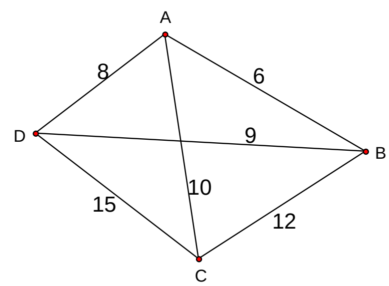
Answer Key –Nearest Neighbor

The Nearest Neighbor Algorithm

1. From your starting city, visit the *nearest* city.
2. From that city, visit the *nearest* city you have not already visited.
3. When you have visited all the cities, return to your starting city.

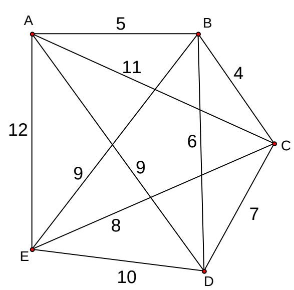
1. Given the table of distances between cities A, B, C, and D and the map, find the shortest round-trip starting at city A.

	A	**B**	**C**	**D**
A	—	6	10	8
B	6	—	12	9
C	10	12	—	15
D	8	9	15	—



A to [B] to [D] to [C] to **A** Total Distance: [40]

2. Given the map of cities A, B, C, D, and E, find the length of the round-trip starting at city B using the nearest neighbor algorithm.



B to [C] to [D] to [A] to [E] to **B** Total Distance: [41]

3. Given the table of distances between cities A, B, C, D, E, and F, find the length of the round-trip starting at city C using the nearest neighbor algorithm.

	A	**B**	**C**	**D**	**E**	**F**
A	—	10	12	4	6	20
B	10	—	2	15	9	18
C	12	2	—	8	13	5
D	4	15	8	—	17	21
E	6	9	13	17	—	3
F	20	18	5	21	3	—

C to [B] to [E] to [F] to [A] to [D] to **C** Total Distance: [46]

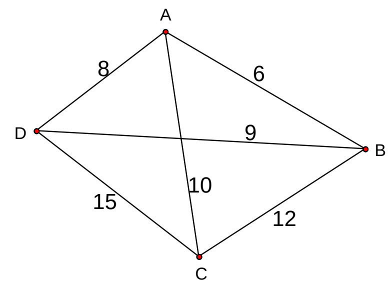
Answer Key –Cheapest Link

Note: For this activity, "route" refers to a path from one city to another, and "mini-tour" refers to a tour that does not include all cities.

The Cheapest-Link Algorithm

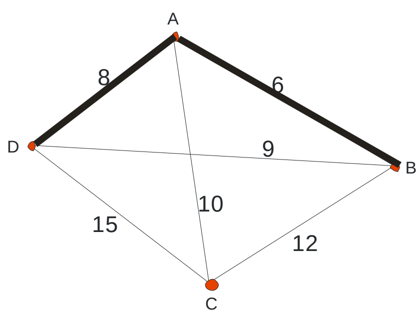
1. Sort the distances of all the routes between each pair of cities from shortest to longest.
2. Select the shortest route available on the list as long as:
   a) It does not cause three routes going to and from the same city.
   b) It does not form a "mini-tour."
3. Continue the process until there is a tour that includes all the cities.

1. Refer to the map on the right to complete this example.

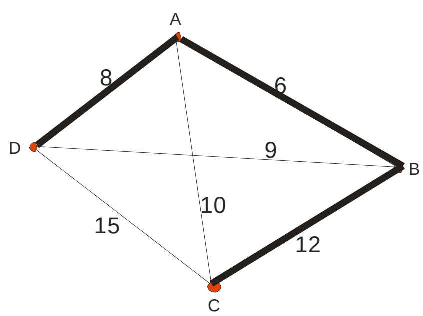


a) Complete the table of routes listed from shortest to longest.

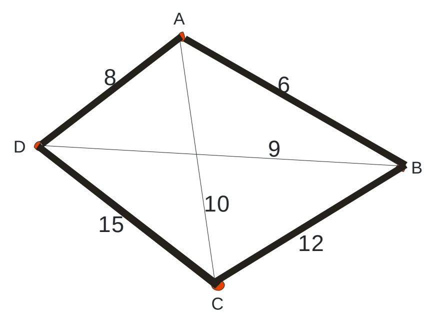
ROUTE	DISTANCE
AB	6
AD	8
[BD]	[9]
[AC]	[10]
[BC]	[12]
[CD]	[15]



Select route AB since it is the shortest routes. Next, select AD because it is the next shortest.



BD and AC are the next cities in the sorted list. However, BC would create a "mini-tour" of ABD, and AC creates three routes at A. Therefore, BC is selected.

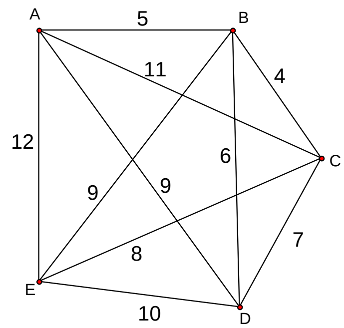


Finally, select CD to complete a tour that includes all the cities.

b) Fill in the distances between the routes used in the tour, then find the sum.

Distance = 6 + [8] + [12] + [15] = [41]

2. Use the Cheapest Link Algorithm to find the shortest round-trip using the map below.



Routes selected in order: BC, AB, CD, DE, AE
Distance: 38

Note: You could not select these routes for the following reasons:
BD because of rule 1 CE because of rule 1
BE because of rule 1 AD because of rule 2
AC because of both rules

3. Use the Cheapest Link Algorithm to find the shortest round-trip using the table below.

	A	B	C	D	E	F
A	—	10	12	4	6	20
B	10	—	2	15	9	18
C	12	2	—	8	13	5
D	4	15	8	—	17	21
E	6	9	13	17	—	3
F	20	18	5	21	3	—

Routes selected in order: BC, EF, AD, CF, AE, BD
Distance: 35

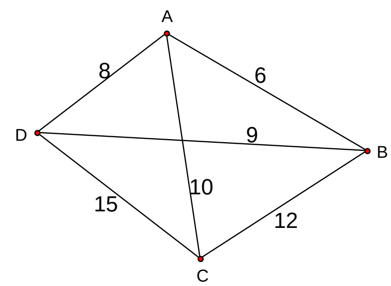
Note: You could not select these routes for the following reasons:
CD because of rule 1 BE because of rule 1
AB because of rule 1 AC because of rule 1
CE because of both rules

Answer Key – Brute Force

The Brute Force Algorithm

1. Beginning with your starting city, list all the possible round-trips. **Hint:** Try drawing a tree diagram.
2. Determine the distances of each round-trip.
3. Pick the shortest round-trip.

1. Refer to the map on the right to complete this question.



a) Starting with city A, list all the possible round-trips.

ABCDA
ABDCA
[ACBDA]
[ACDBA]
[ADBCA]
[ADCBA]

b) Determine the total distance of each round-trip.

ABCDA = AB + BC + CD + DA = 6 + 12 + 15 + 8 = [41]
ABDCA = AB + BD + DC + CA = 6 + 9 + [15] + [10] = [40]
ACBDA = [AC + CB + BD + DA] = [39]
ACDBA = [AC + CD + DB + BA] = [40]
ADBCA = [AD + DB + BC + CA] = [39]
ADCBA = [AD + DC + CB + BA] = [41]

c) Pick the shortest round trip.

ACBDA or ADBCA

2. For four cities, how many round trips were possible?

6

3. Did all the road trips result in different distances? If not, is there anything unique about the ones that gave the same distances?

No, there are only 3 different distance. The ones that have the same distances were just round-trips in reverse order (i.e., ACBDA and ADBCA).

4. Using your answer from Question 3 and the fact that 5 cities have 12 unique round-trips (URTs) and 6 cities have 60 URTs, can you find a mathematical formula that gives you the number of URTs for *n* cities?

Number of URTs = $\frac{(n-1)!}{2}$

The Number of Unique Round-Trips for *n* Cities

Given *n* cities, the number of unique round-trips is given by the following formula:

$$\text{Number of URTs} = \frac{(n-1)\cdot(n-2)\cdot(n-3)\cdots 3\cdot 2\cdot 1}{2} = \frac{(n-1)!}{2}$$

5. In Question 4 above, you are told that a tour of 5 cities has 12 URTs. If you are given cities S, T, U, V, and W, list all the URTs starting from city S.

STUVWS
STUWVS
STVUWS
STVWUS
STWUVS
STWVUS
SUTVWS
SUTWVS
SVTUWS
SVTWUS
SWTUVS
SWTVUS

6. You are a traveling salesperson for a local company. Determine the number of URTs if you need to visit:

a) 10 cities (including your starting city)

$$\frac{(10-1)!}{2} = \frac{362{,}880}{2} = 181{,}440$$

b) 25 cities (including your starting city)

$$\frac{(25-1)!}{2} = \frac{6.2045\times 10^{23}}{2} = 3.1022\times 10^{23}$$

c) The capitals of the lower 48 states (including your starting city)

$$\frac{(48-1)!}{2} = \frac{2.5862\times 10^{59}}{2} = 1.2931\times 10^{59}$$

Answer Key – Best Route

Use the information given on the Road Trip! overhead to solve these problems.

1. Using the Nearest Neighbor Algorithm, find the shortest round-trip starting in Cleveland.

Cle to [Pitt] to [Bal] to [Bos] to [Cin] to **Cle** Total Distance: [1,629]

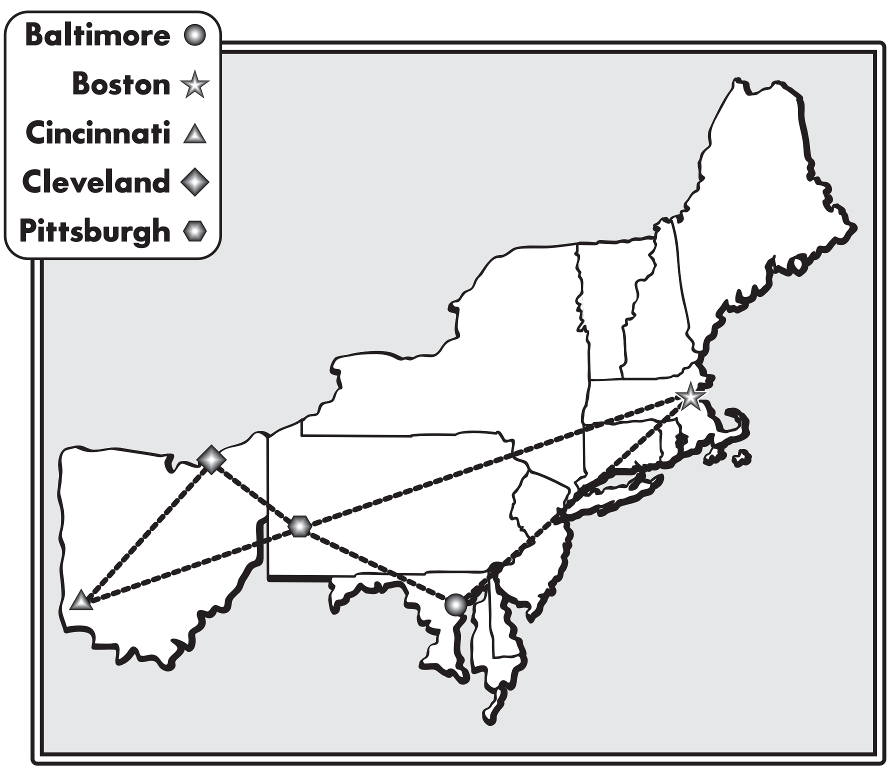
2. Use the Nearest Neighbor steps using each of the other cities as the starting point.

Cin to [Cle] to [Pitt] to [Bal] to [Bos] to **Cin** Total Distance: [1,629]
Pitt to [Cle] to [Cin] to [Bal] to [Bos] to **Pitt** Total Distance: [1,598]
Bos to [Bal] to [Pitt] to [Cle] to [Cin] to **Bos** Total Distance: [1,629]
Bal to [Pit] to [Cle] to [Cin] to [Bos] to **Bal** Total Distance: [1,629]

3. Were the total distances in Questions 1 and 2 the same or different? Is this what you expected? Why do you think the results turned out this way?

The route starting in Pitt did not use the 739 between Bos and Cin or the 200 between Pitt and Bal, but instead used 428 from Cin to Bal and 480 from Bos to Pitt.

4. Using the Cheapest Link Algorithm, find the shortest round-trip. Draw the route on the map.



Route	Distance
[Cle-Pitt]	[115]
[Bal-Pitt]	[200]
[Cle-Cin]	[211]
[Pitt-Cin]	[259]
[Cle-Bal]	[314]
[Bos-Bal]	[364]
[Bal-Cin]	[428]
[Bos-Pitt]	[480]
[Cle-Bos]	[554]
[Bos-Cin]	[739]

5. What is the total distance of the route found using the Cheapest Link Algorithm?

1,629

6. Using the Brute Force Algorithm, how many unique round-trips are possible?

$$\frac{(5-1)!}{2} = \frac{4\cdot3\cdot2\cdot1}{2} = 12$$

7. One of the possible round-trips results in a total distance of 1588 miles. Determine the tour that begins and ends at Cleveland for this round trip.

Cle-Cin-Pitt-Bal-Bos-Cle

Note: This is also the shortest tour possible.

8. What is the best tour to follow on your road trip? Explain your reasoning.

Answers will vary.

9. Which algorithm was the easiest to implement? Explain your reasoning.

Nearest Neighbor

10. Which algorithm was the hardest to implement? Explain your reasoning.

Brute Force, because it requires listing all possible routes. With many cities, this would be very difficult.

11. Which algorithm will always find the shortest distance? Explain your reasoning.

Brute Force, because it finds all possible distances. However, for a large number of cities, this algorithm is too time-consuming even for the fastest computers.