

Facilitating Meaningful Mathematical Discourse

NCTM Interactive Institute, 2015

Name

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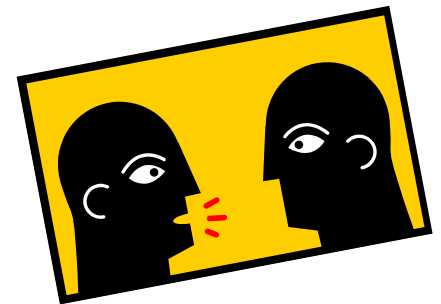
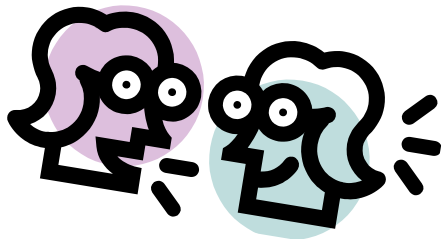
Email Address

Facilitating Meaningful Mathematical Discourse

At your tables, discuss the following question.

How would you define discourse?

- What “IS” discourse?
- What “IS NOT” discourse?



Facilitate Meaningful Mathematical Discourse

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Facilitate Meaningful Mathematical Discourse

“Students who learn to articulate and justify their own mathematical ideas, reason through their own and others’ mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields.” (p.4)

Carpenter, Thomas P., Meagan Loef Franke, and Linda Levi. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary Schools*. Portsmouth, N.H.: Heinemann, 2003.

Facilitate Meaningful Mathematical Discourse

The role of the teacher is
to engage students in tasks
that promote reasoning and problem solving
and
facilitate discourse
that moves students toward
shared understanding of mathematics.

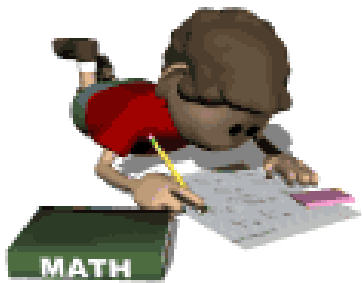
Five Practices for Effective Discourse

1. **Anticipating** student responses prior to the lesson.
2. **Monitoring** students' work on and engagement with the tasks.
3. **Selecting** particular students to present their mathematical work.
4. **Sequencing** students' responses in a specific order for discussion.
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

Smith, Margaret S., and Mary Kay Stein. *5 Practices for Orchestrating Productive Discussions*. Reston, VA: National Council of Teachers of Mathematics, 2011.

(2) Monitoring...

Review the student work from the *Zaggy Zs* problem.



Imagine that this is what you are “*seeing*” while students are working on the problem.

(3) Selecting...

Teacher controls discussion by determining:

- **WHICH** strategies based on clear, explicit goal of lesson
- **WHO** will show their mathematical ideas and competence (equity)

(3) Selecting...

- Teacher records who has used anticipated strategies while monitoring.

Strategy	Who / What	Order
Table of values		
Use center square: $n^2 + 2(n + 1)$		
Use center rectangle plus two end pieces: $n(n + 1) + 2$		
Move pieces or make rectangle around "Z"		

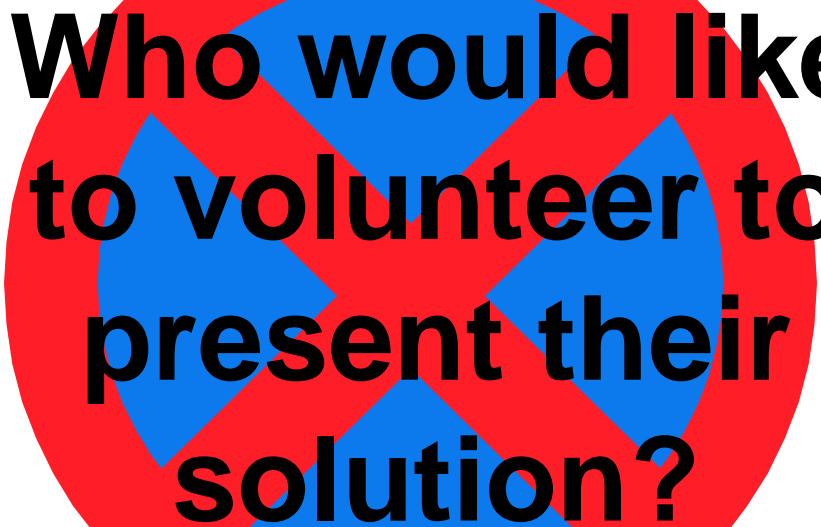
(4) Sequencing...

- Order of student/group presentations to achieve lesson goal
- Purpose is to make mathematics accessible to students and build story line

(4) Sequencing...

It is NOT....

**Who would like
to volunteer to
present their
solution?**



(3) Selecting and (4) Sequencing...

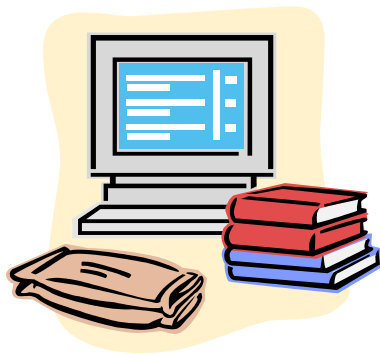
With your table members and the student work:

- Which students will present their strategy?
- Which order will students present their strategy?

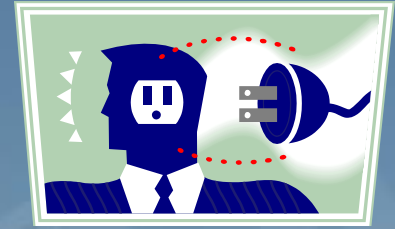
Explain your thinking about your choice.

(4) Sequencing...

- Strategy used most often to novel approach.
- Concrete – Representation – Abstract
- Errors and incorrect solutions



(5) Connecting...



- Teacher crafts questions to make mathematics visible to students.
Questions go beyond clarifying and probing.
- Teacher bridges what students know and what to be learned.
Goal is to link the student generated ideas, relationships, and representations.

Facilitating Meaningful Mathematical Discourse



Framework for moving toward a classroom community centered on discourse with teachers and students

- How teacher supports student engagement
- Who serves as questioner and what questions are posed
- Who provides what kinds of explanations
- How mathematical representations are used
- How much responsibility students share for learning with peers and themselves

Formats for Discourse



Whole Class Discourse

- Teacher “*in charge*” to guide learning
- Provide students with practice in mathematical reasoning
- Focus on students’ ideas, pathways, answers
- Make sense of new ideas
 - Reveal students’ confusion
 - Use misconceptions, errors for teachable moment

Formats for Discourse



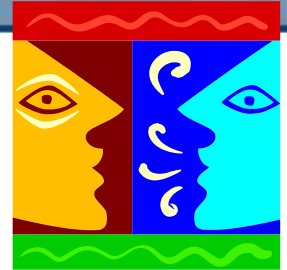
Small Group Discourse

- Teacher gives students a question/problem to discuss in groups of three to six.
- Teacher circulates among groups to observe, gather information, and possibly interject when needed within groups.
- Students control discussions among themselves.

Formats for Discourse

Partner Talk Discourse

- Teacher asks question
- Students respond to partner/neighbor to put thoughts into words.
 - Practice contribution before large group discussion. (ELL)
 - Hear ideas from someone else (struggling students)
 - Bring up questions with partner



Teacher's Role in Meaningful Discourse

Establish a Safe Classroom Environment

- Students take risks by presenting ideas/errors
- Everyone respects ideas/errors presented
- Listen to discussion
- Critique ideas, not people



Mathematics Teaching Today (NCTM),
Standard 5 Discourse, pp. 45-48.

Teacher's Role in Meaningful Discourse

Teacher has the responsibility for:

- Using a non-judgmental tone
- Allowing presentation by student not to be interrupted
- Student's ideas not overlapped with your ideas



Mathematics Teaching Today (NCTM), Standard 5 Discourse, pp. 45-48.

Teacher's Role in Meaningful Discourse

Teacher Moves:

- Revoicing
- Repeating / Restating
- Reasoning
- Adding on
- Waiting



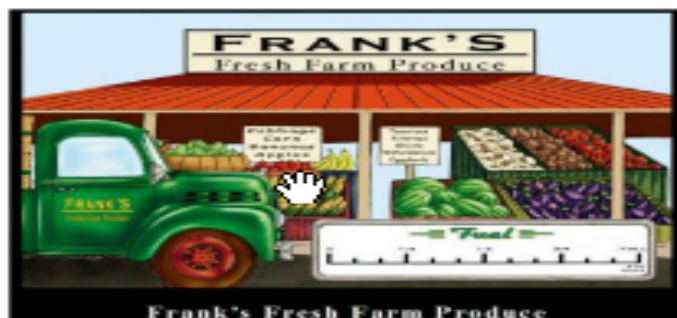
When is it appropriate to use any of the above “teacher moves” for classroom discourse?

Framework for Classroom Discourse

Principles to Action (page 31)
Hufford-Ackles, Fuson, and Sherin (2014)

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.



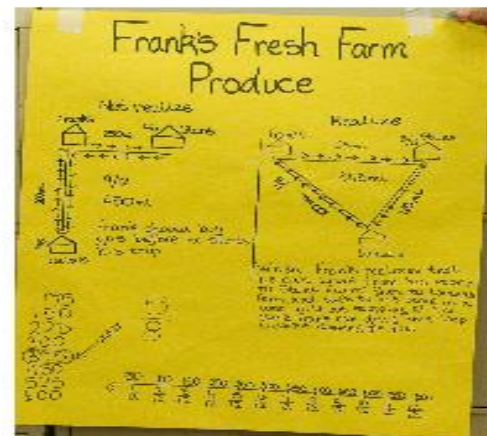
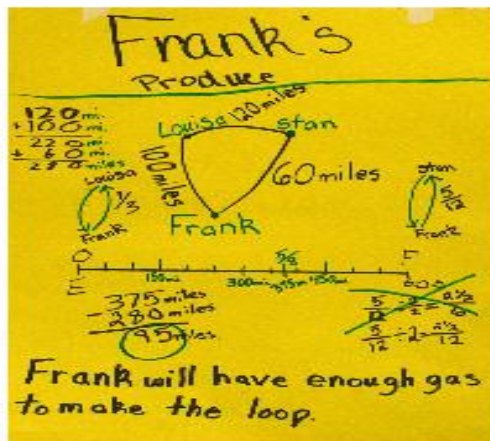
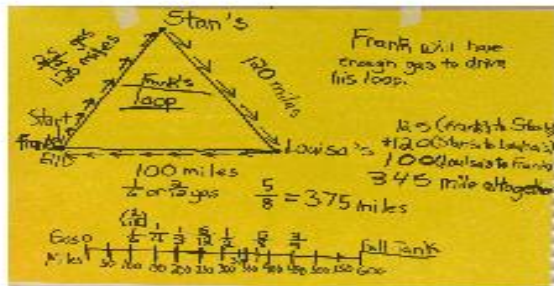
Frank runs a business called Frank's Fresh Farm Produce. Once a week he drives north of the city to farms where he buys the best possible fresh produce for his customers. Frank can travel 600 miles on a full tank of gas. His truck has a fancy, accurate fuel gauge.

Usually Frank has time to visit only one farm on each trip, but this week he decides to visit both Stan's and Louisa's farms. When Frank drives from his store to Stan's farm and back, he knows he uses $\frac{5}{12}$ of a tank of gas. When he drives to Louisa's farm and back, he uses $\frac{1}{3}$ of a tank. From a map of the area, he learns that there is a road from Stan's farm to Louisa's farm that is 120 miles long. He realizes that he can drive from his store to Stan's farm, then to Louisa's farm, and then back to his store in one loop.

Frank can tell by looking at the fuel gauge that he has $\frac{5}{8}$ of a tank of gas. Can he drive this loop without having to stop for fuel? Or should he buy gas before he starts his trip?

Facilitating Meaningful Mathematical Discourse

Frank's Fresh Farm Produce—Madison Elementary School, Washington



Student work for Frank's Farm problem

Facilitating Meaning Mathematical Discourse

Use the *Framework for Levels of Classroom Discourse* to describe what is happening in this classroom as students present solutions to Frank's Farm problem.

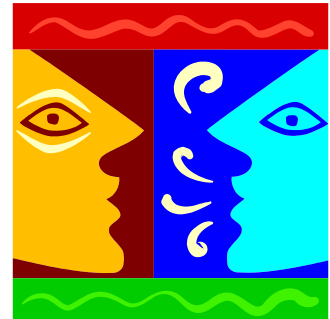


Representing a Problem Visually

Facilitating Meaningful Mathematical Discourse

Discuss with tablemates:

- At what level would you place this teacher within each of the five domains for mathematical discourse? Why?
- What suggestions can you offer to increase meaningful mathematical discourse in this classroom?



Standards of Mathematical Practices



Which of the Standards of Mathematical Practices are used when creating discourse?

- Reason abstractly.
- Construct viable arguments and critique the reasoning of others.
- Attend to precision.



Summary

Discuss with your table members:

When mathematical discourse is effectively facilitated in the classrooms,

what would be the:

Teacher actions?

Student actions?

Teacher and Student Actions

Meaningful Mathematical Discourse

What are teachers doing?	What are students doing?
<p>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</p> <p>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</p> <p>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</p> <p>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</p>	<p>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</p> <p>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</p> <p>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.</p> <p>Identifying how different approaches to solving a task are the same and how they are different.</p>

Disclaimer

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