Mathematical Proficiency: The Five Strands

In 2001, the National Academies released a report on K-8 mathematics, *Adding It Up*, written by the National Research Council. In this report, they noted that for much of the early 20th century, the focus on mathematics learning was computational procedures of arithmetic. The pendulum swung several times over the next 40 years between focusing on understanding and accurate and quick computations.

***Adding It Up* advocates for viewing mathematical proficiency as five interwoven and interdependent strands. A deep understanding of mathematics requires learners to connect pieces of knowledge and use these connections to productively solve problems.**

Below, you will see those five strands, along with some potential teacher and student actions that should be part of all mathematics classrooms. Because these five strands are interwoven and interdependent, you'll likely notice the overlap of some of the actions.

Conceptual Understanding

Procedural Fluency

Strategic Competence

Adaptive Reasoning

Productive Disposition

NCTM Position on Procedural Fluency

Procedural fluency is an essential component of equitable teaching and is necessary to developing mathematical proficiency and mathematical agency. Each and every student must have access to teaching that connects concepts to procedures, explicitly develops a reasonable repertoire of strategies and algorithms, provides substantial opportunities for students to learn to choose from among the strategies and algorithms in their repertoire, and implements assessment practices that attend to all components of fluency.

Procedural Fluency in Mathematics
National Council of Teachers of Mathematics

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1 Conceptual Understanding

Adding It Up describes conceptual understanding as "an integrated and functional grasp of mathematical ideas" (p. 118). Students should be able to understand the importance of concepts and recognize what contexts they are helpful for. This deep understanding helps support retention as they are more likely to remember it. A primary indicator of conceptual understanding is the ability to represent situations in different ways and recognize how different representations are useful for different purposes.

Teacher Actions

- Make mathematics accessible to students through concrete, semi-concrete, and abstract representations.
- Proactively facilitate multiple "access points/modes" for students.
- Use multiple representations when appropriate while using and eliciting student thinking.
- Ask students to give a visual proof of their solution.

Student Actions

- Demonstrate understanding of concepts in multiple ways.
- Make multiple connections among different models and representations.
- Understand that multiple iterations are needed to reach an appropriate solution and persevere in the revision(s) of their thinking.
- Use multiple modes of communication to convey their solution(s) and explain what the solution(s) mean(s) in context.

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Mathematical Proficiency: The Five Strands

2 Procedural Fluency

Adding It Up describes procedural fluency as "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121). Students need to be accurate and efficient in performing basic computations with whole numbers as well as have knowledge of ways to estimate the result of a procedure.

Teacher Actions

- Provide opportunities for distributed practice of procedures.
- Provide multiple opportunities using a variety of formats to assess students' understanding.
- Provide opportunities and experiences for students to connect student-generated strategies with efficient procedures.
- Have students play games that involve strategy and the use of procedural fluency.
- Recognize the need for various modes of explanation, including written, verbal, and diagrams.
- Uses precise mathematical language when describing strategies.
- Provide feedback based on student thinking.
- Provide opportunities for students to find and analyze errors in their work and others' work.

Student Actions

- Accurately use a procedure on a novel problem or new number set.
- Demonstrate flexibility in using strategies and recognize and justify when one strategy is more efficient than others.
- Identify, use, and communicate multiple pathways to a solution.
- Find and analyze errors in their work and others' work to show an understanding of the procedures and why they work.

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Mathematical Proficiency: The Five Strands

3 Strategic Competence

Adding It Up describes strategic competence as "the ability to formulate mathematical problems, represent them, and solve them" (p. 124). Students should recognize that there are a variety of solution strategies and which strategies may be helpful for solving a specific problem.

Teacher Actions

- Develop their own content knowledge to utilize multiple strategies and progressions.
- Develop the pedagogical knowledge to allow for multiple ways of solving problems.
- Provide class discussions on the efficiency of different strategies through questioning.
- Anticipate problems and strategies that may arise during instruction and create intentional questions to engage and develop strategies.
- Provide a thinking task or problem that can be solved in multiple ways with a specific instructional goal in mind.
- Provide feedback on student processes and strategies.

Student Actions

- Ask peers about strategy selected to compare with their own.
- Reflect on which strategies are best used and when to use them when looking at work.
- Recognize that if they arrive at a solution that does not make sense, there is another strategy to try.
- Formulate mathematical problems, represent, and solve them.
- Explain other students' solution strategies and include connecting to your own.

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4 Adaptive Reasoning

Adding It Up describes adaptive reasoning as the "capacity to think logically about the relationships among concepts and situations" (p. 120). Students should be able to justify their reasoning. It includes informal explanations as well as intuitive and inductive reasoning.

Teacher Actions

- Make connections to prior knowledge to further student thinking.
- Utilize protocols for students to justify their mathematical claims and respond to those of others.
- Anticipate student strategies and plan responses.
- Acknowledge, listen, and respond to student thinking and revisit when needed.
- Create a classroom culture where sharing initial thinking, think-alouds, and rough drafts is valued.

Student Actions

- Find and correct errors, recognizing that they should use mistakes to further their development of the concept they are learning.
- Identify multiple strategies to approach a problem and justify the approach taken.
- Justify why they changed their initial approach to a problem.
- Provide informal and formal explanations and justifications.
- Apply what they learned from one idea, task, etc., to a new scenario.
- Ask and answer questions to clarify the task.

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Productive Disposition

Adding It Up describes productive disposition as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). Students see mathematics as understandable, that it can be learned and used, and that they are capable of figuring it out.

Teacher Actions

- Believe every student is capable of learning mathematics.
- Develop perseverance in students through culturally sustainable contextual thinking tasks and encouragement.
- Provide opportunities to grow and show mastery by connecting math learned in the classroom to their community and the world.
- Utilize protocols for students to share their thinking and make sense of others' thinking.
- Model utilizing mistakes and multiple strategies through think-alouds to demonstrate the learning process. Allow students opportunities to identify and correct errors.

Student Actions

- Believe that they can use math to solve problems in their community and the world.
- Persevere in learning new concepts, recognizing that understanding concepts takes time.
- Accept the challenge of the task and recognize that there is always a place to start when trying to solve a problem.
- Recognize that there are a variety of strategies to choose from when approaching a math problem.
- Share their thinking and make sense of others' thinking.
- Make connections with the math learned in the classroom to their community and the world.
- Use think-alouds to share how they identified and corrected mistakes as part of developing understanding concepts.

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