

Elicit and Use Evidence of Student Thinking

NCTM Webinar Panel: What Should Math Learning
Look Like When We Get Back to School?

July 27, 2020
7:00 PM

Shawn Towle, Falmouth Middle School
Falmouth, Maine

@towlemath

stowle@falmouthschools.org



Elicit and Use Evidence of Student Thinking



“Effective teaching of mathematics uses **evidence of student thinking** to **assess progress** toward mathematical understanding and to adjust instruction continually in ways that **support and extend learning**.”

PtoA p 53

“Each lesson needs intentional and systematic plans to elicit evidence that will provide “a constant stream of information about how student learning is evolving toward the desired goal” (Heritage 2008, p.6)

Two Recommendations...



- 1) Use **high level tasks** to reveal students' thinking and reasoning

Explain, Represent, Justify
Mathematical Thinking and Reasoning

- 2) Carefully construct key questions, prior to teaching, to draw out **specific understandings**, **conceptual gaps** or **common errors**, with the goal of making them visible and accessible for examination and discussion.

Desmos



Google Drawing



Jamboard



Google Slides



A few tools for collaborative workspace...Desmos

teacher.desmos.com

desmos

Search for an activity

Math Tools Resources Create Account or Sign In

Desmos Classroom Activities

At Desmos, our mission is to help every student learn math and love learning math. Explore and enjoy our collection of free digital math activities for you and your students.

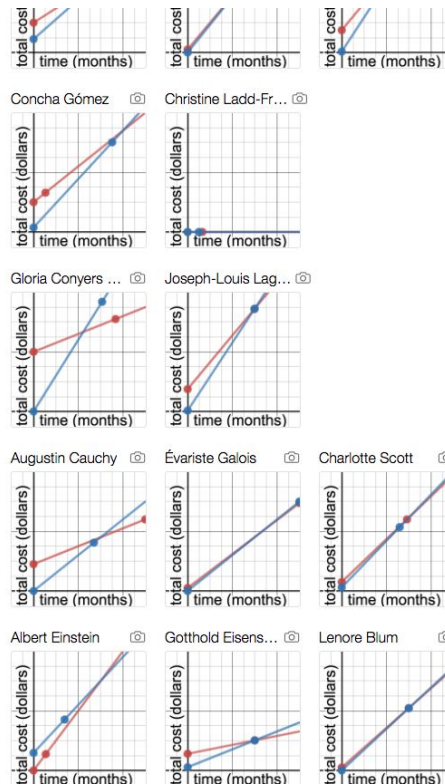
Watch the video

What We Offer

Challenge Slide #1

Meaningful Feedback

We show students what their answers mean, then give them the opportunity to improve their thinking and revise their work.



the cost to join plan A is 20 and the cost per month is 10 dollars. The cost to join plan B is 164 dollars and the cost per month is 2 dollars. at 18 months they will both equal 200 dollars.

Grace Chisholm Young

PLAN A: 300 dollars to join, no monthly cost

PLAN B: 100 dollars to join, \$33.33 dollars monthly

Mary Jackson

Cost to join A \$100 and \$8 a month

Cost to join B \$45 and \$11 a month

Johannes Kepler

10 to join plan A and 0 to join plan B

Plan B costs 15.5 each month and Plan A costs 15 each month

Michael Atiyah

Plan A: \$75 to join and \$15/per month

Plan B: \$3 to join and \$19/per month

Concha Gómez

Plan A has a starting cost of 100 dollars and a monthly cost of 10 dollars, but gives full access to all of the equipment.

Plan B has a starting cost of 15 dollars and a monthly cost of 13.55 dollar, but has a limited access of equipment.

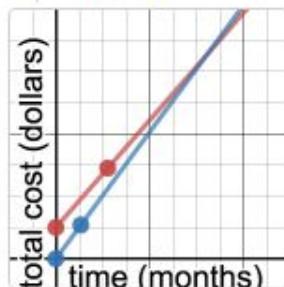
What question might you ask?

≡ 1

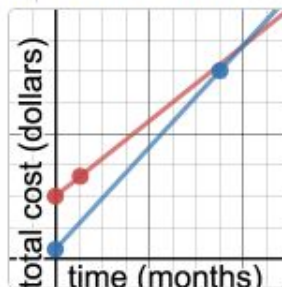
Ask a question (optional)

Present

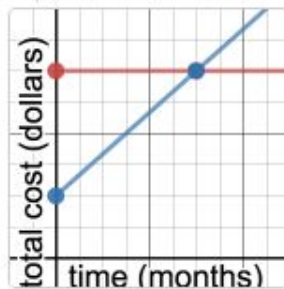
7 John Wallis X



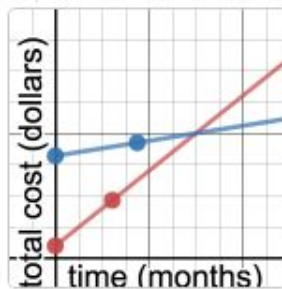
7 Concha Gómez X



7 Grace Chisho... X



7 Emmy Noether X



What question might you ask?

Cost to join A \$100 and \$8 a month
Cost to join B \$45 and \$11 a month

For Plan A, it would be \$56 to join and \$2.50 each month. For Plan B, it would be \$11 to join and \$5 each month.

$$y=2.5x+56$$

$$y=5x+11$$

Plan A costs 18 dollars upfront, plus an additional \$1.00 a month for the membership, and \$0.78 per month for the cleaning fee.

Plan B costs nothing to join, but it costs \$2.00 per month plus an additional \$0.78 per month for cleaning fees.

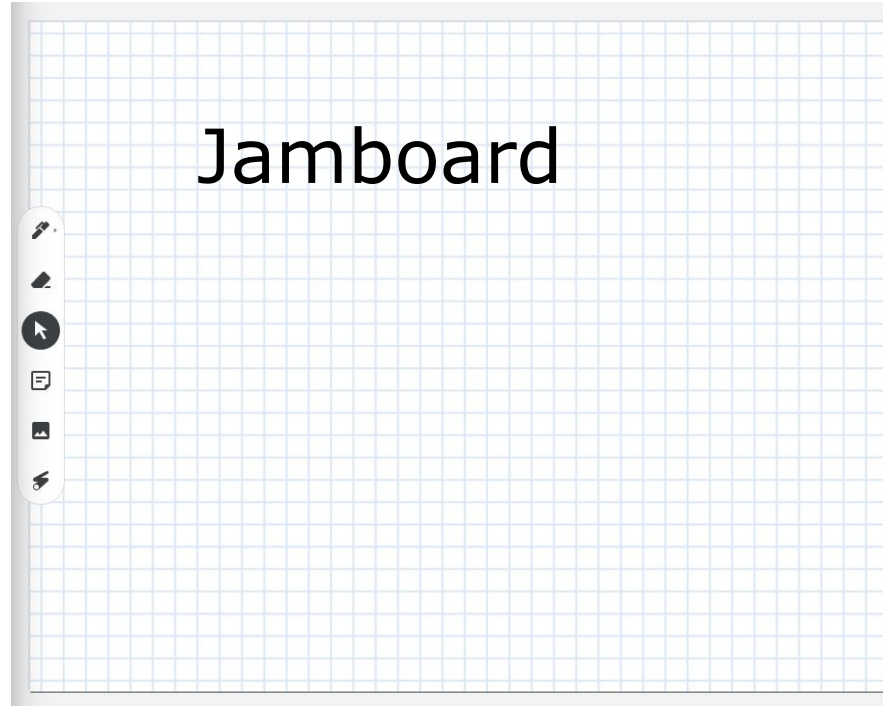
Plan A has a starting cost of 100 dollars and a monthly cost of 10 dollars, but gives full access to all of the equipment.

Plan B has a starting cost of 15 dollars and a monthly cost of 13.55 dollar, but has a limited access of equipment.

Tools for collaborative workspace...



Google
Slides



Two Recommendations...



- 1) Use **high level tasks** to reveal students' thinking and reasoning

Explain, Represent, Justify
Mathematical Thinking and Reasoning

- 2) Carefully construct key questions, prior to teaching, to draw out **specific understandings**, **conceptual gaps** or **common errors**, with the goal of making them visible and accessible for examination and discussion.

Mathematics Assessment Project

CLASSROOM CHALLENGES

Formative Assessment Lessons for Grade 6



[Home](#) [About](#) [News](#) [Lessons](#) [Tasks](#) [Tests](#) [PD Modules](#) [TRU Framework](#) [Standards](#)

Index of Classroom Challenges

Classroom Challenges are lessons that support teachers in formative assessment. There are 100 lessons in total, 20 at each grade from 6 to 8 and 40 for 'Career and College Readiness' at High School Grades 9 and above. Some lessons are focused on developing math concepts, others on solving non-routine problems.

The [Brief Guide for teachers and administrators \(PDF\)](#) is recommended reading before using these lessons for the first time.

Each lesson is downloadable as an all-in-one PDF to print out, and a supporting PowerPoint presentation.

Finding a Lesson

There are several ways of navigating:

1. Use the menu on the left to browse by grade and topic area, or search for a key phrase.
2. Go to the [Standards](#) tab to find lessons linked to a particular content standard or practice.
3. The complete set of lessons is listed below in alphabetic order.

Grade 6

[Adding and Subtracting Directed Numbers](#)
[Creating a Measure of Slope](#)
[Designing 3D Products: Candy Cartons](#)
[Evaluating Statements About Number Operations](#)
[Evaluating Statements: Consecutive Sums](#)
[Finding Factors and Multiples](#)
[Interpreting Equations](#)
[Interpreting Multiplication and Division](#)
[Maximizing Profit: Selling Soup](#)
[Modeling Relationships: Car Skid Marks](#)
[Optimizing Coverage: Security Cameras](#)
[Representing Data With Grouped Frequency Graphs and Box Plots](#)
[Representing the Laws of Arithmetic](#)
[Representing Variability with Mean, Median, Mode, and Range](#)
[Sharing Costs Equitably: Traveling to School](#)
[Translating between Fractions, Decimals and Percents](#)
[Using Coordinates to Interpret and Represent Data](#)
[Using Proportional Reasoning](#)
[Using Space Efficiently: Packing a Truck](#)
[Using Standard Algorithms for Number Operations](#)

Grade 8

[Applying Properties of Exponents](#)
[Building and Solving Linear Equations](#)
[Classifying Solutions to Systems of Equations](#)
[Comparing Fuel Consumption: Buying Cars](#)
[Comparing Lines and Linear Equations](#)
[Comparing Value for Money: Baseball Jerseys](#)
[Defining Lines by Points, Slopes and Equations](#)
[Discovering the Pythagorean Theorem](#)
[Estimating Length Using Scientific Notation](#)
[Finding the Shortest Route: A Schoolyard Problem](#)
[Generalizing Patterns: The Difference of Two Squares](#)
[Identifying Similar Triangles](#)

Grade 7

[Analyzing Games of Chance](#)
[Applying Angle Theorems](#)
[Classifying Proportion and Non-Proportion Situations](#)
[Comparing Data Using Statistical Measures](#)
[Comparing Strategies for Proportion Problems](#)
[Describing and Defining Quadrilaterals](#)
[Describing and Defining Triangles](#)
[Designing a 3D Product in 2D: A Sports Bag](#)
[Designing: A Game of Chance](#)
[Drawing to Scale: A Garden](#)
[Estimating Volume: The Money Munchers](#)
[Evaluating Statements About Probability](#)
[Finding Areas of Circles](#)
[Increasing and Decreasing Quantities by a Percent](#)
[Maximizing Area: Gold Rush](#)
[Modeling: Hot and Cold](#)
[Representing: Road Race](#)
[Sampling and Estimating: Counting Trees](#)
[Solving Linear Equations](#)
[Using Positive and Negative Numbers in Context](#)

High School

[Building and Solving Complex Equations](#)
[Calculating Arcs and Areas of Sectors of Circles](#)
[Calculating Volumes of Compound Objects](#)
[Classifying Equations of Parallel and Perpendicular Lines](#)
[Classifying Rational and Irrational Numbers](#)
[Deducting Relationships: Floodlight Shadows](#)
[Devising a Measure: Correlation](#)
[Evaluating Conditions for Congruency](#)
[Evaluating Statements About Enlargements](#)
[Evaluating Statements About Length and Area](#)
[Evaluating Statements about Radicals](#)
[Evaluating Statements about Rational and Irrational Numbers](#)

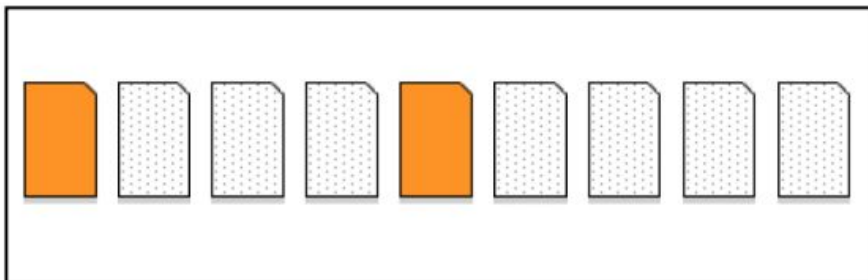
<https://www.map.mathshell.org/lessons.php>

A possible learning goal...



Use proportional reasoning to solve a real world problem involving a ratio relationship between two quantities, expressed in different ways.

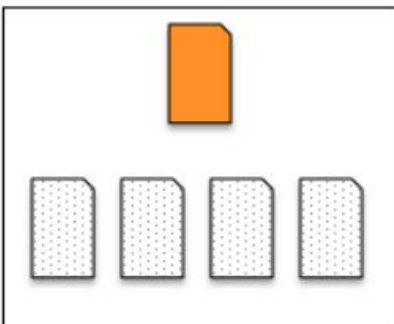
Card 1:



Card 2:

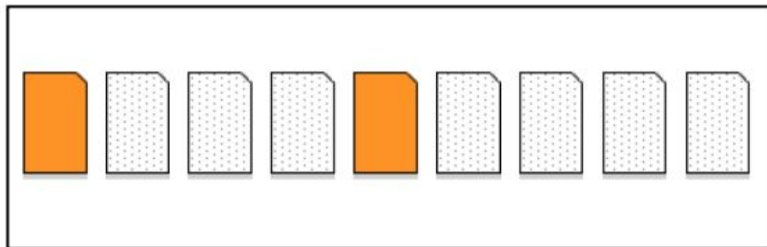
$\frac{1}{4}$ of the mixture is orange

Card 3:



Which is strongest?

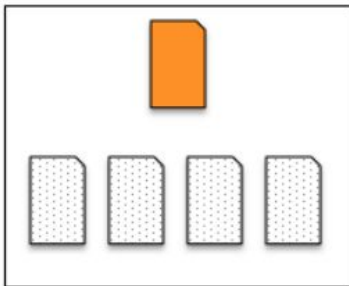
Card 1:



Card 2:

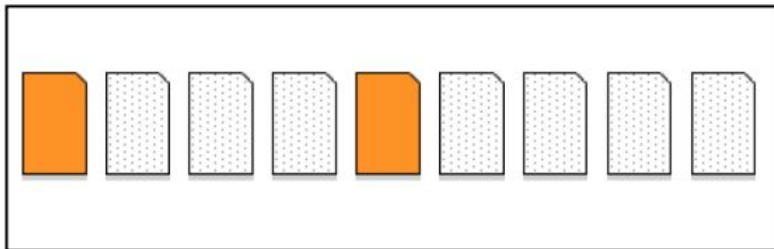
$\frac{1}{4}$ of the mixture is orange

Card 3:



Which is strongest?

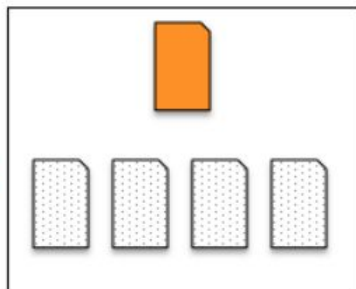
Card 1:



Card 2:

$\frac{1}{4}$ of the mixture is orange

Card 3:



Arrange the cards in order from **least** orangey to **most** orangey.

Type your order **and how you decided** in the chat.

Working together

1. Work together to put the cards in order of strength, taking turns with the work.
 - a. Explain decisions to your partner.
2. If you think more than one card describes the same fizzy orange mixture, group them together.
 - a. If a group of cards does not contain a juice box card, then shade in one of the Cards M - P.
3. When you both agree where each card should go and why, glue them onto your poster. On your poster, explain your decisions.

Virtual Ideas...

Synchronous online:

Students are paired in breakout rooms and share one document they edit together.

Asynchronous online:

Have an assigned “working partnership” and they schedule time to work together.

Google Drawing for Fizzy Orange Card Sort

<https://bit.ly/NCTMfizzy>

A [Diagram: 4 juice boxes, 2 orange, 2 soda]

B [Diagram: 4 juice boxes, 3 orange, 1 soda]

D [Diagram: 4 juice boxes, 2 orange, 2 soda]

E [Diagram: 4 juice boxes, 3 orange, 1 soda]

F Half of the mixture is orange

G For every orange there are 2 sodas

H Orange : Soda = 4 : 5

I One fourth of the mixture is orange

J $\frac{2}{3}$ of the mixture is soda

K For every orange there is $1\frac{1}{3}$ soda

L [Diagram: 4 juice boxes, 3 orange, 1 soda]

M Shade in: [Diagram: 4 juice boxes, 2 orange, 2 soda]

N Shade in: [Diagram: 4 juice boxes, 3 orange, 1 soda]

O Shade in: [Diagram: 4 juice boxes, 2 orange, 2 soda]

P Shade in: [Diagram: 4 juice boxes, 3 orange, 1 soda]

Your task is to work with your partner to put the cards in order of strength, from least orangey to most orangey.

1. Work together to put the cards in order of strength, taking turns with the work. Explain decisions to your partner.
2. If you think more than one card describes the same fizzy orange mixture, group them together.
 - a. If a group of cards does not contain a juice box card, then shade in one of the cards M-P
3. When you both agree where a card should go and why, explain your decision on the google drawing.

Sharing work

1. One person from each group get up and visit a different group.
2. If you are staying with your poster, explain your card order to the visitor, justifying the placement of each card.
3. If you are the visitor, look carefully at the work and challenge any cards that you think are in the wrong place.
4. If you agree on the placement of the cards, compare your methods used when ordering.

Virtual Ideas...

Synchronous online:

Have one student leave each breakout room and teacher re-assigns the one student to a new room.

Asynchronous online:

Have an assigned sharing partner or another pair and invite them to google meet or zoom to work live.

Support and Extend Learning

Support

- Partner Dialogue and opportunity to revise
- Encourage turn taking
- Listening for Mathematical

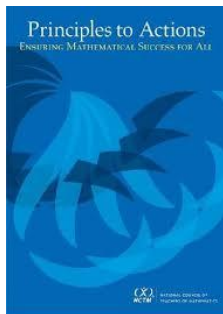
Explanations

- What mixture is most orangey and why?
- How do you know this is more orangey?
- Why is this card placed here?
- After class discussion are you still happy with where your cards are?
Make changes?

Extend Learning

- Invent a card that belongs between two mixtures.
- Invent a card that would go in the same place as a particular mixture.
- What might you add to this mixture to make it taste more like this mixture?

Eliciting and Using Evidence of Student Thinking



“Effective teaching of mathematics uses **evidence of student thinking** to **assess progress** toward mathematical understanding and to adjust instruction continually in ways that **support and extend learning**.”
PtoA p 53

Session Links:

<http://bit.ly/TOWLENCTM> Session Slides

<https://bit.ly/NCTMfizzy> Fizzy Drink Task (will prompt “make a copy”)