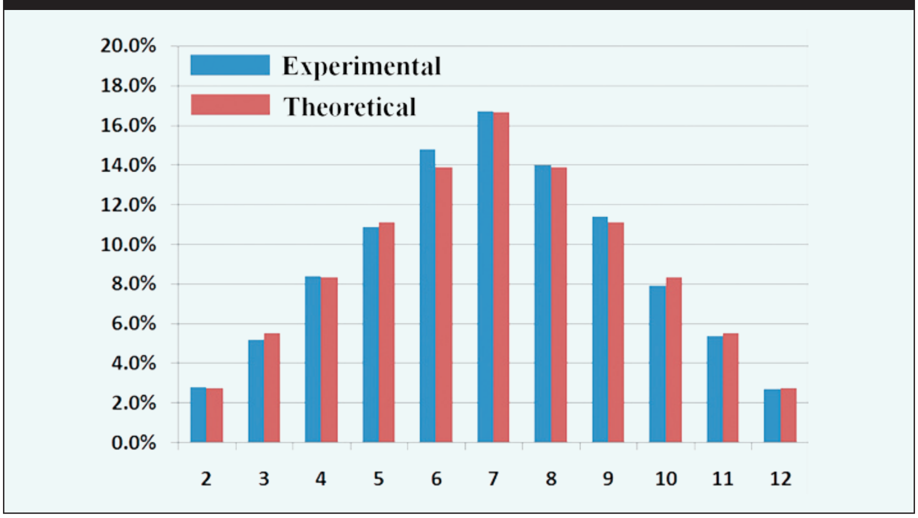
ROLLING A 4-SIDED DIE AND AN 8-SIDED DIE

After a convincing agreement between theory and experiment with two 6-sided dice, students later investigate the case of rolling a 4-sided die with an 8-sided die. This lesson begins with the experiment. For homework, each student performs 100 rolls, makes a bar graph of their experimental results (see **fig. 4**), and enters their data on a similar shared spreadsheet. In class, the students compare their individual bar graphs with one another and with the shared results. Students see noticeable variations in individual data, as compared with the shared data (see **fig. 5**), as shown by these comments:

- Overall there are no patterns other than the fact that the sums 5–9 usually rolled higher than 2–4 and 10–12.
- The individual 4 × 8 graphs were nowhere near consistent. Some kept on going up, down; up, down.
- I was surprised by the shared results, because all the individual results were so different. The variation from the individual results really balanced out how the percentages in the shared results turned out.

Following their observations on the experimental results for the 4 × 8 case, students study the theoretical probability to make sense of their experimental findings. Comparing the 4 × 8 with the 6 × 6 graphs, students see the basic contrast as being a plateau in the result of the 4 × 8 experiment versus a peak in the 6 × 6 experiments. In addition, they attempt (a bit awkwardly, perhaps) to connect these shapes to the outcome in the underlying theory. As with the first set of experiments, we show the students a double-bar graph for side-by-side comparison of theoretical and experimental results (see **fig. 5**). For this specific set of

Fig. 3 A side-by-side comparison of the theoretical and combined experimental probability for throwing two 6-sided dice shows that when a large number of experiments are counted, the experimental probability more closely resembles the theoretical.



results, students recognized a dip in the experimental graph. They were encouraged to consider it part of a minor oscillation about the theoretical plateau spanning the sums of 5 through 9.

CONCLUSION

A shared spreadsheet allows students to achieve a powerful collective result that they would not be able to achieve individually. The fuzzy, unconvincing statistics of a hundred rolls is clarified when several hundred rolls are combined. The payoff is a striking illustration of the clean matches that are found between theory and experiments.

I like to impress on my students that there is invisible structure bristling throughout our world. Performing probability experiments using shared spreadsheets convinces students of this structure as shared data come to life. The information gathered through shared technology provides a powerful enhancement for determining pattern and structure. For example, combining data sets from an algebra experiment can result in a group scatter plot with a more definitive fitted line or curve. Using shared technology allows students to collectively analyze, discover, and create in new and robust ways.

REFERENCES

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Solutions to **activity sheets 1** and **2** are appended to the online version of this article at **www.nctm.org/mtms**.

Fig. 4 Students roll a 4-sided and an 8-sided die 100 times and record the sum on each roll. The experimental probability results for four student teams, which barely resemble the theoretical probability, are shown.

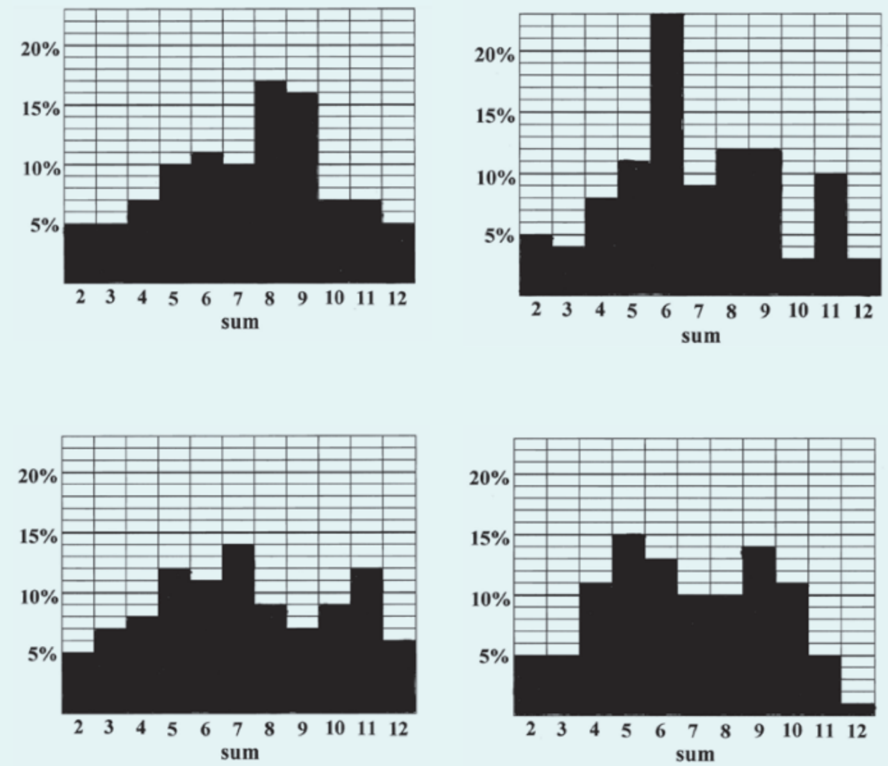
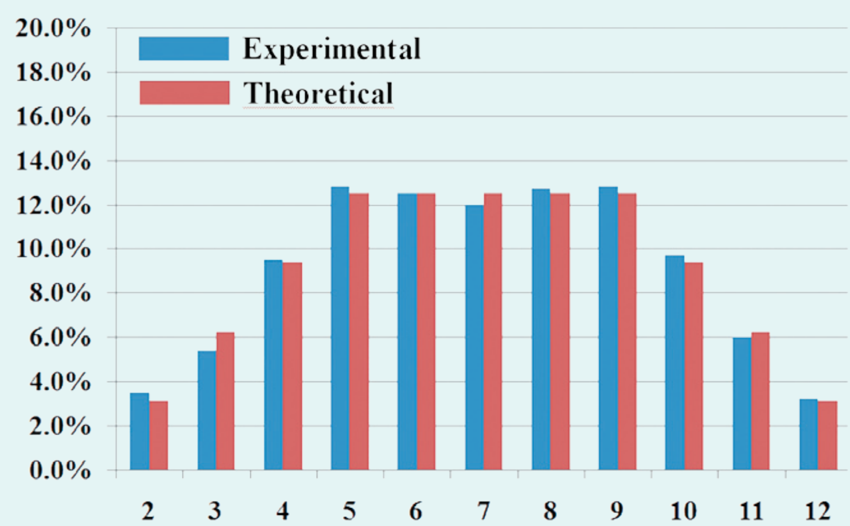


Fig. 5 Again, a side-by-side comparison of the theoretical and combined experimental probability for throwing a 4-sided die and an 8-sided die shows that with a large number of experiments, the experimental results resemble the theoretical results.



activity sheet 1

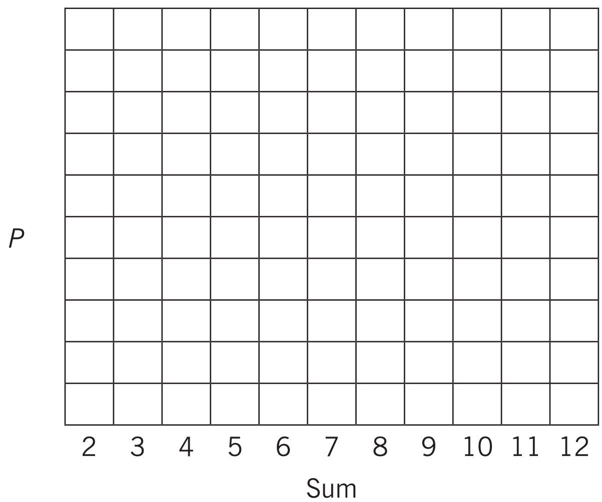
Name ______________________________

THEORETICAL PROBABILITY: DICE ______

Complete the chart to determine the theoretical probability of rolling each possible sum for the pair of dice. Write the probabilities in fraction form (do not simplify for sake of comparison).

Sum	Outcomes	Probability
2	1 + 1	1/36
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Make a bar graph of your probability *P*. Label the vertical axis in fractional form (1/36, 2/36, 3/26 … 8/36).



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activity sheet 2

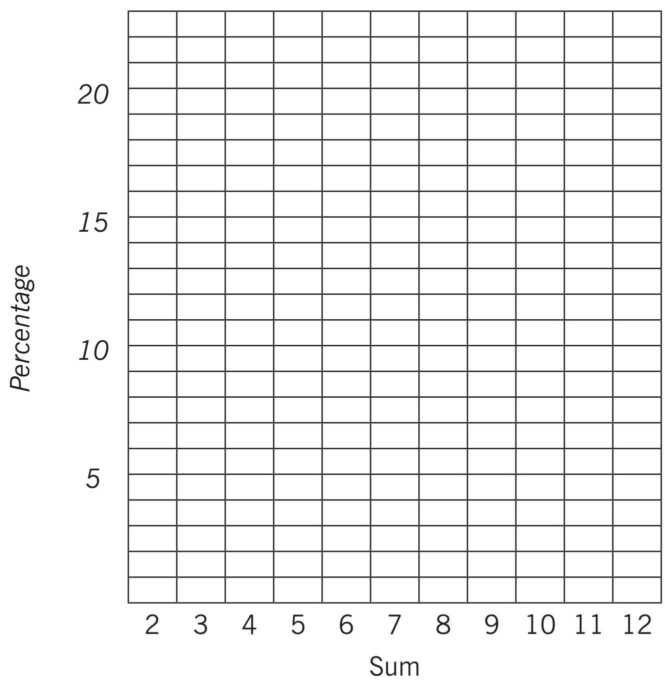
Name ______________________________

EXPERIMENTAL PROBABILITY: DICE ______

Roll a pair of dice 100 times and tally how many times you get each of the possible sums. To make the tally, put a check in the appropriate row each time you roll that sum. Give your answers in the form of a percent.

Sum	Number of Times Rolled	Percentage
2		/100 = ____ %
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Make a bar graph of your experimental percentages.



solutions to mathematical explorations

(Continued from page 489)

SOLUTIONS TO ACTIVITY SHEET 1

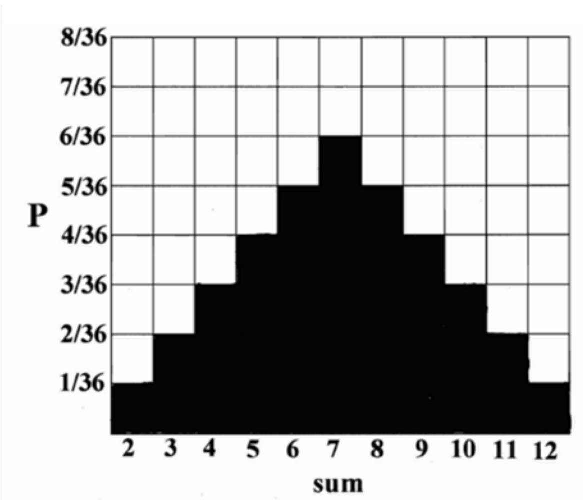
The theoretical chart for rolling two 6-sided dice:

Sum	Outcomes	Probability
2	1 + 1	1/36
3	1 + 2, 2 + 1	2/36
4	1 + 3, 3 + 1, 2 + 2	3/36
5	1 + 4, 4 + 1, 2 + 3, 3 + 2	4/36
6	1 + 5, 5 + 1, 2 + 4, 4 + 2, 3 + 3	5/36
7	1 + 6, 6 + 1, 2 + 5, 5 + 2, 3 + 4, 4 + 3	6/36
8	2 + 6, 6 + 2, 3 + 5, 5 + 3, 4 + 4	5/36
9	3 + 6, 6 + 3, 4 + 5, 5 + 4	4/36
10	4 + 6, 6 + 4, 5 + 5	3/36
11	5 + 6, 6 + 5	2/36
12	6 + 6	1/36

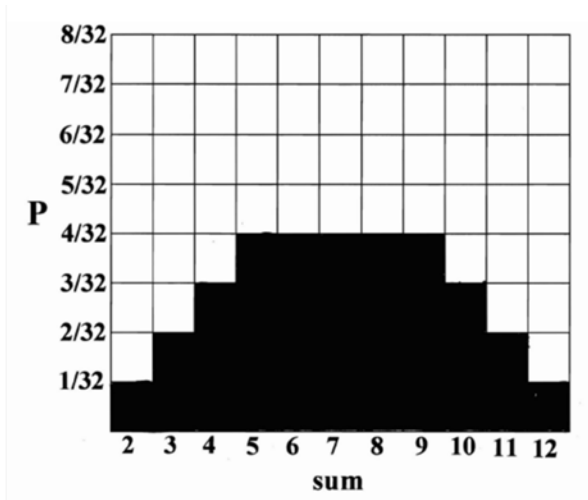
The theoretical probability chart for rolling a 4-sided die and an 8-sided die:

Sum	Number of Times Rolled	Percentage
2	1 + 1	1/32
3	1 + 2, 2 + 1	2/32
4	1 + 3, 3 + 1, 2 + 2	3/32
5	1 + 4, 4 + 1, 2 + 3, 3 + 2	4/32
6	1 + 5, 2 + 4, 4 + 2, 3 + 3	4/32
7	1 + 6, 2 + 5, 3 + 4, 4 + 3	4/32
8	1 + 7, 2 + 6, 3 + 5, 4 + 4	4/32
9	1 + 8, 2 + 6, 3 + 6, 4 + 5	4/32
10	2 + 8, 3 + 7, 4 + 6	3/32
11	3 + 8, 4 + 7	2/32
12	4 + 8	1/32

The bar graph of the theoretical probability of rolling two 6-sided dice:



The bar graph of the theoretical probability of rolling a 4-sided die and an 8-sided die:



SOLUTIONS TO ACTIVITY SHEET 2

Experimental results will vary.