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Recommendations for Statistics and Probability in the Secondary Curriculum: Implications for Teachers

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We must help students make sense of data that surround them

–

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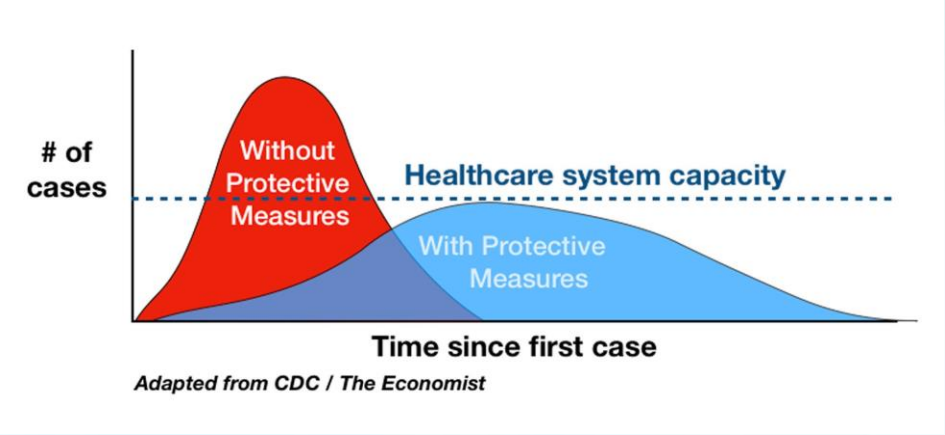
Global Pandemic – What Questions Might Students Ask?



Adapted from CDC / The Economist

Knowing how to ask the right questions – Screening Tests

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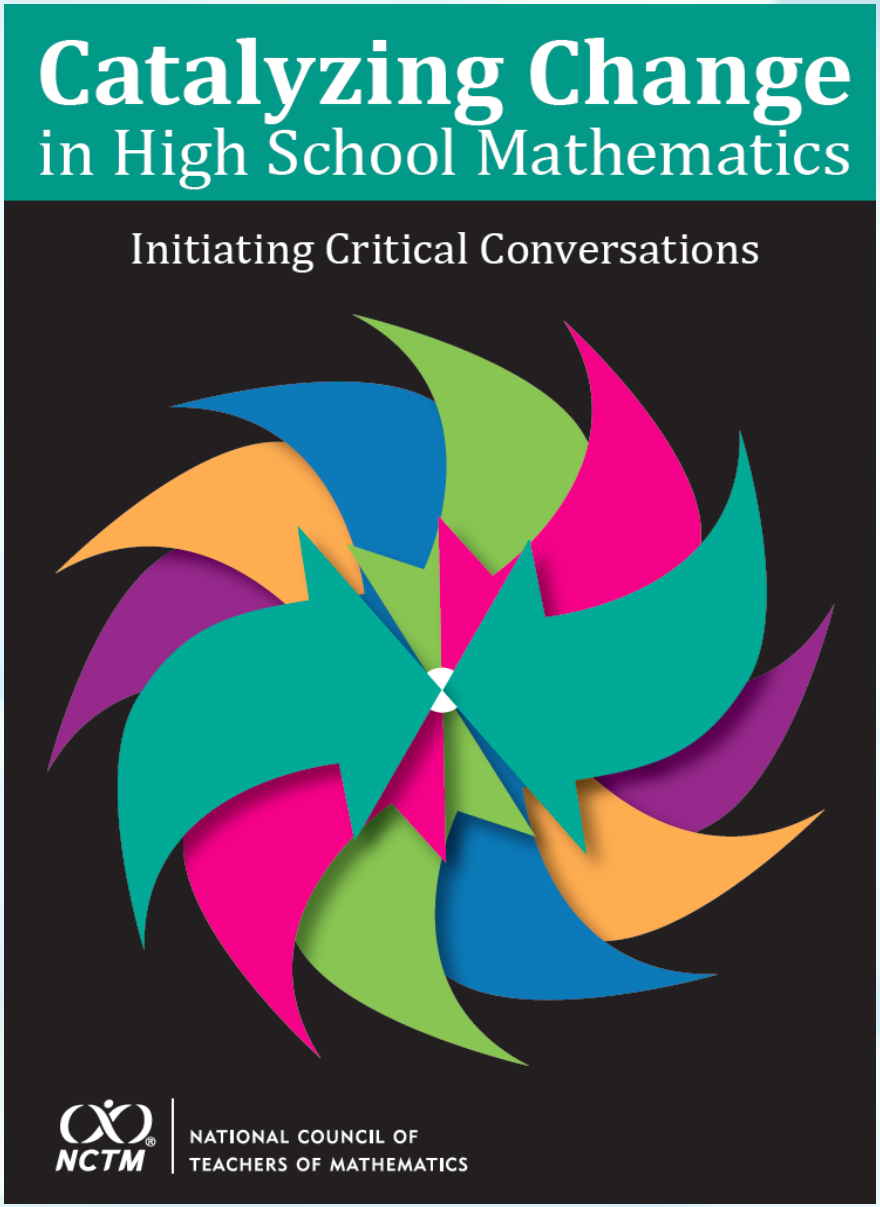
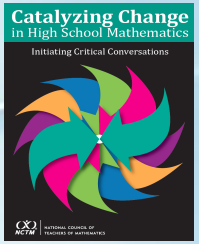
What is *Catalyzing Change in High School Mathematics*?

Why now?



The steady improvement in mathematics learning at the elementary and middle levels has not been shared to the same degree at the high school level.

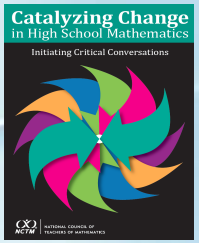
Large numbers of high school students do not have access to the mathematics they need either for their personal or for their professional lives.



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

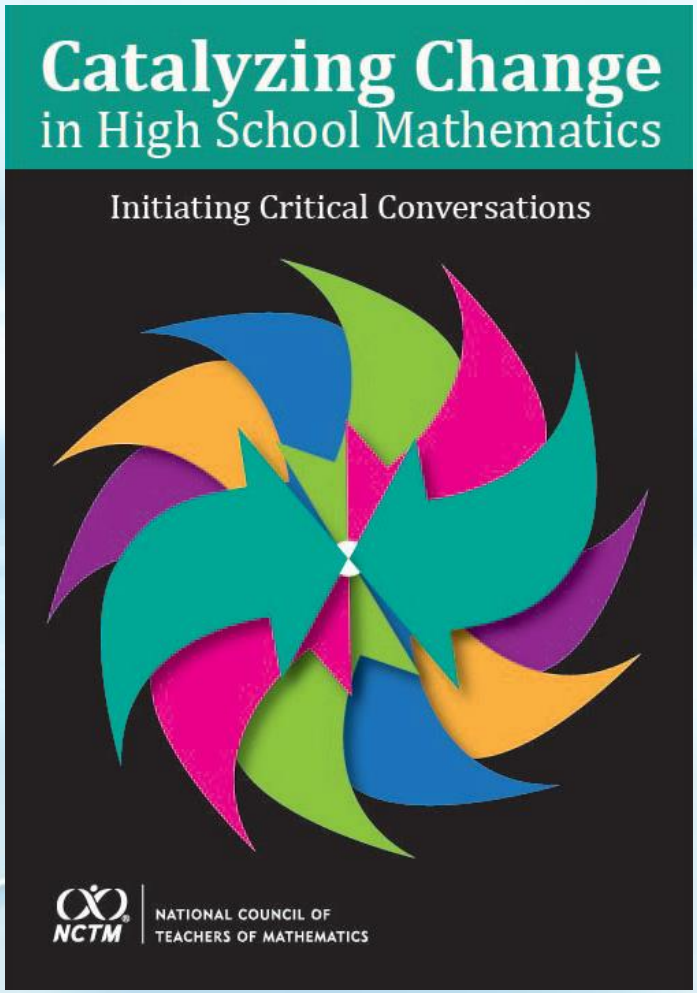
Four Key Recommendations

- The purpose of learning mathematics and Essential Concepts
- Equitable Structures
- Equitable Instruction
- A Common Essential Concepts Pathway



NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

The Purposes of School Mathematics and Statistics



- Expand professional opportunities
- Understand and critique the world, and
- Experience the joy, wonder, and beauty of mathematics and statistics.

NCTM, 2018

Outline

Catalyzing Change

- Reasons for teaching mathematics and statistics
  - To make sense of the world
    - our world as teachers
    - the world in which we live
- What statistical concepts should we teach?
- How can we make this happen in our classrooms?

Why quantitative literacy and statistics in my world?

QL: To understand numbers in everyday contexts

Our scores are improving!

Michigan NAEP grade 8 scores

Year	Average scale score
2017	280
2015	278
2013	280
2011	280
2009	278
2007	277
2005	277
2003	278
2000	273
2000[1]	270
1996[1]	272
1992[1]	268
1990[1]	264

NAEP Data Explorer

Our scores are improving! But what about the gaps?

Michigan NAEP grade 8 scores

Year	Average scale score
2017	280
2015	278
2013	280
2011	280
2009	278
2007	277
2005	277
2003	278
2000	273
2000[1]	270
1996[1]	272
1992[1]	268
1990[1]	264

Year	Average scale score	Standard deviation
2017	280	39
2015	278	36
2013	280	36
2011	280	35
2009	278	36
2007	277	36
2005	277	36
2003	278	36
2000	273	38
2000[1]	270	37
1996[1]	272	36
1992[1]	268	36
1990[1]	264	34

NAEP Data Explorer

QL: Apply probabilistic thinking to decision making

What do you notice? Wonder?

	Scale Score	Margin of Error	Perf. Level
A	2140	(±16)	A
B	2110	(±16)	P
	2126	(±16)	A
C	2118	(±16)	P
	2107	(±16)	P

Margin of Error— "The margin of error around the student score is an estimate of the range or scores one would expect if the same student was to be measured repeatedly with parallel assessments." (Michigan M-Step Final Reports Webcast 2016)

QL: Apply probabilistic thinking to decision making

What do you notice? Wonder?

Margin of Error— "The margin of error around the student score is an estimate of the range or scores one would expect if the same student was to be measured repeatedly with parallel assessments." (Michigan M-Step Final Reports Webcast 2016)

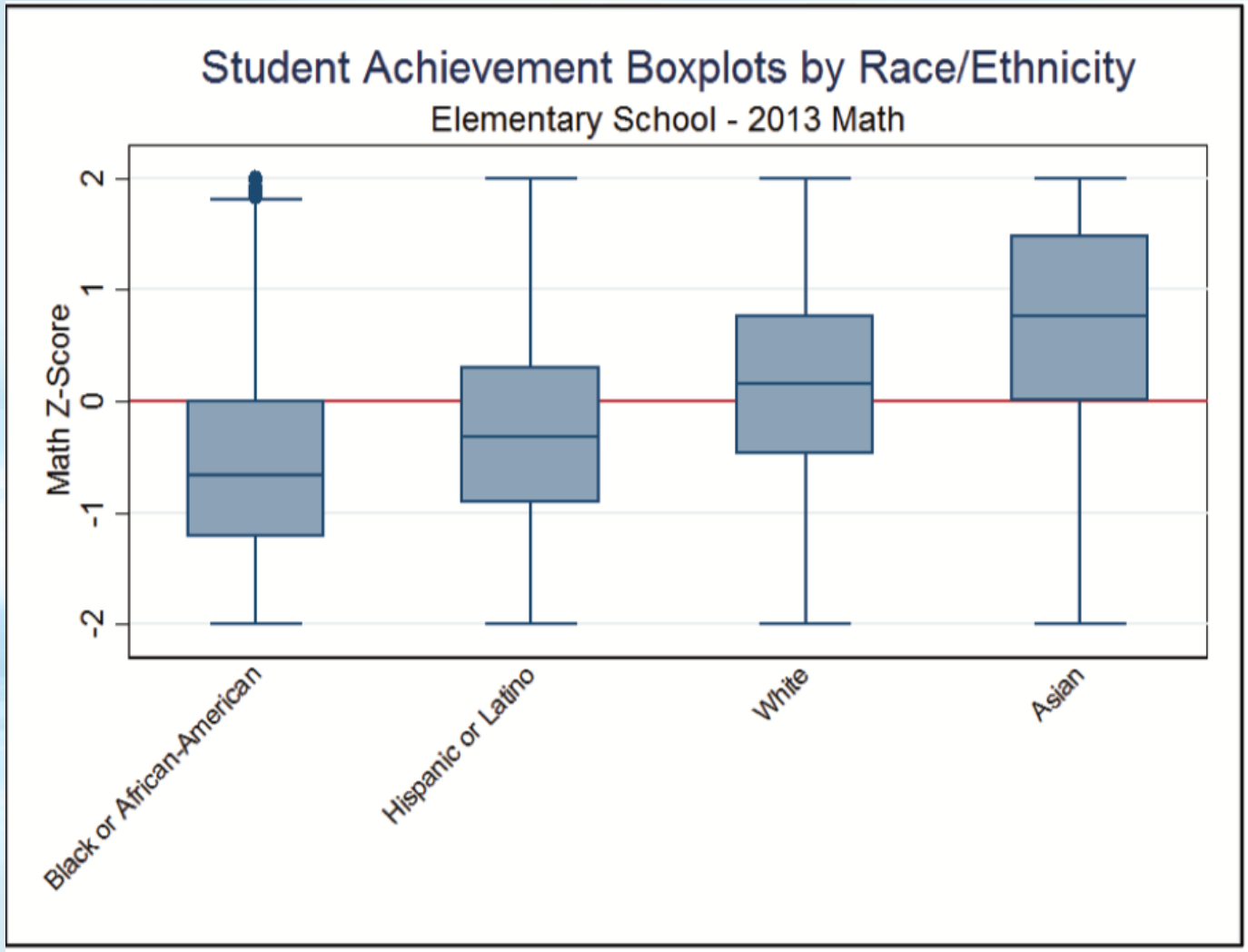


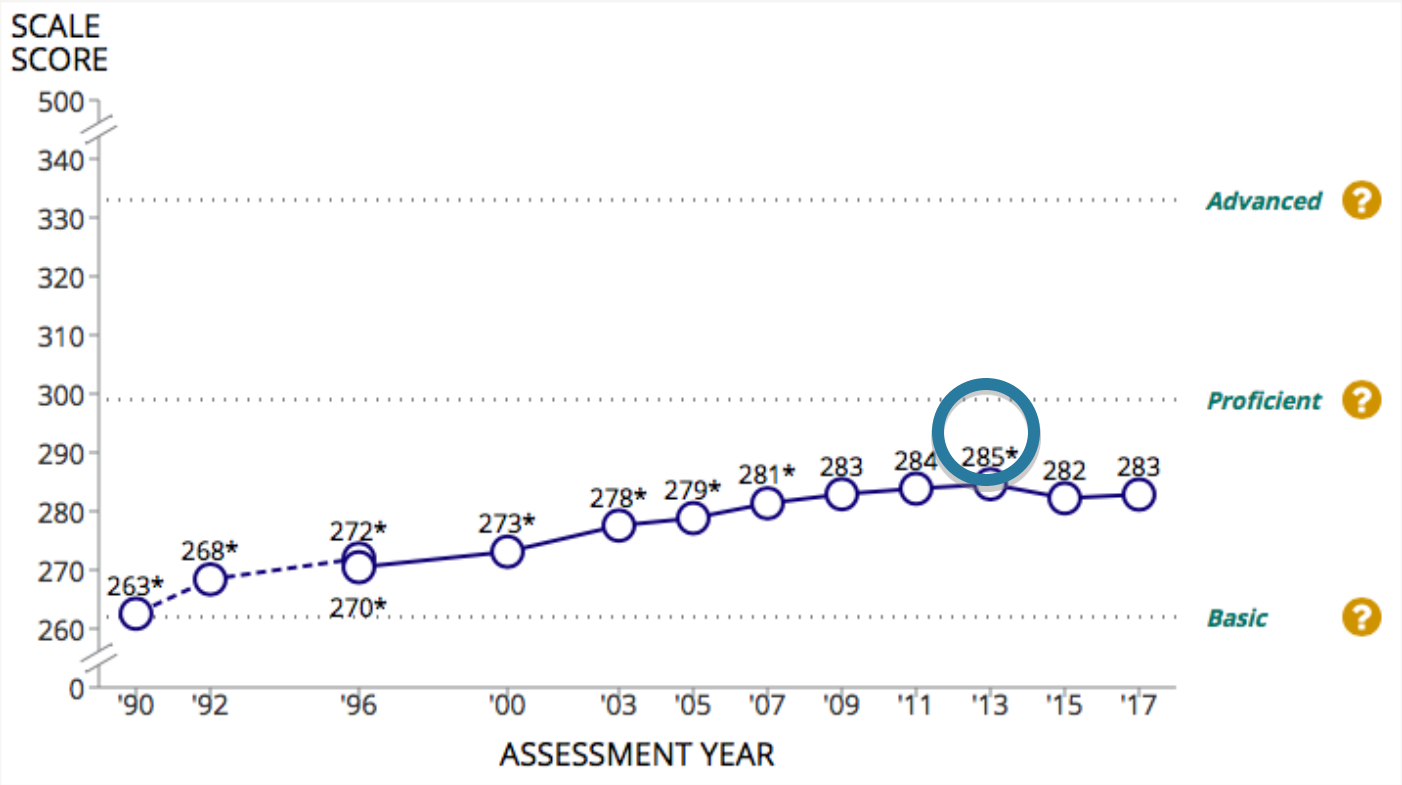
Figure 4. Student achievement box plots by race/ethnicity, elementary school math

QL: Interpret visual representations of data.

Flores, S. (2014). Quantifying the Achievement Gap: Baseline characteristics of African-American Student Achievement in Michigan

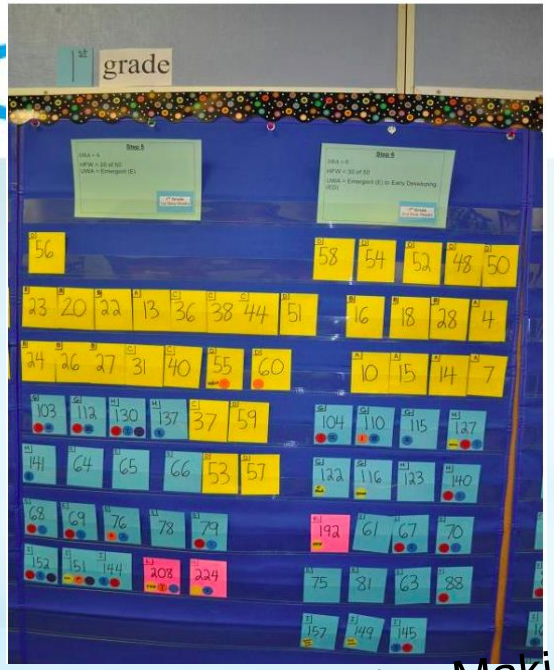
QL: Apply probabilistic information to decision making, and understand the limitations of such reasoning

Trend in eighth-grade NAEP mathematics average scores



No significant score change
Score increase
* Significantly different ($p < .05$) from 2017.
Accommodations not permitted
Accommodations permitted

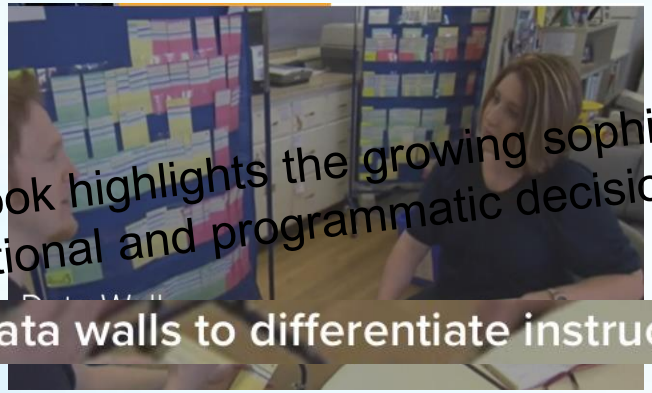
"We are completely data driven"



DATA WALLS - Continuous Improvement

Data Walls Demoralize Students.
NEPC, 2019

"CSD1 data wall encourages student success"



Data-Based Decision Making 2.0 This book highlights the growing sophistication in the use of data by school leaders for instructional and programmatic decisions.

- "Guidel[illegible] for data walls - Connecticut State Department of Education"

Six Steps for School Leaders to Use Data Effectively

USING DATA WALLS TO TURN DATA INTO INSTRUCTION

Analyzing student assessment data shows no evidence of effectiveness in raising student test scores (Hill, 2020)

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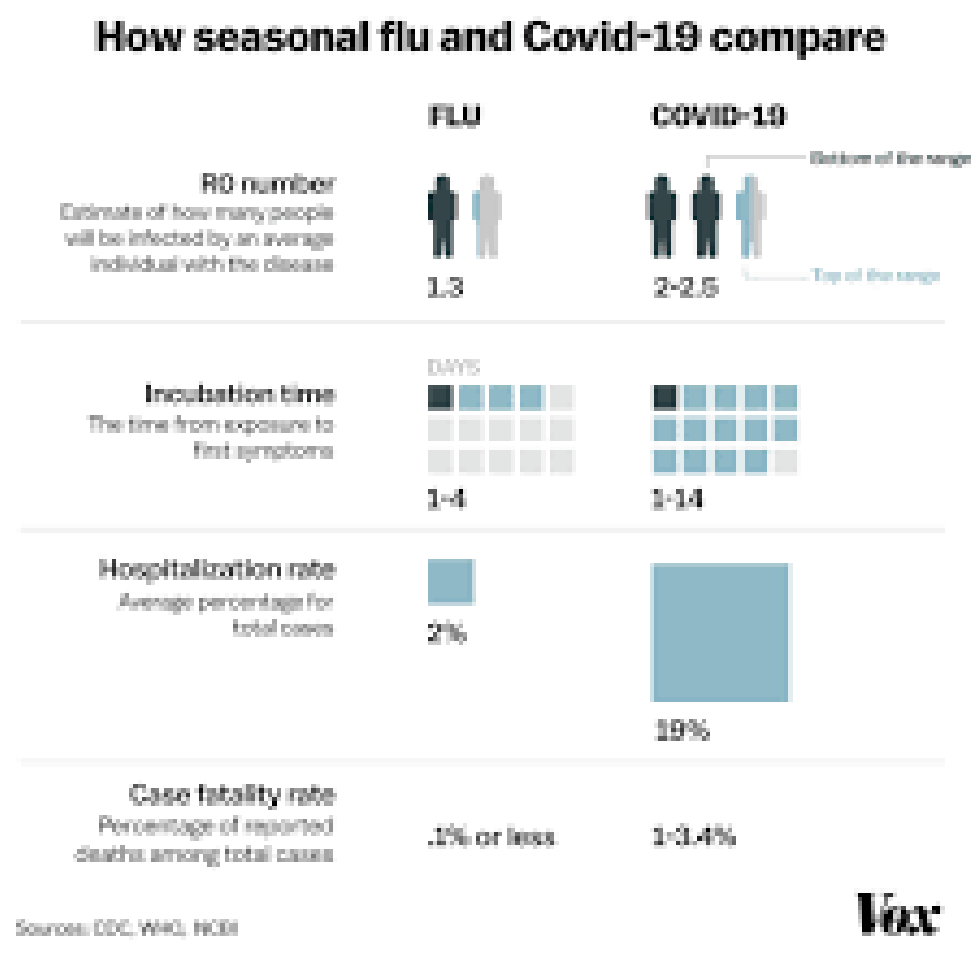
To make sense of the world in which we live

Help teachers bring in math in "life"

- "K–8 mathematics is visible in students' daily lives; numbers and geometry in the curriculum are more directly evident in the world—students find examples in their daily activities of counting, sharing, using money, building, …"
- The mathematics in the high school curriculum is less immediately evident in the world that students encounter in their daily lives—but it is no less important
- Make secondary school "number sense" and mathematical thinking important….

NCTM, 2018

Making Decisions Related to One's Health



Quantitative literacy in Medicine

- In one study, over 75% of gynecologists estimated that a woman whose mammogram was positive had a higher than 80 percent chance of having breast cancer; the reality is her chance is about 7 percent, less than 10 percent. (Morgan, D. Washington Post, Oct 2018)
- What does 7% represent? What is the difference in a false positive rate and a positive predictive value? For a screening mammogram, false positive rate is estimated to be about 14%. What happens to a false positive patient? Further testing…. What happens if have false negative with estimated rate of 13% percent.
- At any given time of testing, it is estimated a female will have breast cancer at about 1%. This is the prevalence rate for the population.

Percents vs Counts – Finding False Positive

	Diseased	Not Diseased	(totals)
Test result positive	86	1188	1274
Test result negative	14	8712	8726
(totals)	100	9900	10,000

- False Positive: 1188/9900 = 14%
- Chance have cancer given a positive = 86/1274 = 6.75%

Quantitative literacy in Medicine

- For the Coronavirus screening test, what is the sensitivity of the test – P(positive given have coronavirus)?
- What is the positive predictive value?
- False negative rate? False positive rate?
- Is it more important to minimize the false negative or false positive rate?
- How does antibody blood test differ from common screening test? Why is it used?

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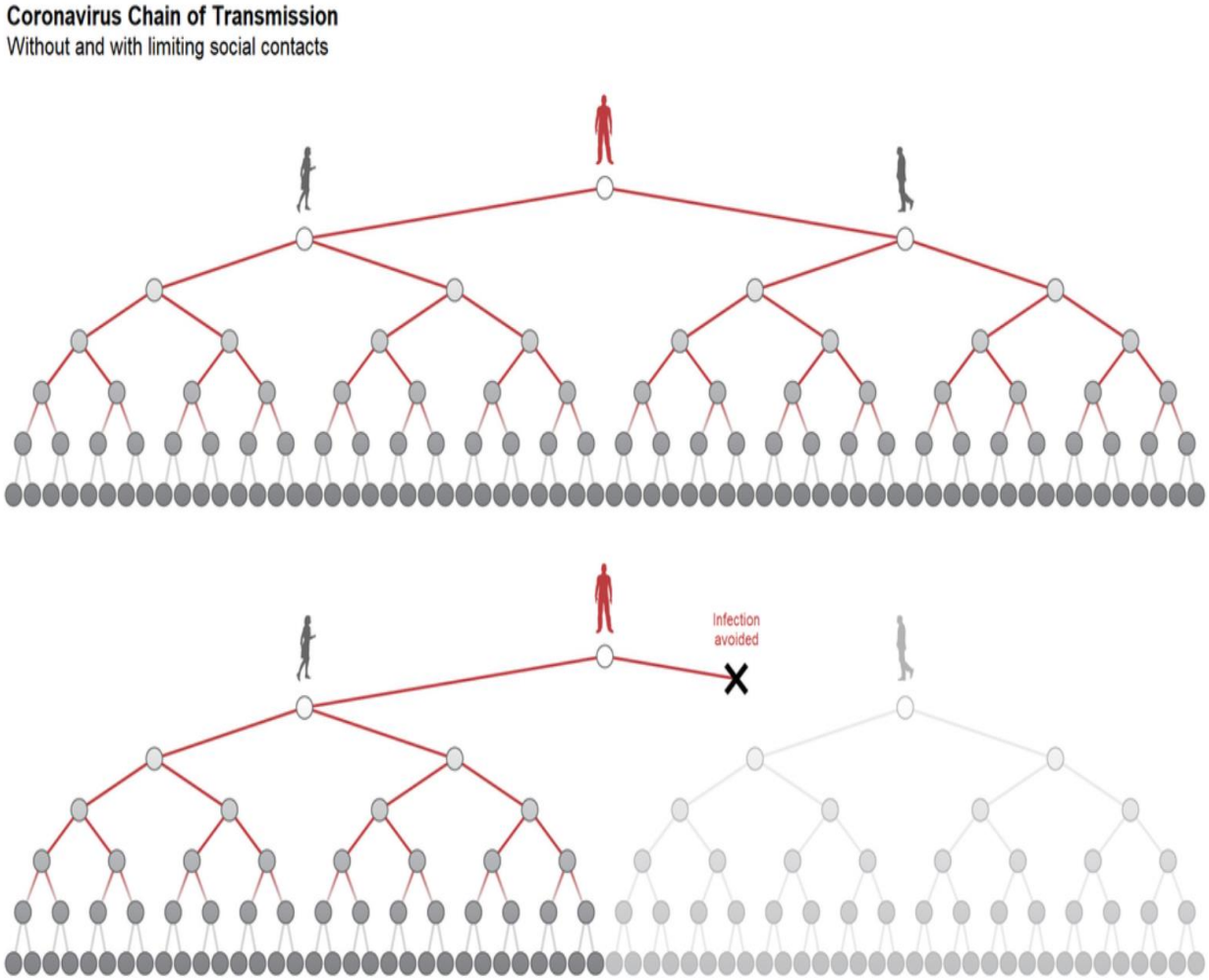
American Statistical Association/New York Times Partnership: What's going on in this graph?

What's Going On in This Graph? | April 1, 2020

How can social distancing affect the chain of coronavirus transmission?

March 26, 2020 · By THE LEARNING NETWORK



https://www.nytimes.com/column/whats-going-on-in-this-graph

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Statistics Teacher Journal

ASA NCTM

STATISTICS TEACHER

SUPPORTING THE TEACHING AND LEARNING OF STATISTICS

About | Grade Level | Announcements | Assessment | Technology | Lesson Plans | ST Publications | Events

COVID-19: A Teachable Moment

March 25, 2020

At a time when nothing good can be said about COVID-19, Sara Brown, Patrick Hopfensperger, and Henry Kranendonk—authors of ***Focus on Statistics: Investigations for the Integration of Statistics into Grades 9–12 Mathematics Classrooms***—have made available for free an investigation that can at least help students understand how the virus spreads. Investigation 12: Chances of Getting the Flu? develops a probability distribution through the design and use of a simulation involving the spread of flu in an apartment building. It follows the four components of statistical problem-solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*: formulate a statistical question; design and implement a plan to collect data; analyze the data by measures and graphs; and interpret the results in the context of the original question.

Download Investigation 12
Download Investigation 12 Worksheets

Search

SUBSCRIBE FOR UPDATES

ASA EDUCATION RESOURCES

NCTM EDUCATION RESOURCES

LOCUS
Levels of Conceptual Understanding in Statistics

ASA Community TEACHER GROUP

LESSON PLAN ARCHIVE (2012-2016)

EDUCATION

Understand and critique the world

"Students should be able to identify, interpret, evaluate, and critique the mathematics embedded in social, scientific, commercial, and political systems, as well as the claims made in the private and public sectors and in public interest group pronouncements (Ernest 2010)."

Students should leave our educational institutions with the ability to reason and make sense of information, know what questions to ask when confronted with data and conclusions from data, understand what "evidence" is and why it is important, and know how to deal with alternate truths as well as inconvenient facts.

Essential Concepts

- "… outline a **common shared pathway**- a progression of courses **that all students take**- as part of high school mathematics education." p. 9
- "… experience the **foundational mathematics** that [all students] will need for whatever future path they pursue." p. 9
- **Domains**: Number, Algebra & Functions, Geometry & Measurement, Statistics & Probability

NCTM, 2018

Essential Concepts Statistics & Probability build from middle grades

- Knowledge of different types of data (e.g., quantitative, categorical)
- Knowledge of shapes, measures of center and of variability
- Familiar with graphical representations of data
- Fitting linear models to data
- Use of simulation to investigate behavior of samples
- Basic probability concepts

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High School Statistics & Probability

- Focus 1: Quantitative Literacy
  - 2 Essential Concepts
- Focus 2: Visualizing and Summarizing Data
  - 6 Essential Concepts
- Focus 3: Statistical Inference
  - 7 Essential Concepts
- Focus 4: Probability
  - 2 Essential Concepts

NCTM, 2018

Quantitative Literacy includes the ability to

- use estimation and scale to place quantities in context;
- understand numbers as used in everyday discussions;
- create and interpret visual representations of data;
- engage with real data and assess its validity;
- understand the difference between association and causation and the different ways variables might be linked;
- explore and analyze statistical models, and
- generate and apply probabilistic information to decision making, and understand the limitations of such reasoning.

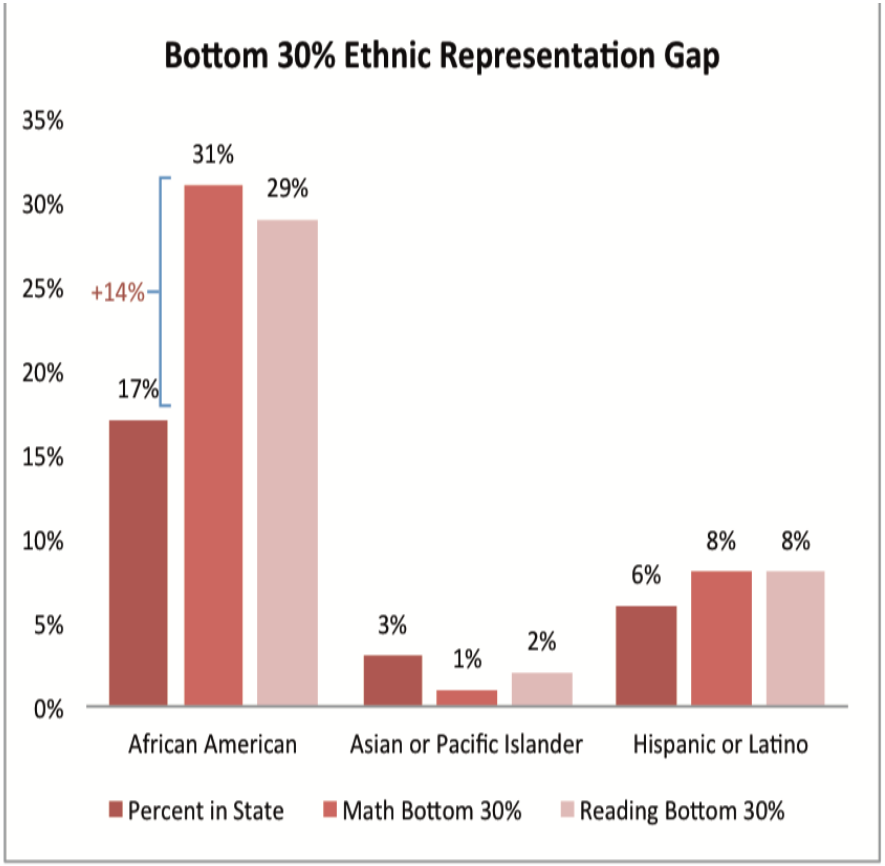
Adapted from National Council of Teachers of Mathematics, 2018

Gender	Age	Handedness	Height (cm)	Foot length (cm)	Arm span (cm)	Favorite subject	Languages spoken	Texts sent yesterday
Male	17	Left	175	24.75	170	Science	2	59
Male	15	Right	173	27	172	Math/stat	1	1
Female	17	Right	175	26.5	178	Science	1	20
Female	16	Right	161	27	162	Math/stat	1	10
Male	18	Right	178	27	169	Phy ED	2	70
Female	18	Right	158	21	164	Art	2	50
Female	17	Right	163	23.5	168	English	1	61
Male	16	Right	169	26	174	Comp Sci	1	70
Female	17	Right	162.6		157.5	Music	2	45
Female	18	Right	166.5	26	170	History	1	100
Male	17	Right	193.4	27.8	189.4	Comp Sci	1	26
Male	16	Left	177	67	128	Comp Sci	1	18
Female	17	Right	154	23	156	Other	1	200
Female	17	Right	148	22.5	149	Comp Sci	2	120
Male	16	Right	183	25	192	Music	2	12
Female	16	Right	165	22	156	Science	1	200
Female	17	Left	154.4	23	124	Science	2	45
Female	16	Right	179	26	174	History	1	16
Female	16	Right	174	25	171	History	2	36
Female	16	Right	158	20.5	165	English	1	15
Female	16	Right	171	25	168	Music	2	32
Female	17	Left	169	24	175	Science	1	25
Female	16	Right	166	25	168	English	1	0
Female	18	Right	170	23	156	Art	1	5

Table 1: Random Sample of Grade 11 students from the United States from Census at School American Statistical Association

Using real data from real contexts:

not always clean and usable



gure 2. Ethnic representation of students statewide scoring in the Bottom 30 Percent in Math and Reading

QL: Sometimes numbers tell a better story.

Flores, S. (2014). Quantifying the Achievement Gap: Baseline characteristics of African-American Student Achievement in Michigan

Making Sense with Percent

say 1000 students in state

Math:

17% or 170 African American Bottom 30%, 300 students

31% of 300, gives 93 African Americans. So 93/170 or 55% African Americans are in the bottom 30% of the scores in math.

3% or 30 are Asian

1% of 300 is 3. So 3/30 or 10% Asians are in the bottom 30% of the scores in math and likewise, 1/12 or 8% are Hispanic

Statistical modeling: “Is women’s income catching up to men’s?

Year	Men	Women
1960	39941	24234
1965	44656	26760
1970	51888	30805
1975	54291	31933
1980	54152	32578
1985.	53997	34869
1986.	55395	35603
1987.	55016	35858
1988.	54551	36030
1989.	53590	36802
1990.	51720	37040
1991.	53047	37058
1992.	53125	37605
1993.	52179	37318
1994.	51863	37325
1995.	51696	36926
1996.	51391	37907
1997.	52698	39082
1998.	54574	39932
1999.	55018	39786
2000.	54471	40156
2001.	54418	41537
2002.	55189	42275
2003.	55659	42049
2004.	54365	41631
2005.	53345	41063
2006.	53762	40594
2007.	54769	42616
2008.	54210	41791
2009.	55290	42562
2010.	55344	42575
2011.	53934	41532
2012.	54126	41408
2013.	54021	42278
2014.	53493	42067
2015.	54280	43183
2016.	54036	43482
2017.	53459	43658
2018.	55291	45097

Median Income of fulltime workers 15 and older (in 2018 dollars)

Hegewisch, et al, 2018)

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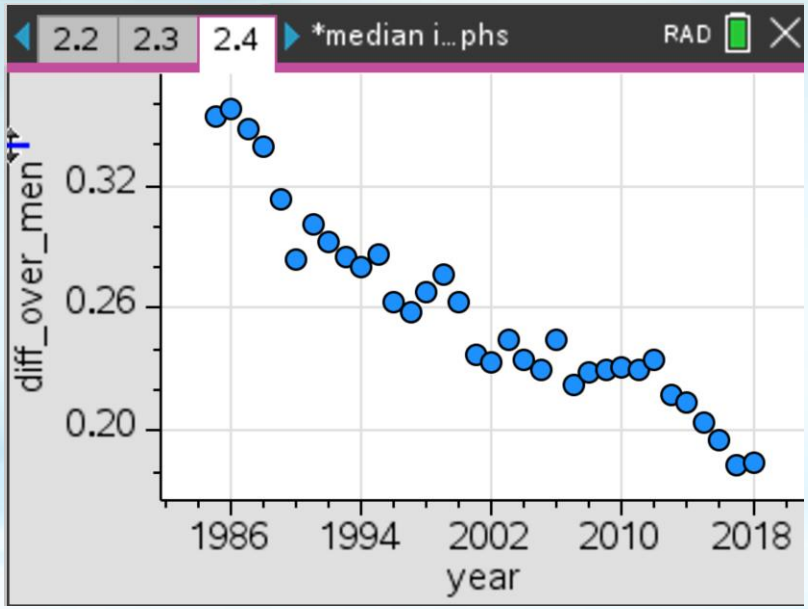
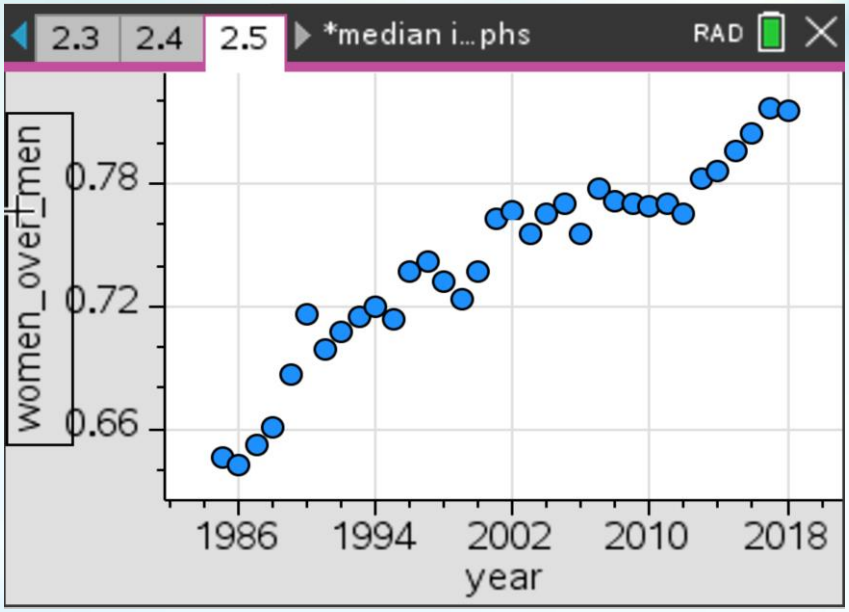
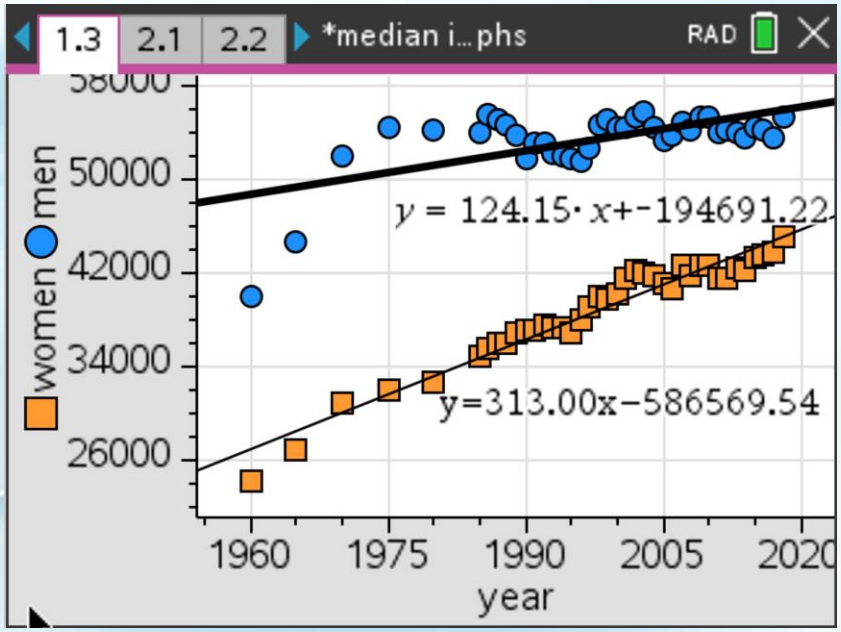
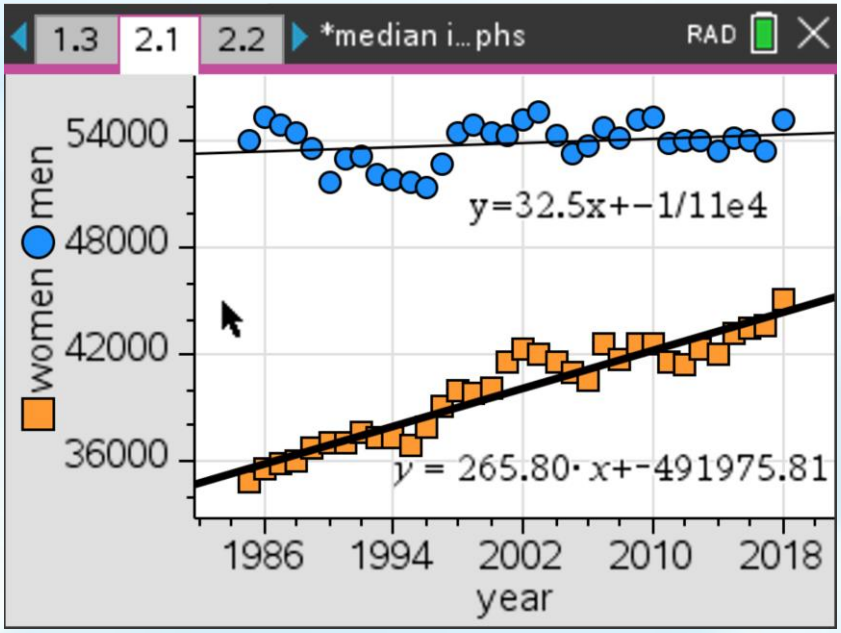
Is women's income catching up?

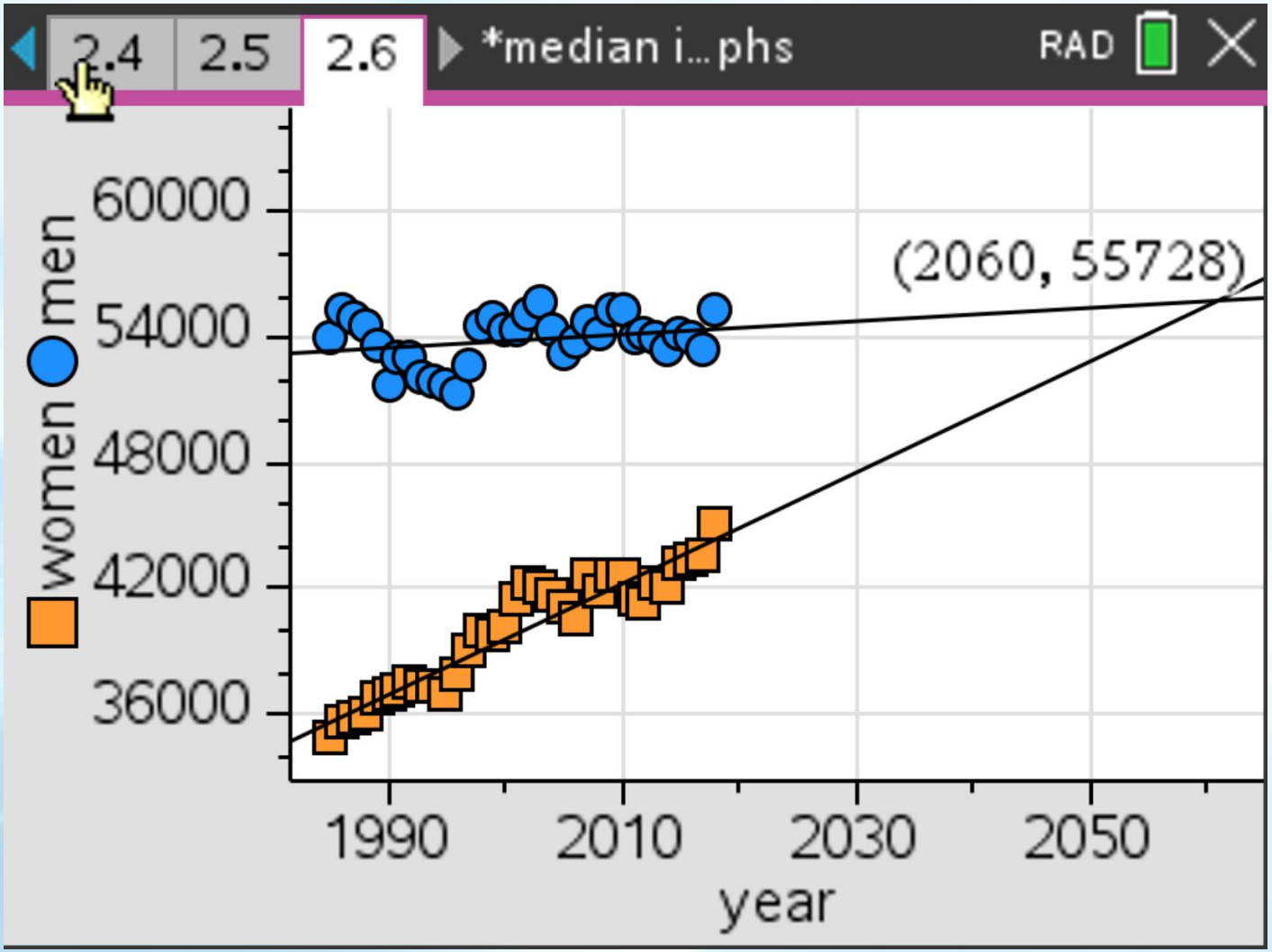




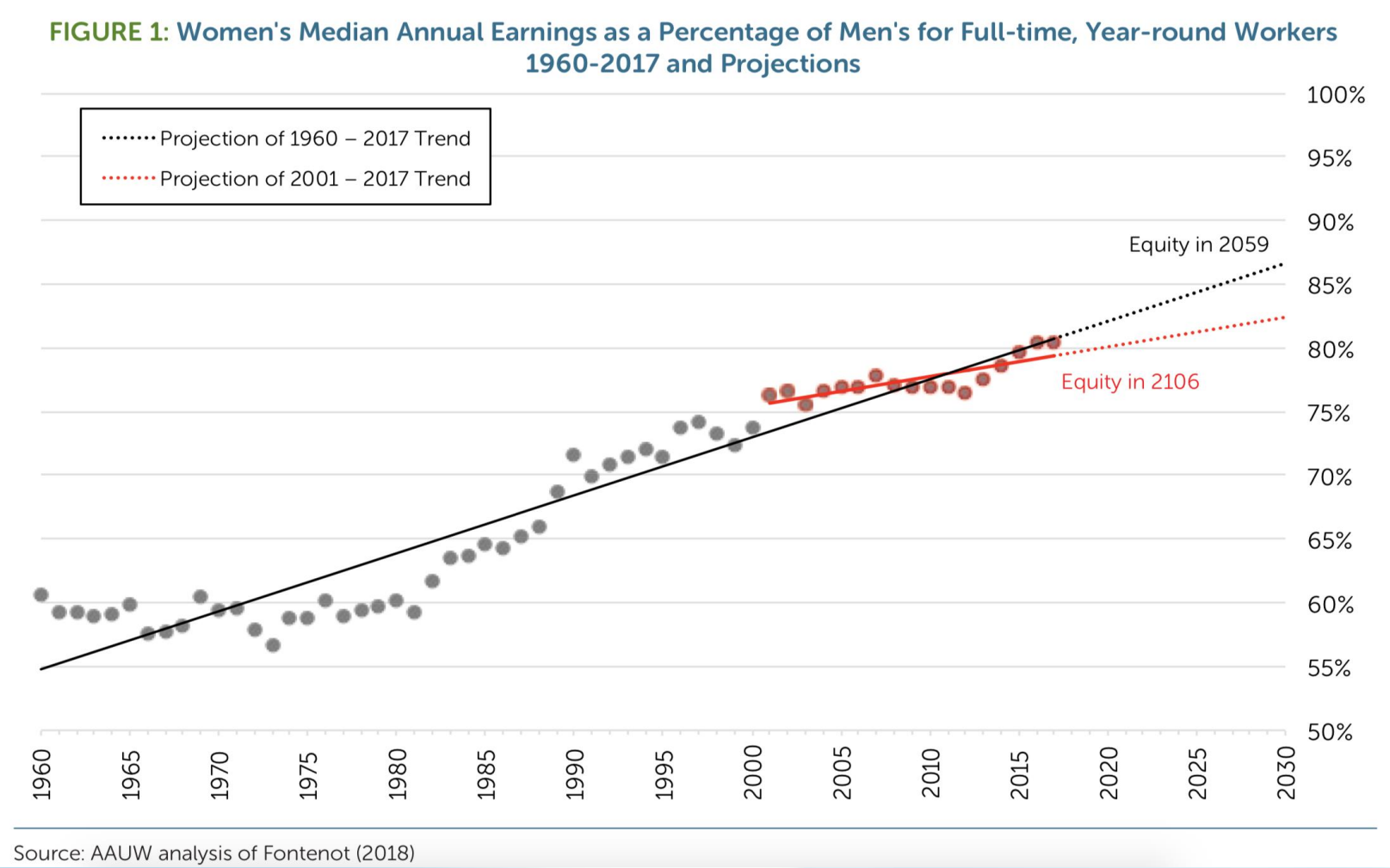




Is women's income catching up to men's?



Is women's income catching up to men's? Yes, but…



Key Recommendation

High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts.

NCTM. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: NCTM.

Pathway A—Geometry First
2 ½ Years

- **Grade 9**: Geometry and Measurement Essential Concepts followed by Statistics and Probability Essential Concepts
- **Grade 10:** Algebra and Functions Essential Concepts
- **First Half of Grade 11:** Integrate Algebra, Geometry, and Statistics with emphasis on practices and processes.

- Note: Need for “targeted instructional support” for those beginning grade 9 without necessary foundation in K-8 mathematics curriculum; Guard against race to calculus

Pathway B—Integrated Approach 3 Year

- **Grade 9**: Integration of geometry and measurement with statistics and probability
- **Grade 10:** Algebra and functions
- **Grade 11:** Integration of functions, modeling and statistical inference

Possible Course Options beyond the Essential Concepts

- Precalculus
- AP Calculus
- AP Statistics
- IB Mathematical Studies
- Quantitative Literacy
- Financial Mathematics
- History of Mathematics
- Mathematical Modeling
- Discrete Mathematics
- Advanced Quantitative Reasoning (AQR) / Advanced Mathematical Decision Making (AMDM) (Dana Center 2017)
- Statway and Quantway (Carnegie Math Pathways 2017)
- Math Ready: Ready for College-Level Math (Southern Regional Education Board 2016)

The Challenge

- Incorporate statistical thinking into your curriculum to give students the power to understand and make sense of the data in their world, making data-based decisions about issues that matter to them and to the larger community in which we all live.
- Questions?

Resources

ASA AMERICAN STATISTICAL ASSOCIATION
Promoting the Practice and Profession of Statistics®

- Pre-K-12 GAISE report (2007)
- SET – Statistical Education of Teachers (2015)
- Pre-K12 GAISE updated – roll out October 2020 at NCTM Fall Conference
- FOCUS on Statistics: Investigations for the Integration of Statistics into Grades 9-12 Mathematics Classrooms (2020) https://www.amstat.org

Go to Education menu

Resources

Essential Understandings for Statistics: 6-8 and 9-12

Catalyzing Change

More 4 U

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Resources for Assessment

LOCUS
Levels of Conceptual Understanding in Statistics

Learn About LOCUS | Use standard LOCUS test | View sample items and commentaries | Make your own test

Looking to take a test? | Looking to give a test? | Already have an account? Login!

LOCUS is an NSF Funded DRK12 (DRL-1118168) project focused on developing assessments of statistical understanding. These assessments measure students' understanding across levels of development as identified in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework. LOCUS assessments measure statistical understanding at two levels: Beginning/Intermediate and Intermediate/Advanced. The intent of these assessments is to provide teachers, educational leaders, assessment specialists, and researchers with a valid and reliable assessment of conceptual understanding in statistics consistent with the Common Core State Standards (CCSS).

1 of 27 next ›

Question:

The student council members at a large middle school have been asked to recommend an activity to be added to physical education classes next year. They decide to survey 100 students and ask them to choose their favorite among the following activities: kickball, tennis, yoga, or dance.

(a) What question should be asked on the survey? Write the question as it would appear on the survey.

(b) Describe the process you would use to select a sample of 100 students to answer your question.

(c) Create a table or graph summarizing possible responses from the survey. The table or graph should be reasonable for this situation.

(d) What activity should the student council recommend be added to physical education classes next year? Justify your choice based on your answer to part (c).

- Overview of the question
- Standards
- Ideal response and scoring
- Sample responses indicating solid understanding
- Common misunderstandings

Part (a) asked students to write an appropriate survey question.

Most students were able to write a reasonable survey question in part (a), but some questions were incomplete or vague. The following two responses illustrate this and were scored as only partially correct for part (a).

(a) What question should be asked on the survey? Write the question as it would appear on the survey.

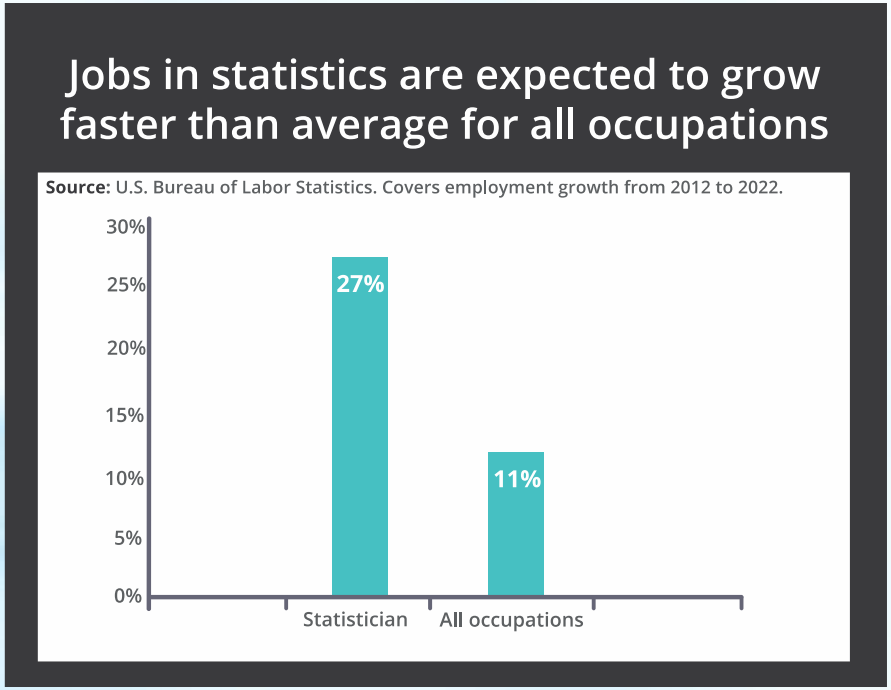
What activity for physical education classes next year should be used?

(a) What question should be asked on the survey? Write the question as it would appear on the survey.

What gym class would you most likely take next year

Some questions, like the one in the student response below, would not have resulted in data that could be

Future Jobs – In Statistics



Data Scientist: Sexiest Job of the 21st Century

- Harvard Business Review

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