Welcome!

NCTM 2017 Winter Institute
San Diego, CA
Middle School Strand
Session 1: Ratio and Proportion

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Introductions

- Please make a nameplate
- Please introduce yourself to your tablemates
  - Name
  - Where you teach
  - Grades/classes you teach
  - How long you have been teaching
  - Two “fun facts” about yourself

Introductions

"In too many schools, professional isolation severely undermines attempts to increase collaboration among colleagues, both between teaching peers internally in the school and among teachers, mathematicians, and mathematics educators externally…Such isolation stands as an obstacle to ensuring mathematical success for all students as well as teachers' continual growth."

(NCTM, 2014, p. 100)

As you engage in the Institute, we encourage you to keep your eyes & ears open for people that you could build professional relationships with!

Challenge

- Challenge for today: Please share your thinking, even if you're not sure about it!

  – It's our mistakes that make us lovable!

After the Institute, all content (e.g., slides, handouts, articles, etc.) will be posted online at:
http://nctm.org/pa17

Math jokes?

Why should you wear glasses to math class?

Math jokes?

Why should you wear glasses to math class?

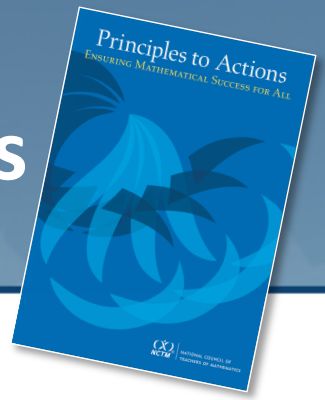
Why did 1/5 go to the psychiatrist?

Today's Agenda

- Content: Ratio and Proportion
  - foundation – why it's important
  - write/define ratio
  - explore a variety of contextual problems involving ratios using "student hat" and "teacher hat"

- Principles to Actions Effective Mathematics Teaching Practices
  - Use and connect mathematical representations

Effective Mathematics Teaching Practices



1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. **Use and connect mathematical representations.**
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

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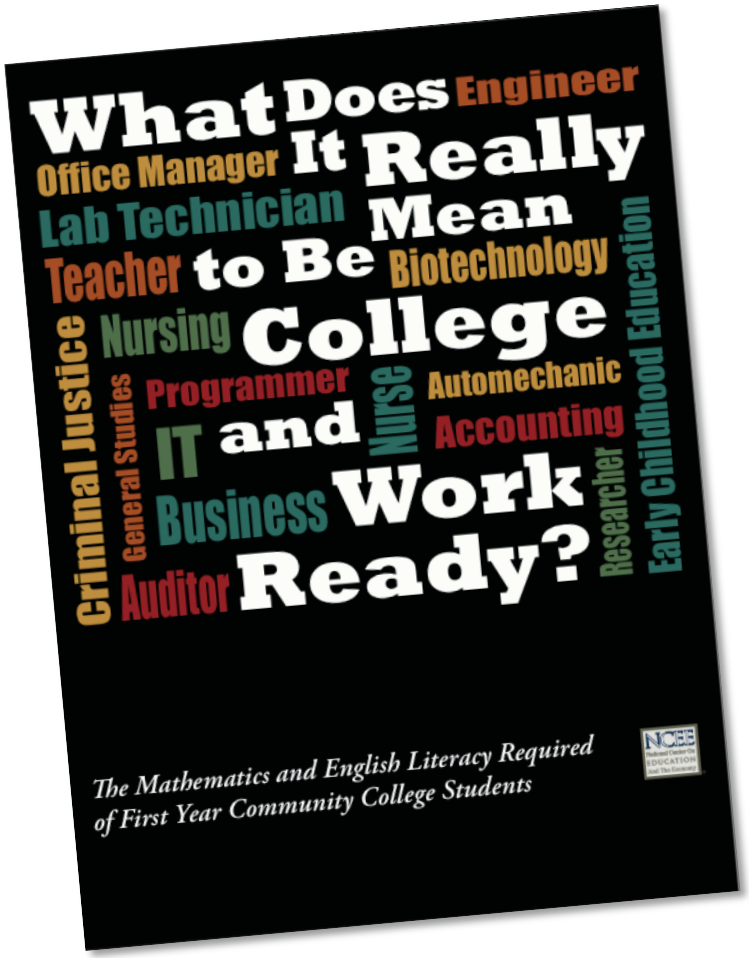
What does it really mean to be college and career ready?



"A very high priority should be given to the improvement of the teaching of proportional relationships…" (p. 12)

http://www.ncee.org/college-and-work-ready/

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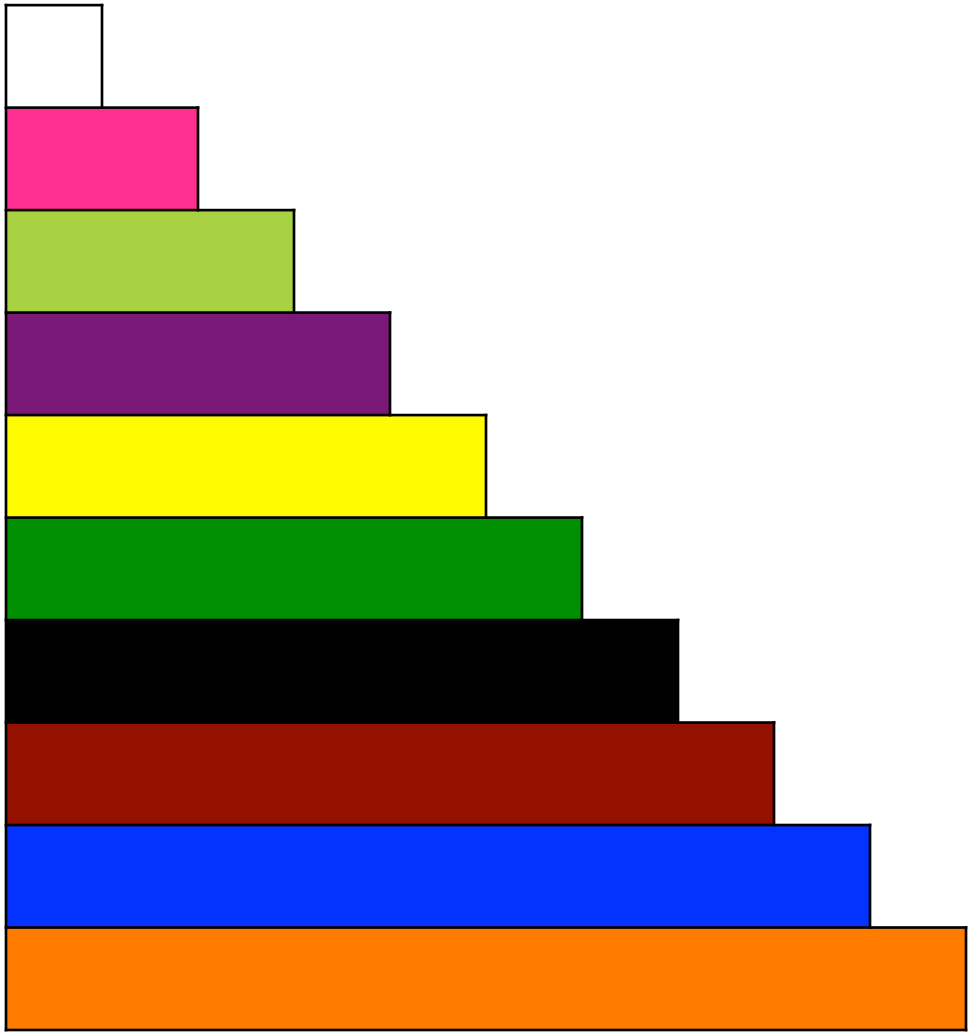
Building on students' work in the elementary grades

- In Grades 4 & 5, students have used multiplication for scaling problems
- Students have solved problems such as, "If Samir earns $3 per hour and Houda earns three times as much, how much does Houda earn?"
- Work with fractions (especially unit fractions) is an important foundational component

Briars, et al, *(2012) Common Core Mathematics in a PLC at Work™*, Solution Tree Press, 93.

Cuisenaire rods

Thanks to ETA hand2mind for supplying these!



Cuisenaire rods

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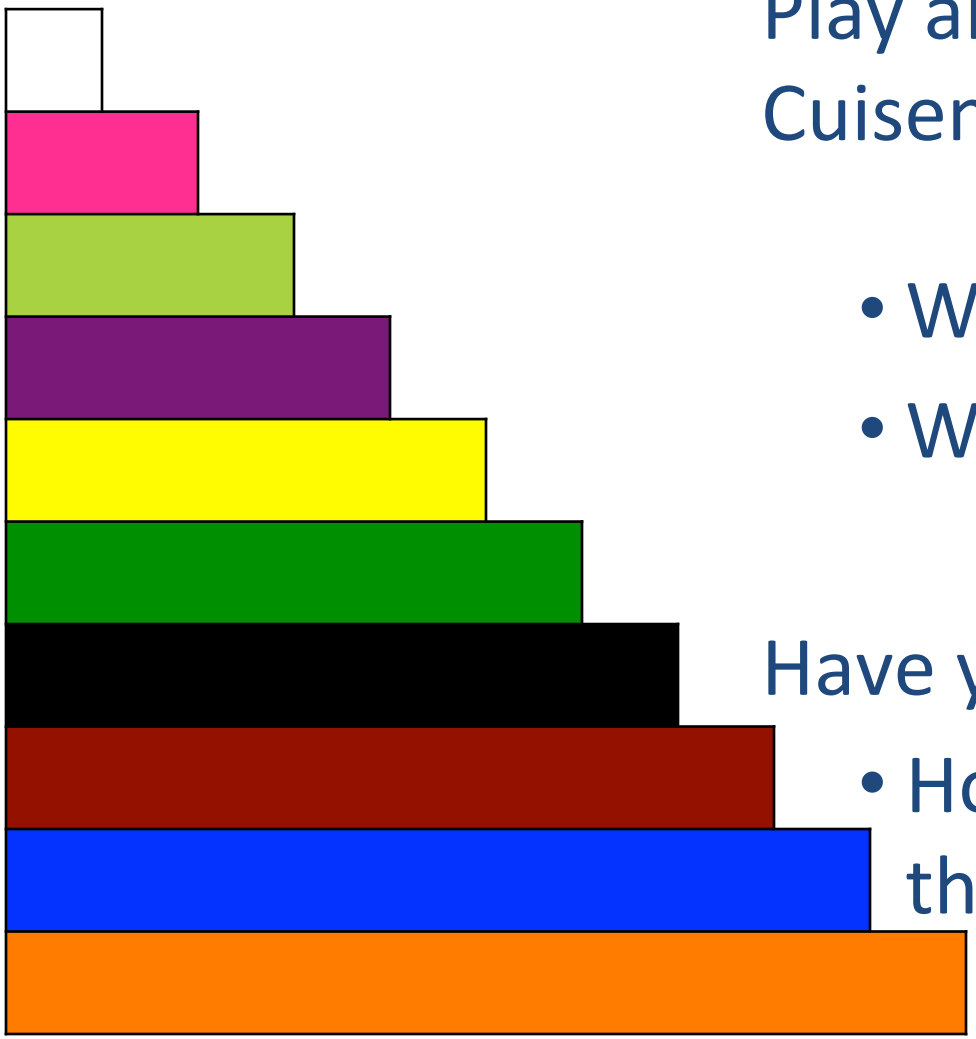


Play around w/ the Cuisenaire rods.

- What do you notice?
- What do you wonder?

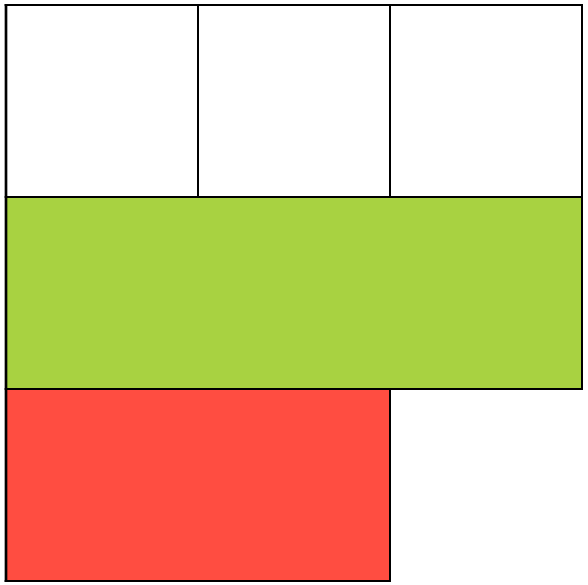
Have you seen these before?

- How have you used them?

Using Cuisenaire rods

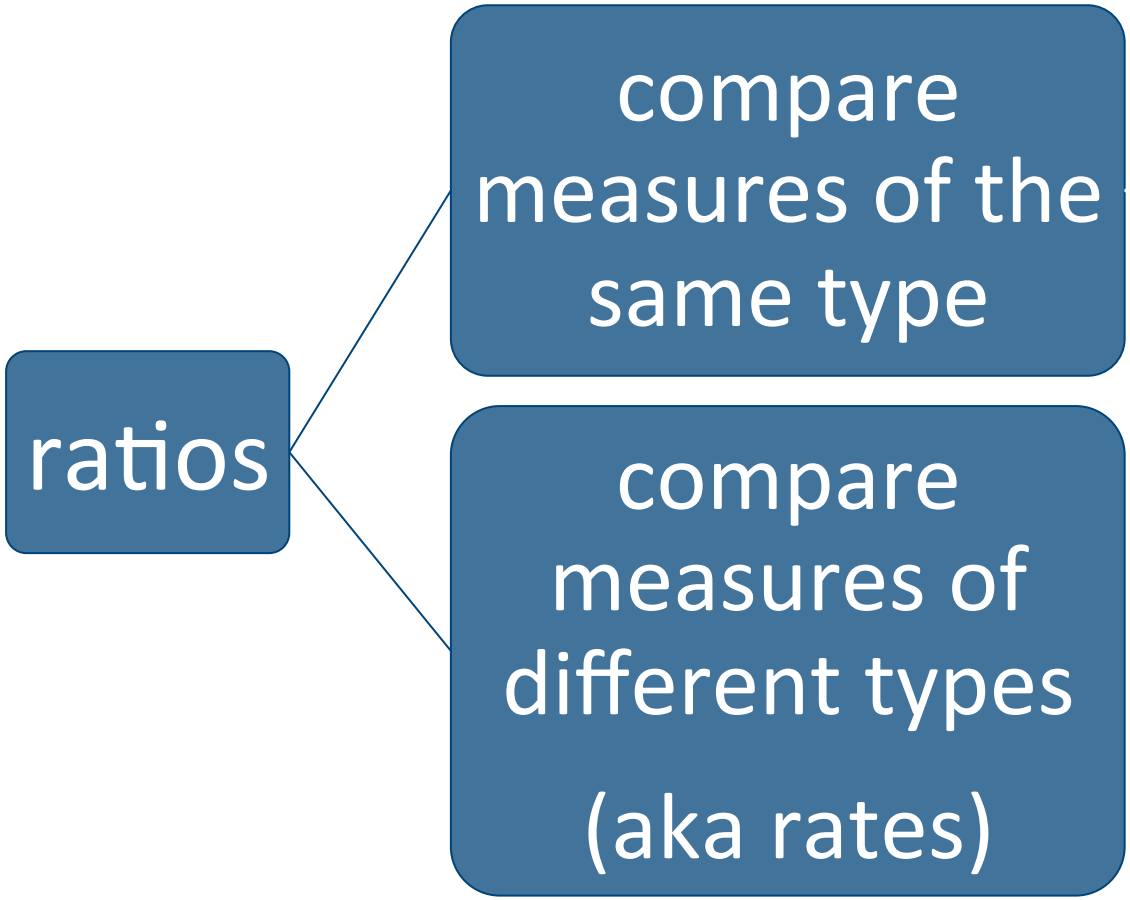
Thanks to ETA hand2mind for supplying these!

Write as many ratios as you can about this arrangement of white, lime green, and red rods



What is a ratio?

A ratio is a comparison of any two quantities.



(Lamon, 1999, p. 164)

Using Cuisenaire rods

- Is the relationship between the purple and blue rod the same as the relationship between the red and yellow rod?
- Try to find multiple ways to explain why or why not.

(Lamon, 1999, p. 136)

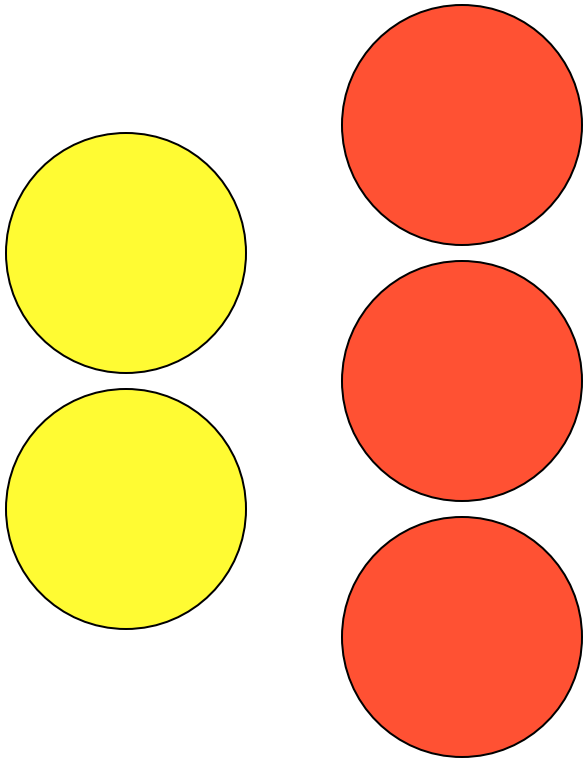
Using Cuisenaire rods

- Is the relationship between the purple and blue rod the same as the relationship between the red and yellow rod?
- Try to find multiple ways to explain why or why not.
- Can you find pairs of rods that have the same relationship?

(Lamon, 1999, p. 136)

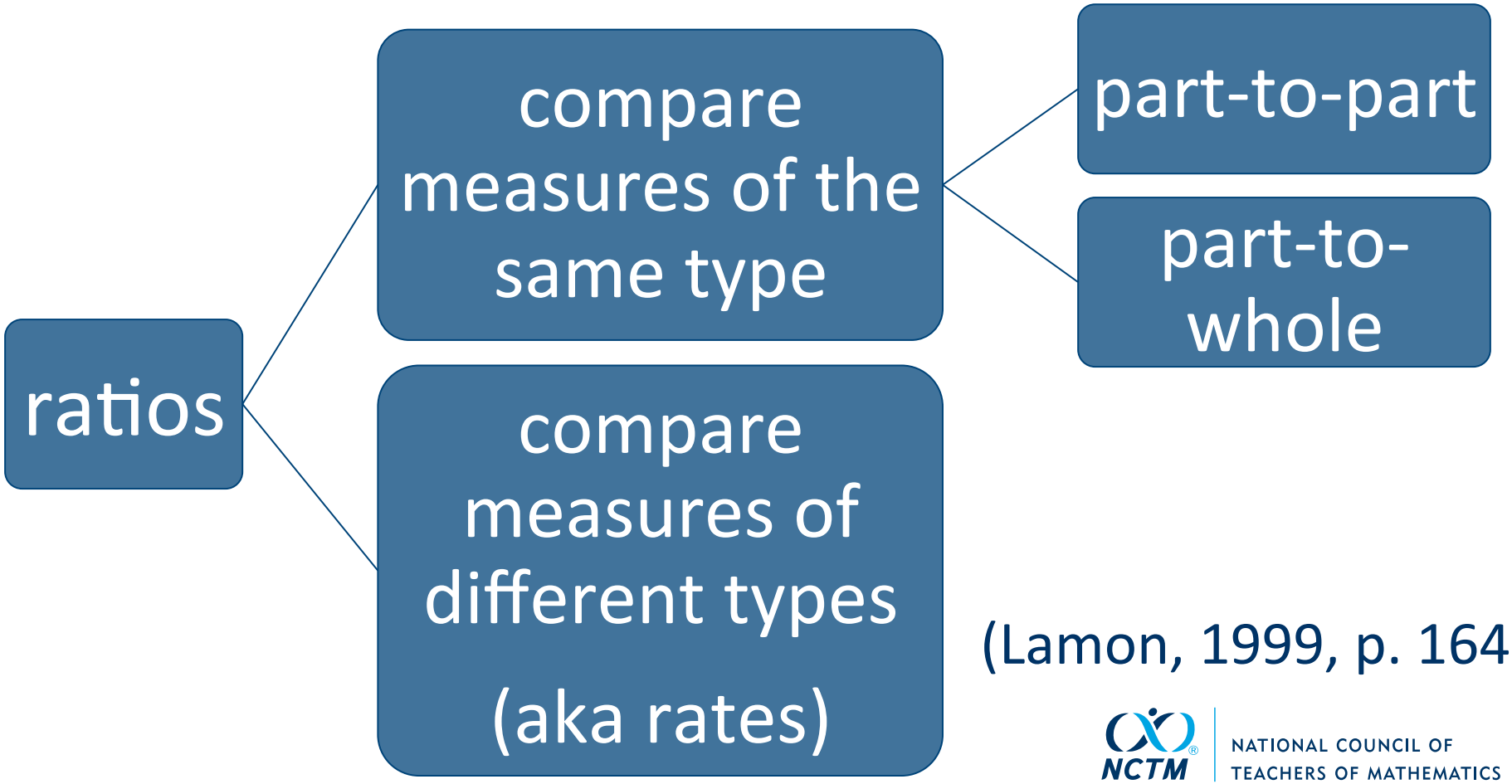
Using counters/chips

Write as many ratios as you can about this arrangement of counters



What is a ratio?

A ratio is a comparison of any two quantities.



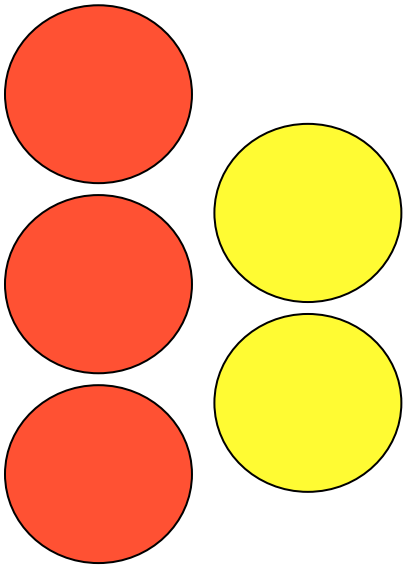
(Lamon, 1999, p. 164)

Comparing two arrangements

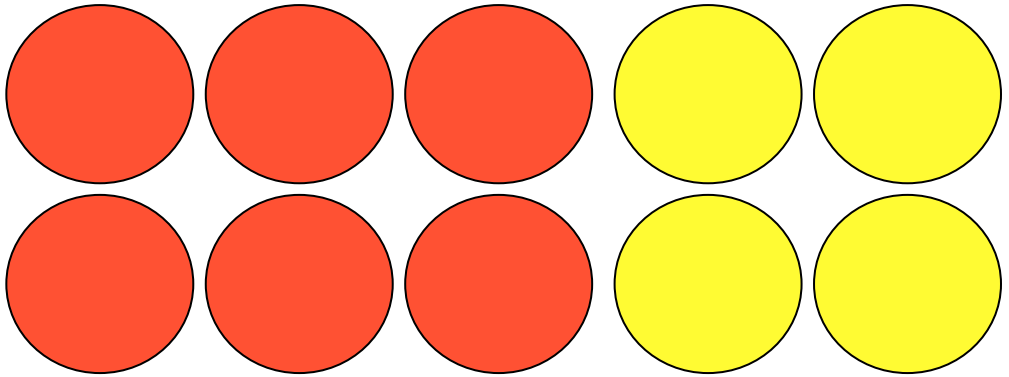
How are Arrangements A and B similar?

How are they different?

A



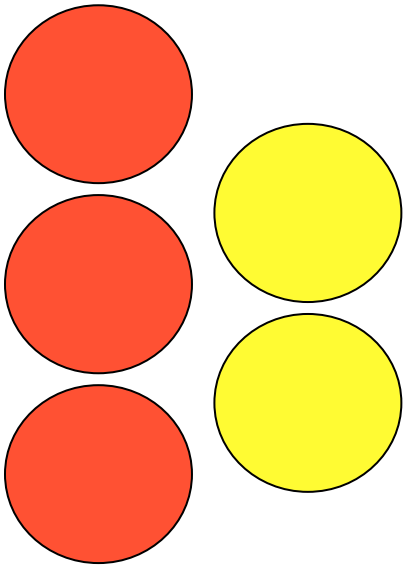
B



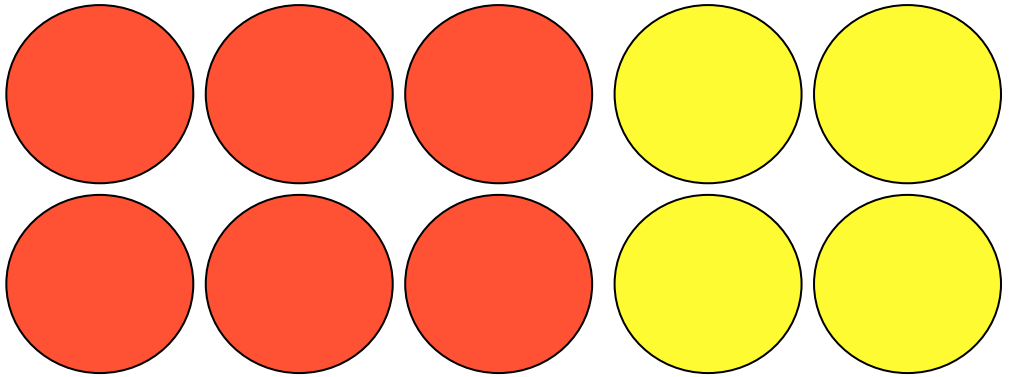
Comparing two arrangements

What other arrangements can you build that have the same relationship?

A

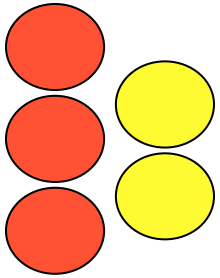


B



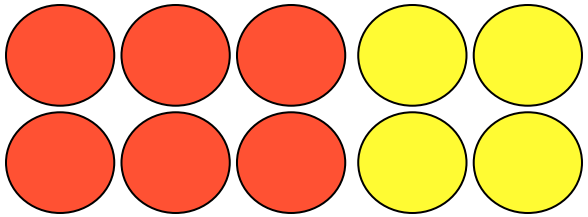
Comparing two arrangements

A



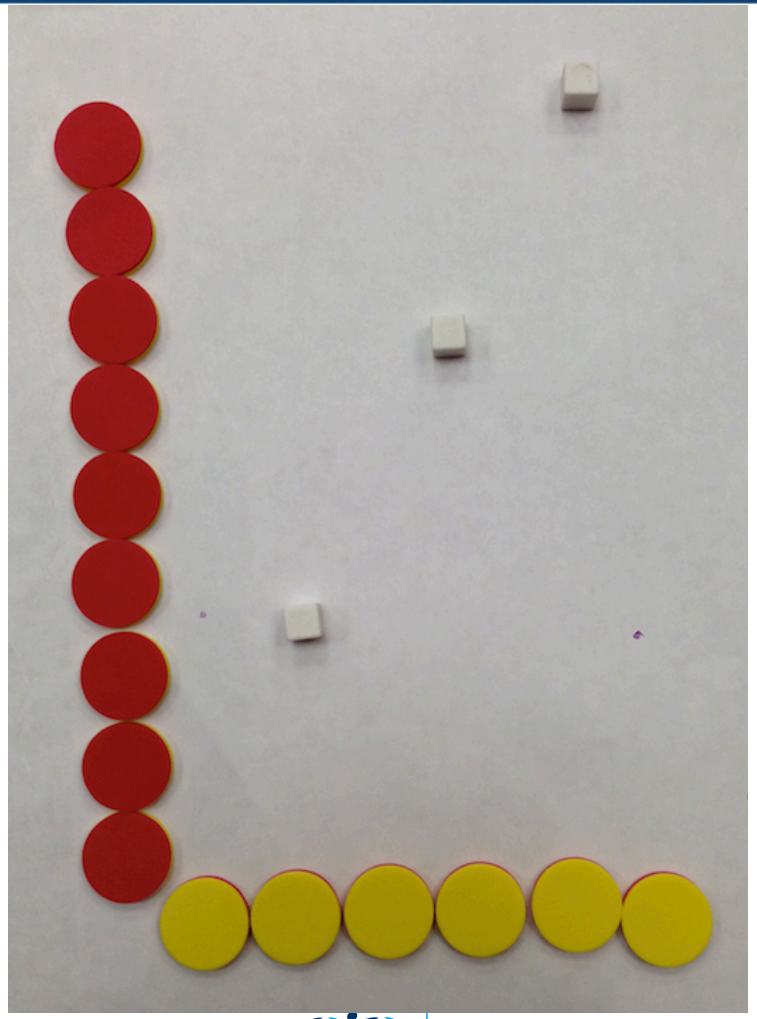
red	yellow	total
3	2	5
6	4	10
9	6	15
12	8	20
15	10	25

B



Considering arrangements of counters/chips

In what ways does this picture relate to the previous pictures?



Making punch

A punch mixture has five parts grape to two parts peach.

What questions could be asked about this situation?

Tinker with some of your questions!

Making punch

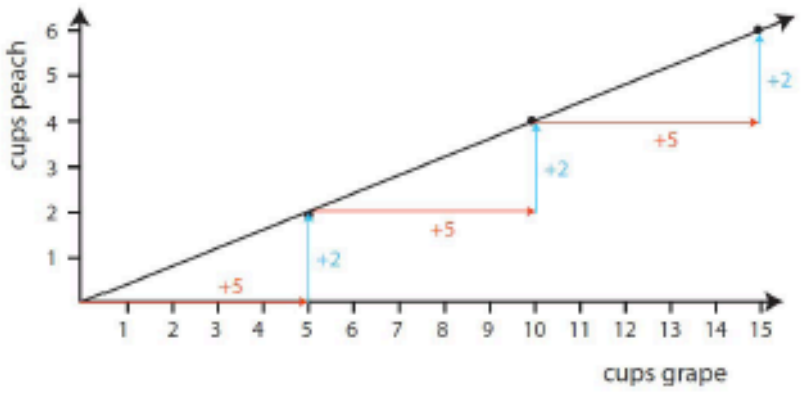
grape	peach
5	2
10	4
15	6
20	8
25	10

Essential Understanding

Showing structure in tables and graphs

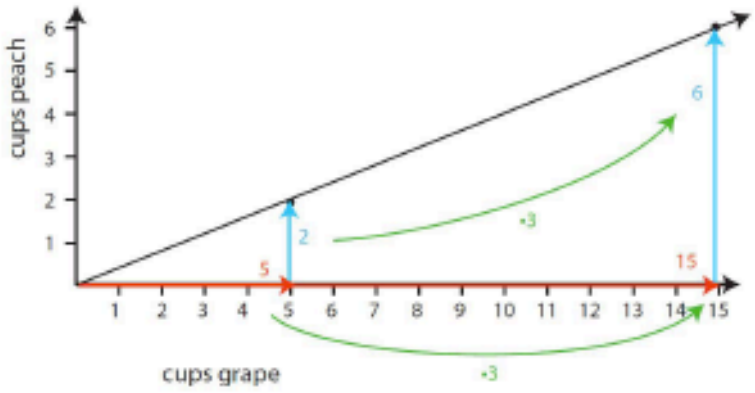
Additive Structure

cups grape	cups peach
5	2
10	4
15	6
20	8
25	10



Multiplicative Structure

cups grape	cups peach
5	2
10	4
15	6
20	8
100	40



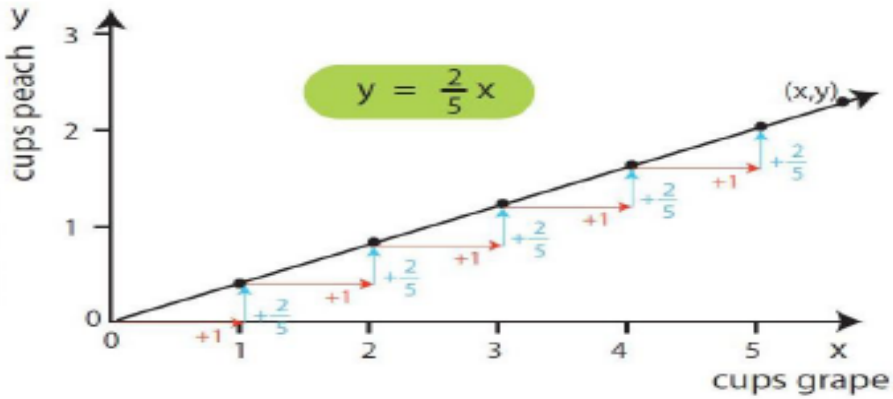
In the tables, equivalent ratios are generated by repeated addition (left) and by scalar multiplication (right). Students might be asked to identify and explain correspondences between each table and the graph beneath it (MP1).

Moving to 7th grade…

Correspondence among a table, graph, and equation of a proportional relationship

For every 5 cups grape juice, mix in 2 cups peach juice.

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$



On the graph: For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

When you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

Starting from $(0, 0)$, to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$ $y = \frac{2}{5}x$

Progressions for the Common Core State Standards in Mathematics, 2011, p. 4

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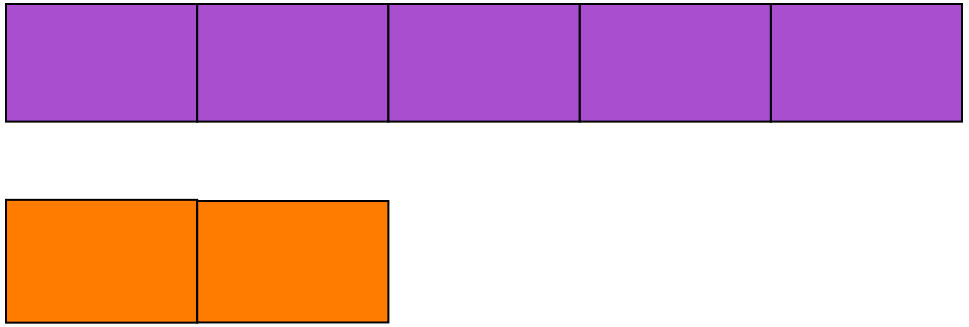
Fred's punch

Fred created a new punch mixture that is made of 7 parts grape and 4 parts peach. Which punch is "peachier?"

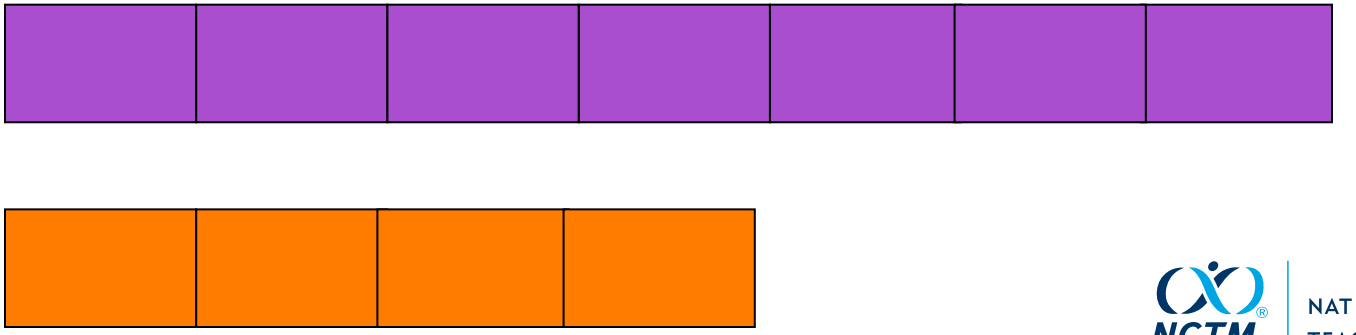
(Recall our original punch mixture has five parts grape to two parts peach.)

Anticipating students' thinking

Original punch:

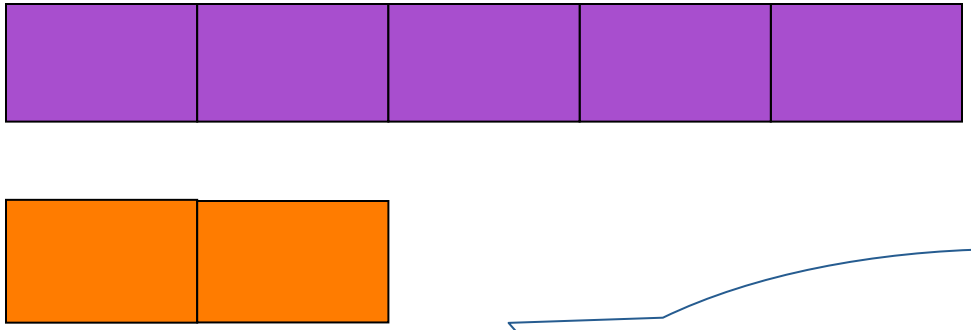


Fred's punch:



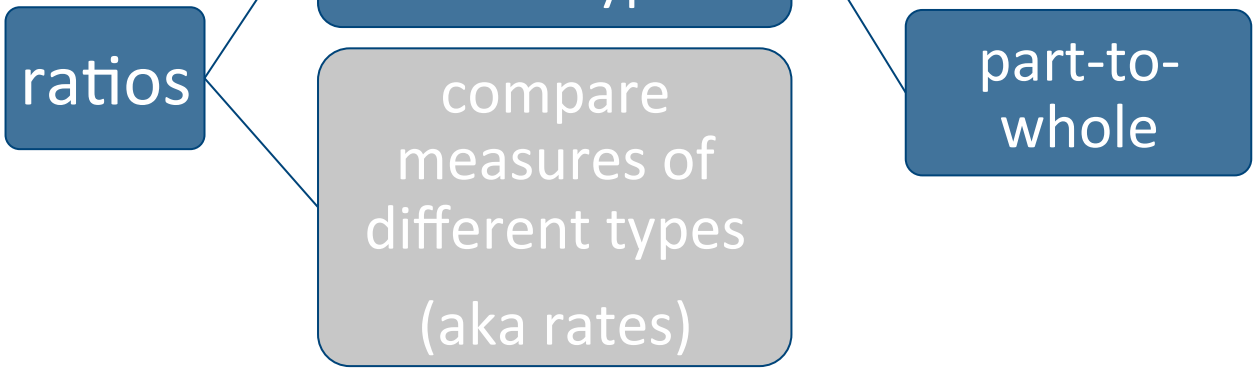
Anticipating students' thinking

Original punch:



"The original punch is two-fifths peach."

All fractions are ratios…
…but not all ratios are fractions!



2 : 5 ratio

but the mixture is not two-fifths peach

2 : 7 ratio

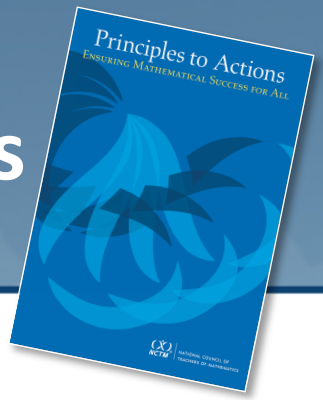
the mixture is two-sevenths peach

All fractions are ratios…
…but not all ratios are fractions!

Ratios are often expressed in fraction notation, but ratios and fractions do not have identical meaning.

$$\frac{1cat}{2dogs} + \frac{3cats}{5dogs} = \frac{4cats}{7dogs} \qquad \frac{1}{2} + \frac{3}{5} = \frac{11}{10}$$

Effective Mathematics Teaching Practices



1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. **Use and connect mathematical representations.**
4. Facilitate meaningful mathematical [illegible]

- What representations have we used this morning?
- What connections did we make between these representations?

8. Elicit and use evidence of student thinking.

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Use and connect mathematical representations
Teacher and student actions

What are teachers doing?

Selecting tasks that allow students to decide which representations to use in making sense of the problems.

Allocating substantial instructional time for students to use, discuss, and make connections among representations.

Introducing forms of representations that can be useful to students.

Asking students to make math drawings or use other visual supports to explain and justify their reasoning.

Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.

Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.

What are students doing?

Using multiple forms of representations to make sense of and understand mathematics.

Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.

Making choices about which forms of representations to use as tools for solving problems.

Sketching diagrams to make sense of problem situations.

Contextualizing mathematical ideas by connecting them to real-world situations.

Considering the advantages or suitability of using various representations when solving problems.

NCTM, 2014, p. 29

Thank you for your perseverance!

Any loose ends, questions, wonderings, etc.?

Please jot them down on an index card and drop it off on your way out.

Next up…Lunch!

Disclaimer

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Pole Problem

There is a swimming pool that has the same depth throughout the entire pool. Three poles are placed vertically in the pool. Their bases are on the bottom of the pool and each pole extends above the surface of the water.

- 3/4 of the first pole is above the surface of the water.
- 2/5 of the second pole is above the surface of the water.
- 4/7 of the third pole is above the surface of the water.

The total length of the three poles combined is 360 cm.

How deep is the water in the pool?

Expressions and Equations

NCTM 2017 Winter Institute
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Session 2: Expressions and Equations

Jennifer Outzs

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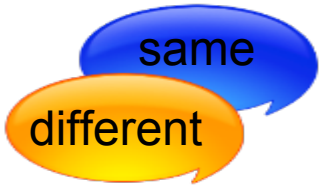
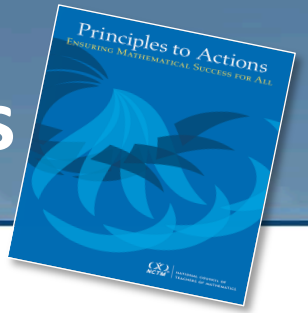
Introductions…

With your table, decide the similarities and differences about the four phrases below:



- **Numerical expression**
- **Numerical equation**
- **Algebraic expression**
- **Algebraic equation**

Effective Mathematics Teaching Practices



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2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
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Algebra Magic

- Think of a number.
- Add 6.
- Multiply by 2.
- Subtract 2.
- Divide by 2.
- Subtract the original number.

Compare your answer to others at your table.

Algebra Magic

Why did this happen? Does it always happen? How could you prove or disprove it?

Explore the problem with some different numbers, being sure the numbers vary in important ways.

Algebra Magic

Fill out an Algebra Magic table with parts of the trick missing.

How is this problem different than the first one we did?

Algebra Magic

Make a separate algebra magic trick with at least five steps that will meet one of the bullets listed below:

- Final result is one more than the original number.
- Final result is 0.
- Uses all four operations.
- Result is same, whether steps are done backwards or forward.

Algebra Magic

The following trick is missing the last step.

- Think of a number.
- Take its opposite.
- Multiply by 2.
- Subtract 2.
- Divide by 2.
- ??????????

Decide what the last step should be for the given condition so final result is:

a) One more than original number.

b) Opposite of original number.

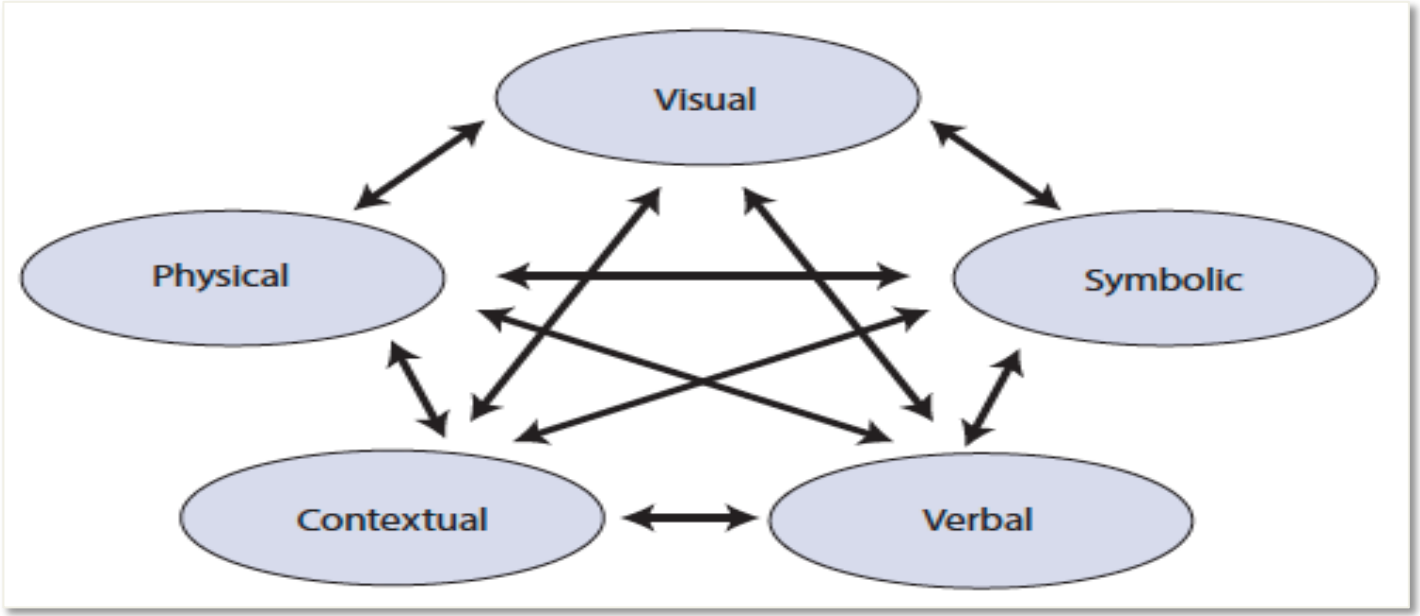
c) Always 0.

d) Always -1.

Algebra Magic

We need to make sure we take kids from the visual (picture) to the verbal (written descriptions of the pictures) to the abstract (abbreviation or algebraic expression).

Connecting to the Effective Teaching Practices: Use and connect representations



(NCTM, 2014, p. 25)

Solving Equations

Cover up method: $5 + \frac{3x - 1}{4} = 7$

Solving Equations

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$5 + \square = 7$	$\square = 2$	$\frac{3x - 1}{4} = 2$

Solving Equations

Cover up method: $5 + \frac{3x - 1}{4} = 7$

$5 + \square = 7$	$\square = 2$	$\frac{3x - 1}{4} = 2$
$\frac{\square}{4} = 2$	$\square = 8$	$3x - 1 = 8$

Solving Equations

Cover up method: $5 + \frac{3x - 1}{4} = 7$

$5 + \square = 7$	$\square = 2$	$\frac{3x - 1}{4} = 2$
$\frac{\square}{4} = 2$	$\square = 8$	$3x - 1 = 8$
$\square - 1 = 8$	$\square = 9$	$3x = 9$

Solving Equations

Cover up method: $5 + \frac{3x - 1}{4} = 7$

5 + ☐ = 7	☐ = 2	$\frac{3x - 1}{4} = 2$
$\frac{\square}{4} = 2$	☐ = 8	3x – 1 = 8
☐ - 1 = 8	☐ = 9	3x = 9
3 * ☐ = 9	☐ = 3	x = 3

Solving Equations

- Practice solving equations using the Cover Up Method with your tablemates.

Solving Equations

- Practice solving equations using the Cover Up Method with your tablemates.
- What will students learn as a result of this activity?

Solving Equations

- Practice solving equations using the Cover Up Method with your tablemates.
- What will students learn as a result of this activity?
- What challenges might student encounter with this activity?

Solving Equations - Goals

- Helps students conceptualize the characteristics of the problem to solve

  Make sense of variable to represent unknown quantity

- Helps students formulate an algebraic equation to solve the problem

  Analyze relationship(s) between components of problem

- Helps empower students

  Develop competence and confidence in using the algebraic method.

Solving Equations Using Pictures

Solve the first problem on the handout in groups.

Consider a variety of ways to solve the problem including at least one visual representation.

"Write up" on poster paper one way your group thought about solving the problem.

“Teacher Hat”:

Orchestrating a Mathematical Discussion

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5 Practices for Orchestrating Productive Mathematics Discussions

Anticipating → Monitoring → Selecting → Sequencing → Connecting

Selecting & Sequencing

Anticipating → Monitoring → **Selecting** → **Sequencing** → Connecting

With your group, examine the posters and consider the following:

1. Which solutions would you want to have students present? Why?
2. What order would you want the solutions

Selecting & Sequencing

Anticipating → Monitoring → **Selecting** → **Sequencing** → Connecting

- What were your "rules of thumb" for ***selecting*** the solutions to be shared?
- What were your "rules of thumb" for ***sequencing*** these solutions?

Connecting

Anticipating → Monitoring → Selecting → Sequencing → **Connecting**

- What questions would you ask in order to help students reach your mathematical goal(s)?

Principles to Actions

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and dominates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students keeping ideas to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to depict their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking in supportive, collegial ways and accept the same support from others.

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.

Facilitate Meaningful Mathematical Discourse

What are teachers doing?

Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.

Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.

Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.

Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.

What are students doing?

Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.

Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.

Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.

Identifying how different approaches to solving a task are the same and how they are different.

NCTM, 2014, p. 29

Thank you for your perseverance!

Any loose ends, questions, wonderings, etc.?

Please jot them down on an index card and drop it off on your way out.

Next up…Keynote with Tim Kanold then team time at 4:30

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Welcome!

Breakout Session 3: Statistics and Probability: Exploring Univariate Data

Jennifer Outzs

After the Institute, all content will be posted online at http://nctm.org/pa17

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Overcoming professional isolation

"In too many schools, professional isolation severely undermines attempts to increase collaboration among colleagues, both between teaching peers internally in the school and among teachers, mathematicians, and mathematics educators externally…Such isolation stands as an obstacle to ensuring mathematical success for all students as well as teachers' continual growth."

(NCTM, 2014, p. 100)

As you engage in the Institute, keep your eyes & ears open for people that you could build professional relationships with!

Session Expectations

- Look at the progression of the data and statistics domain through middle school
- Put on our "student hat" and look at a task as a participant
- Put on our "teacher hat" and look at a task from an implementation standpoint
- Look at the Mathematics Teaching Practices, in particular Implementing Tasks That Promote Reasoning and Problem Solving

Look at Statistics Progression

What do you notice about your Data and Statistics Domain?

- 6th grade focus?
- 7th grade focus?
- 8th grade focus?

Look at Progression – 6th grade

6th grade focus

- Univariate data – number line plots, dot plots, histograms, and box plots
- Measures of central tendency – mean, median, (mode)
- Measures of variability – range, interquartile range, mean absolute deviation
- Shape of the data distribution – mound, skewed, uniform

Look at Progression – 7th grade

7th grade focus

- Random sampling – sample vs. population
- Comparing univariate data using measures of center and measures of variability
- Probability
  - O to 1
  - Experimental Probability and Relative Frequency
  - Theoretical Probability for single and compound events
  - Using organized lists, tables, tree diagrams, and simulations

Look at Progression – 8th grade

8th grade focus

- Bivariate Data – scatter plots, clustering, outliers, positive and negative association, linear and nonlinear association
- Informal line of best fit
- Using linear equations to solve problems in context

“Soccer Tournament” task

- The total number of goals scored by all 9 teams in the tournament is 54.
- Use post-its and the number line on the chart paper to make a graph showing number of goals each team might have scored.
- Assume each team scored at least one goal and no more than 10 goals.

Adapted from Kader & Memer, 2008

Let's take a look!

What are some questions we might want to answer about this collection of data?

How could we answer those questions?

Perfect evenly matched teams

- With your group, rank the tournaments in terms of least to most evenly matched.

Perfect evenly matched teams

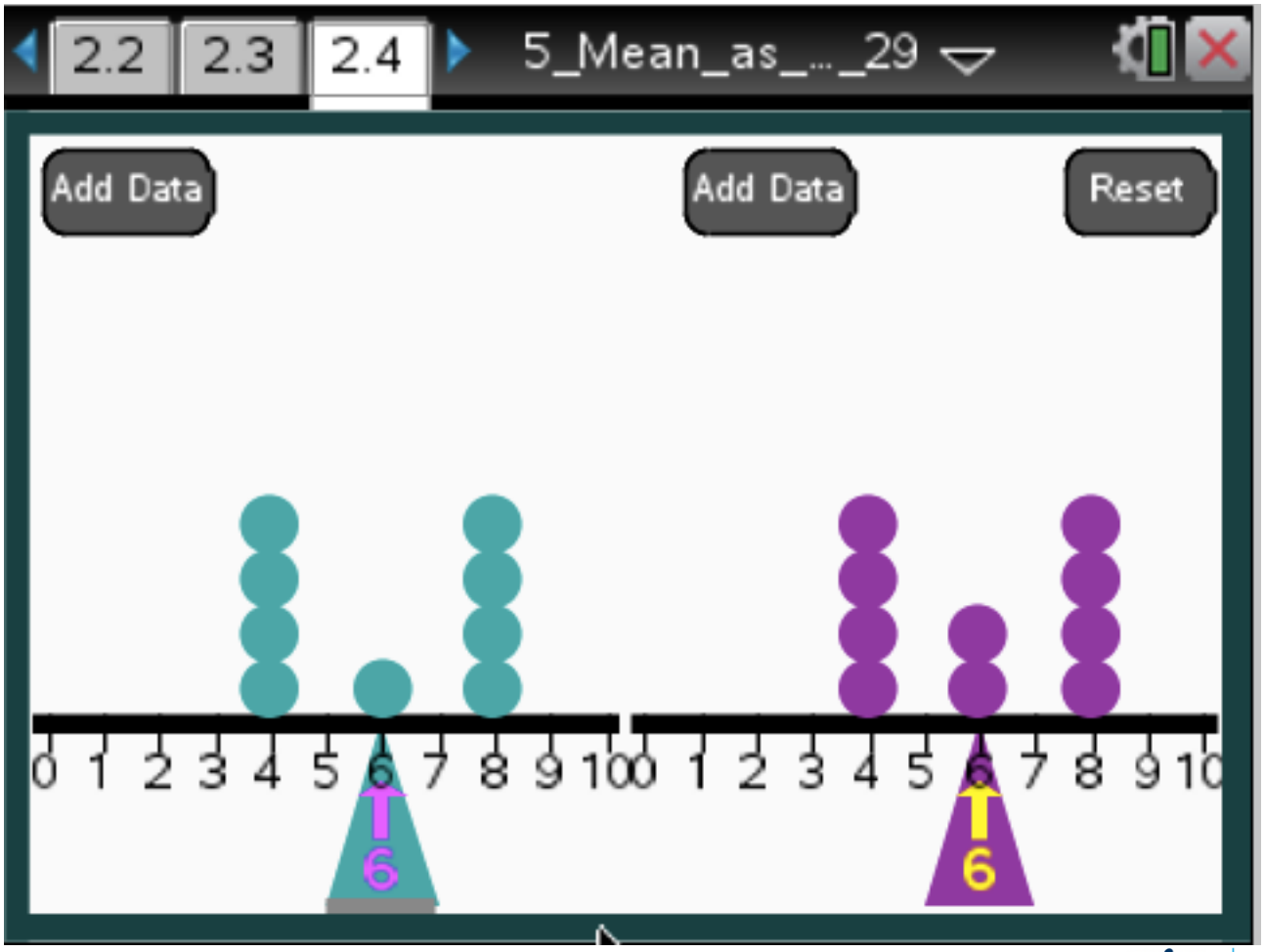
- With your group, rank the tournaments in terms of least to most evenly matched.

- What criteria are you using for your ranking?

Perfect evenly matched teams

- With your group, rank the tournaments in terms of least to most evenly matched.
- What criteria are you using for your ranking?
- Could you find a mathematical way to do the ranking?

Which of the two tournaments has the more evenly matched teams? Why?



Why measures of center & spread?

- The average temperatures in San Francisco and St. Louis are both 57º.
    - Hmmmmmm…

Standard we looked at today

- CCSSM Statistics and Probability, Grade 6: **Summarize and describe distributions.** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Teacher hat

As a teacher…

What did you like about this task?

What would you have changed?

Possible goals for the soccer task:

- Understand mean as balance point for a distribution of data
- Understand deviations and, in particular, the mean absolute deviation as a way to describe the typical distance of the data from the mean
- Justify ideas using mathematical language
- Use statistical thinking to create a measure to describe variability in a distribution
- Develop a mathematical model to describe a context
- Recognize that the deviations above the mean are the same in magnitude as the deviations below the mean

Formative Assessment

At your tables, choose one of the goals on the slide. What "success criteria" could you use to figure out whether you met your goal?

Principles to Actions – Levels of Demands

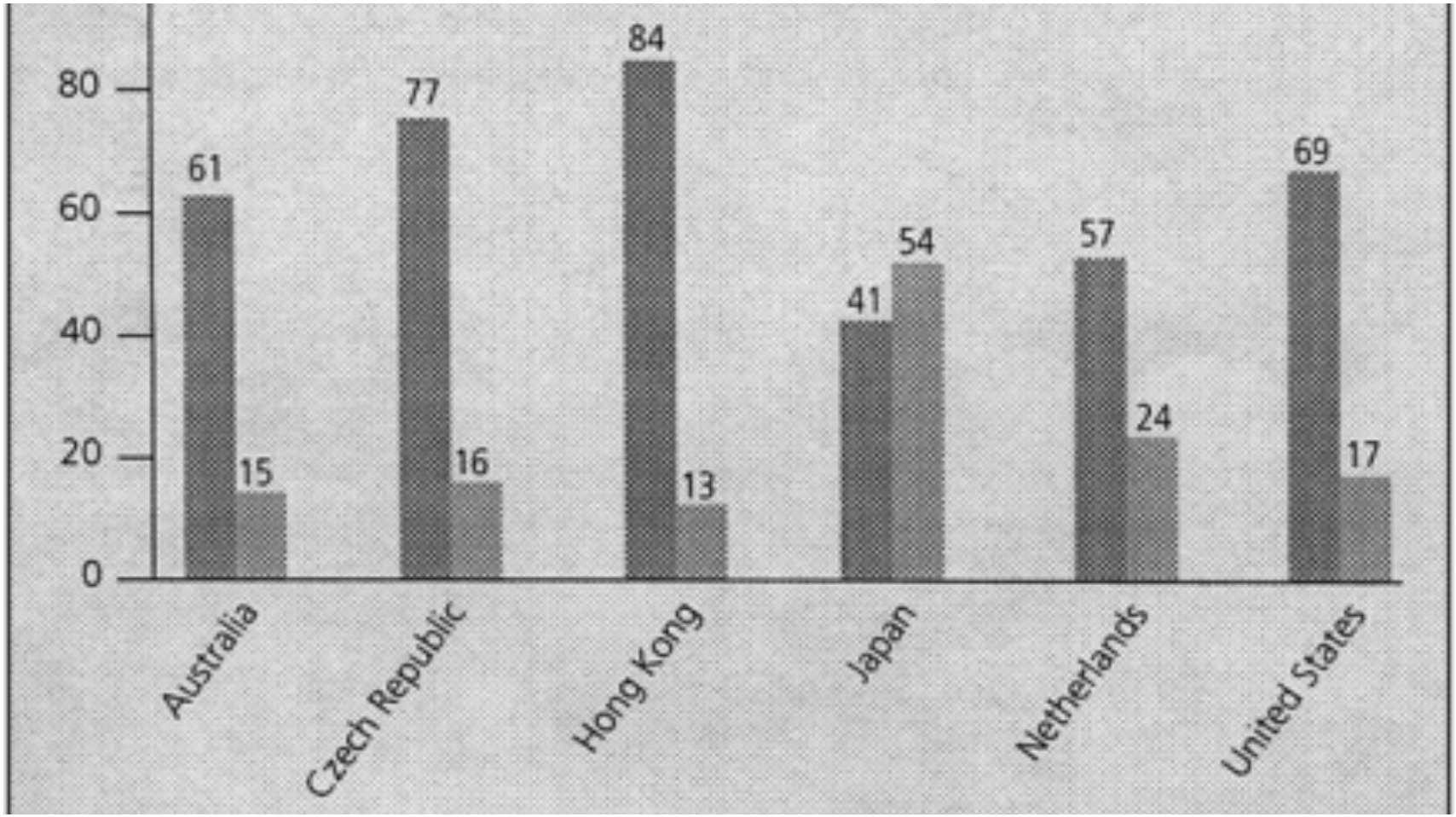
Characteristics of mathematical tasks at four levels of cognitive demand – Figure 3, p.18

- Lower-level demands (memorization)
- Lower-level demands (procedures without connections)
- Higher-level demand (procedures with connections)
- Higher-level demands (doing mathematics)

Implement Tasks

Not only do we need to find tasks that fit into these high-level student thinking categories but we also need to implement them with fidelity.

Improving Mathematics Teaching – Stigler and Hiebert, 2004



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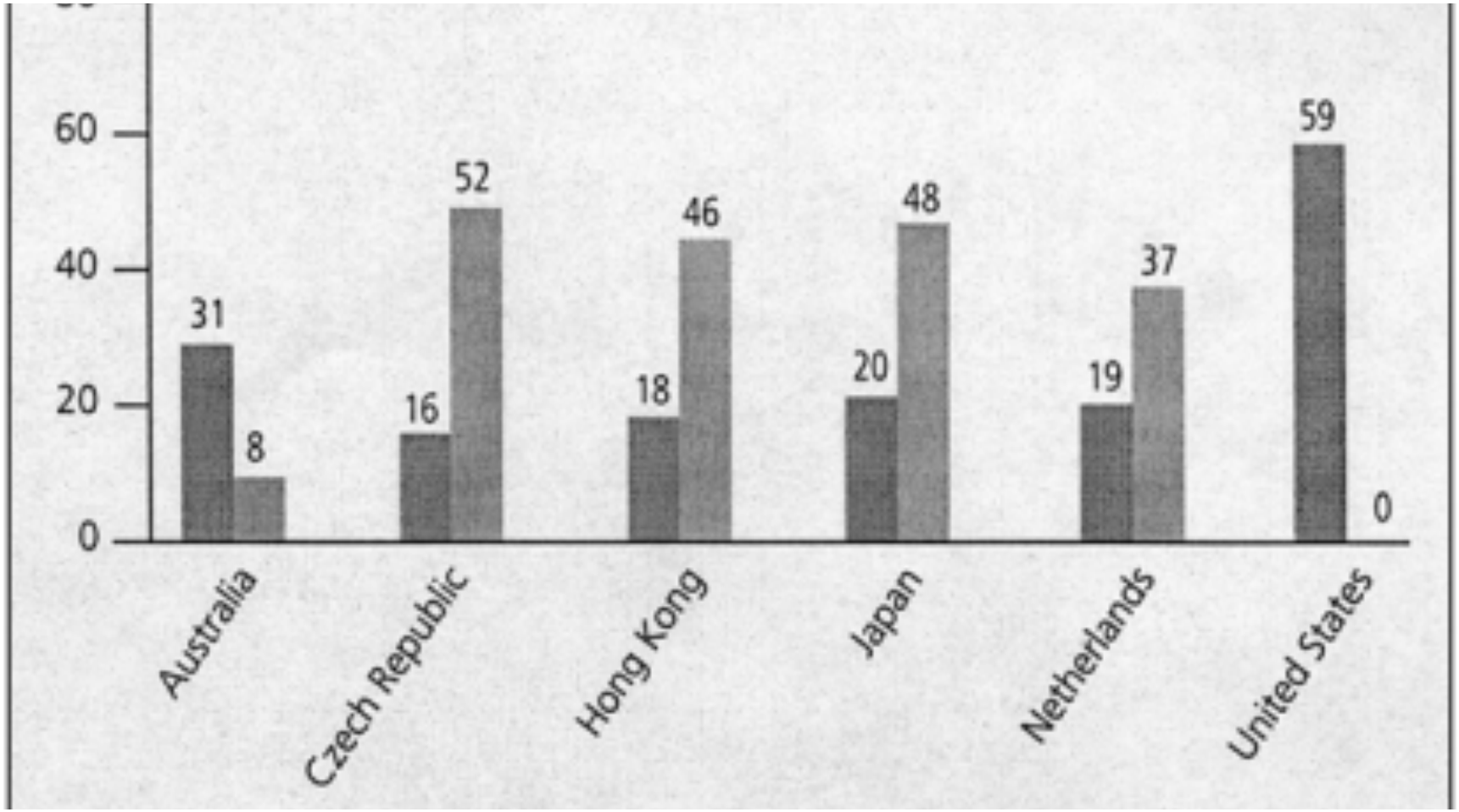

Implement tasks that promote reasoning and problem solving

What are teachers doing?

Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.

Selecting tasks that provide multiple entry points through the use of varied tools and representations.

Posing tasks on a regular basis that require a high level of cognitive demand.

Supporting students in exploring tasks without taking over student thinking.

Encouraging students to use varied approaches and strategies to make sense of and solve tasks.

NCTM, 2014, p. 24

What are students doing?

Persevering in exploring and reasoning through tasks.

Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.

Using tools and representations as needed to support their thinking and problem solving.

Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Thank you!

- Please drop off any questions, noticings, and/or wonderings to The Parking Lot
- Please share your favorite resources (websites, books, articles, etc.) on the posters on the wall
- Have a great evening!
- Tomorrow…
  - Keynote 8:30am
  - Breakout 3 (expressions & equations) will be in here

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Statistics and Probability

Effective Teaching with *Principles to Actions*: Implementing College- and Career-Readiness Standards

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Session Expectations

- Look at the progression of the statistics and probability domain through middle school
- Investigate problems that facilitate understanding of both theoretical and experimental probabilities
- Investigate chance processes and the role of simulation
- Look at the Mathematics Teaching Practices, in particular Implementing Tasks That Promote Reasoning and Problem Solving

Look at Statistics Progression

What do you notice about your Statistics and Probability Domain?

- 6th grade focus?
- 7th grade focus?
- 8th grade focus?

Probability Cluster: Depending on the curricula you use, the probability standards may be new (Briars, Asturias, Foster, & Gale, 2013).

Look at Progression – 6th grade

6th grade focus

- Univariate data – number line plots, dot plots, histograms, and box plots
- Measures of central tendency – mean, median, (mode)
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Look at Progression – 7th grade

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- Random sampling – sample vs. population
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Look at Progression – 8th grade

8th grade focus

- Bivariate Data – scatter plots, clustering, outliers, positive and negative association, linear and nonlinear association
- Informal line of best fit
- Using linear equations to solve problems in context

Why probability in 7th grade?

"Because statistically sound data production is based on random sampling, a probabilistic concept, students must develop some knowledge of probability before launching into sampling. Their introduction to probability is based on seeing probabilities of chance events as long-run relative frequencies of their occurrence, and many opportunities to develop the connection between theoretical probability models and experimental probability approximations."

Roller Derby

Place your 12 chips on the numbers 1 to 12 in any arrangement you choose. Two dice will be rolled and one chip may be removed from the sum of the dice. The object of the game is to be the first person to remove all your chips.

Roller Derby

Play again!!

Place your 12 chips on the numbers 1 to 12 in any arrangement you choose. Two dice will be rolled and one chip may be removed from the sum of the dice. The object of the game is to be the first person to remove all your chips.

Experimental Probability

Work with your table group to calculate the experimental probabilities of landing on each number in roller derby based on the data we've collected.

Experimental Probability

Work with your table group to calculate the experimental probabilities of landing on each number in roller derby based on the data we've collected.

Have we collected enough data to feel comfortable with our results? If not, how much more data should we collect?

Theoretical Probability

Work with your table group to calculate the theoretical probabilities of landing on each number in roller derby.

Try to come up with multiple ideas on how you could organize your work to solve this problem.

Reflection on Activity: Unpacking Key Points for Teaching

Prediction first: Why?

Data collection next: Why?

Find Experimental Probability for the event: Why?

Find Theoretical Probability for the event: Why?

Reflection on Activity: Differentiation

How would this activity work with your struggling learners?

Reflection on Activity: Differentiation

How would this activity work with your struggling learners?

How could you extend this activity with your accelerated learners?

Reflection on Activity: Differentiation

How could you extend this activity with your accelerated learners?

- roll the two dice and find out the difference
- roll the two dice and find the product
- roll the two dice and find the highest number
- roll the two dice and find if the product is even or odd
- what if we rolled three dice

The Hare and the Tortoise

The tortoise and the hare start each turn at S.

Each turn consists of three moves. An odd roll moves one place to the left. An even roll moves one place to the right.

Scoring: The tortoise gets the point if landing is on P or X. The hare gets a point if the landing is on M, N, S, Y, or Z.

The Hare and the Tortoise

Predict:

Who you think is going to win? Why?

The Hare and the Tortoise

Predict:

Who you think is going to win? Why?

Collect Data – Play the game ☺

The Hare and the Tortoise

Predict:

Who you think is going to win? Why?

Collect Data – Play the game ☺

Find the experimental probabilities.

- how much data is needed?

Theoretical Probability

What are the theoretical probabilities of landing on each spot?

What kind of models could the kids use to find the theoretical probabilities?

Decisions from Data – The Role of Simulation

True False

- What is the probability of passing a ten-question true/false test by guessing?

Decisions from Data – The Role of Simulation

True False

- What is the probability of passing a ten-question true/false test by guessing?

- If the number of questions is increased to 20, will your chances of passing by guessing increase, decrease, or stay the same?

Investigate chance processes and develop, use, and evaluate probability models

- Approximate the probability of a chance event by collecting data and observing it's long-run relative frequency

- Develop a probability model and use it to find probability of events

(CCSSM, Grade 7)

Principles to Actions – Levels of Demands

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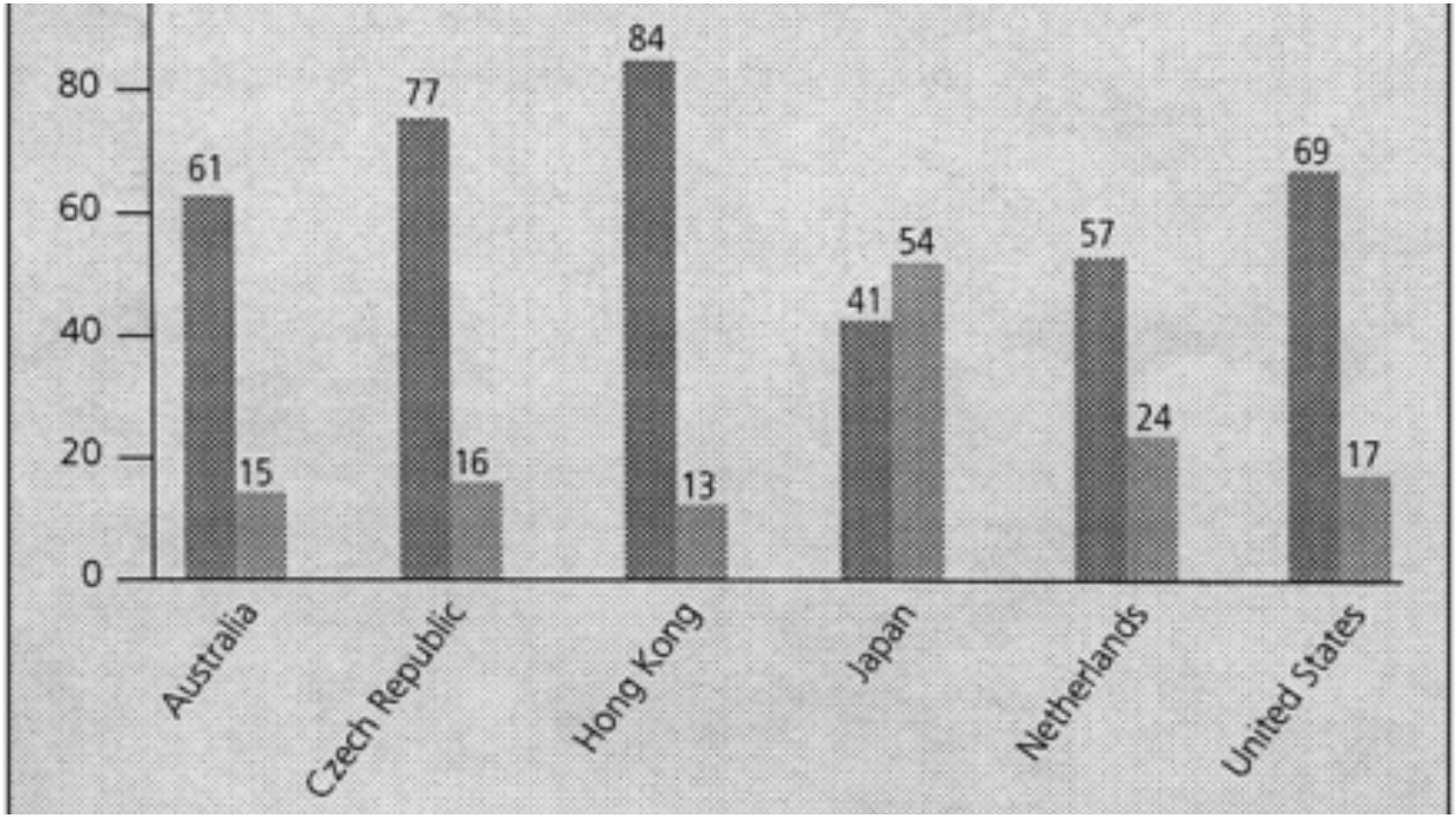

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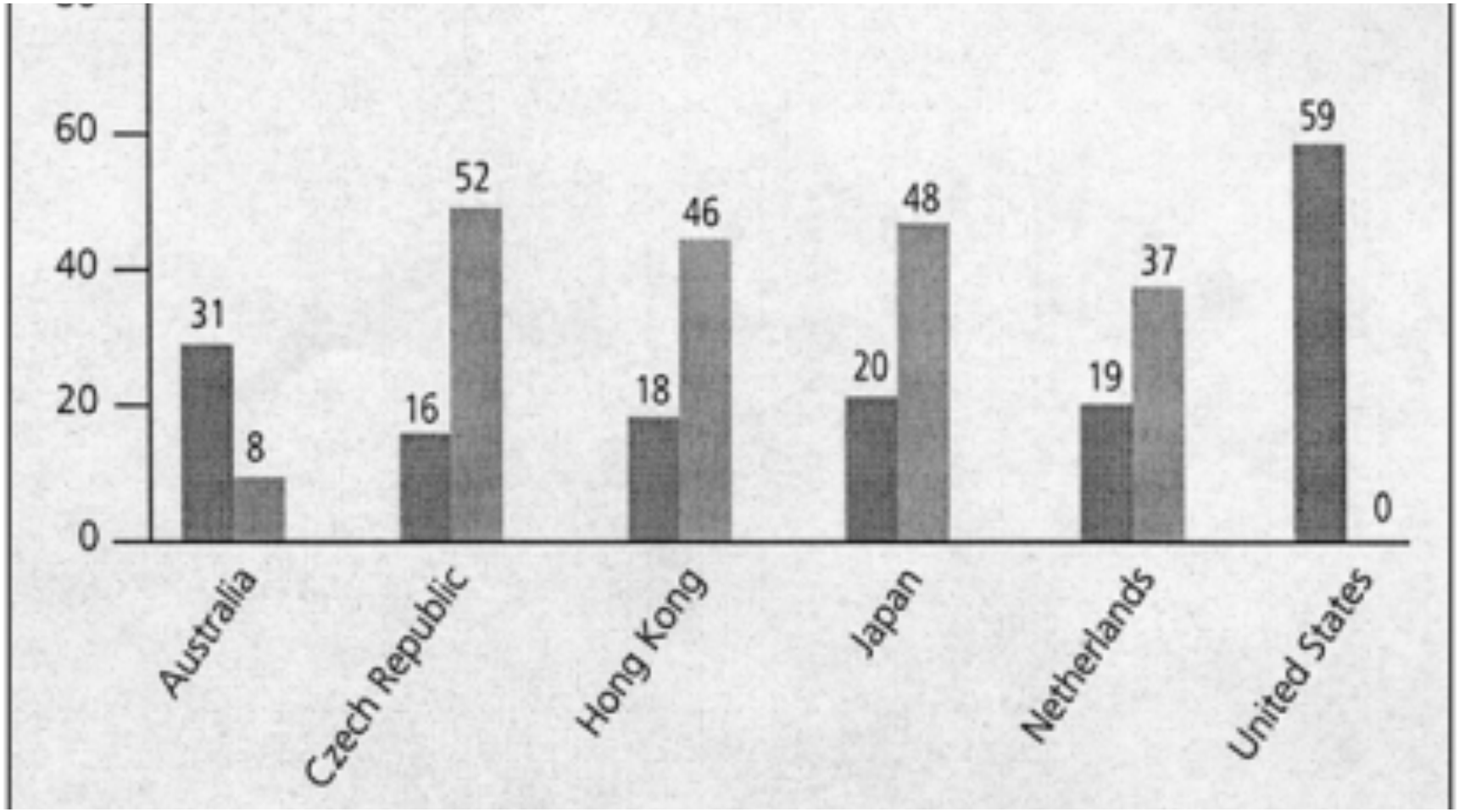

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Using tools and representations as needed to support their thinking and problem solving.

Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Thank you!

- One more session this afternoon

- 2:15 in the bigger room we were in yesterday (Edge) with the entire middle school group

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