Improving Archimedes' Method $\qquad$


1. Calculate the area of each regular n-gon inscribed in the unit circle. Round the result to the nearest ten-thousandths. Use exact values for the sine and cosine functions throughout the calculations, rounding only in the last step. The regular triangle solution is provided to check your work before calculating the areas for larger values of $n$.

| $n$ | AREA EXPRESSION | AREA APPROXIMATION |
| :---: | :---: | :---: |
| 3 | $3 \cos 30^{\circ} \sin 30^{\circ}$ | 1.2990 |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  | $\ldots$ |
| 10 |  |  |
| $\ldots$ |  |  |
| $\ldots$ |  |  |
| 200 |  |  |

2. As $n$ increases, what value will the areas approach? Explain your answer.

3. Now, consider circumscribed regular polygons with the unit circle inside of each polygon, as pictured above. Complete the table for the circumscribed areas.

| $n$ | AREA EXPRESSION | AREA ApProximation |
| :---: | :---: | :---: |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  | $\ldots$ |
| 10 |  |  |
| 5 |  |  |
| $\ldots$ | $\ldots$ | $\ldots$ |
| 200 |  |  |

4. As $n$ increases, what value will the areas approach? How do the circumscribed approximations differ from the inscribed approximations? Explain your answer.
