Linking Essential Concepts from Catalyzing Change to Mathematical and Statistical Practices Cluster from *High School Mathematics Reimagined, Revitalized, and Relevant*

Essential Concepts in Number	Mathematical and Statistical Practices Primary Cluster
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line.	Seeing, describing, and generalizing structure
Quantitative reasoning includes, and mathematical modeling requires attention to units of measurement.	Habits of a productive mathematical and statistical thinker
Essential Concepts in Algebra and Functions	
Algebra	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	Seeing, describing, and generalizing structure
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.	Habits of a productive mathematical and statistical thinker
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	Seeing, describing, and generalizing structure
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	Explaining, reasoning, and proving

Connecting Algebra to Functions	
Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	Seeing, describing, and generalizing structure
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	Modeling and using tools and representations
Functions	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f (x) = x2), recursive definitions, tables, and graphs.	Modeling and using tools and representations
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	Seeing, describing, and generalizing structure
Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change, and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	Seeing, describing, and generalizing structure
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	Modeling and using tools and representations

Essential Concepts in Number	Mathematical and Statistical Practices Primary Cluster
Essential Concepts in Statistics and Probability	
Quantitative Literacy	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	Modeling and using tools and representations
Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.	Explaining, reasoning, and proving
Visualizing and Summarizing Data	
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to “clean” and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data.	Modeling and using tools and representations
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	Modeling and using tools and representations
The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.	Modeling and using tools and representations
Scatterplots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables.	Modeling and using tools and representations
Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.	Modeling and using tools and representations
Data-analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	Modeling and using tools and representations
Statistical Inference	

Study designs are of three main types: sample survey, experiment, and observational study.	Habits of a productive mathematical and statistical thinker
The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.	Habits of a productive mathematical and statistical thinker
The scope and validity of statistical inferences are dependent on the role of randomization in the study design.	Modeling and using tools and representations
Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.	Habits of a productive mathematical and statistical thinker
The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.	Seeing, describing, and generalizing structure
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of error as a measure that describes the sampling variability.	Seeing, describing, and generalizing structure
Simulation of sampling distributions by hand or with technology can be used to determine whether a statistic (or statistical difference) is significant in a statistical sense or whether it is surprising or unlikely to happen under the assumption that outcomes are occurring by random chance.	Habits of a productive mathematical and statistical thinker
Probability	
Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.	Seeing, describing, and generalizing structure
Conditional probabilities—that is, those probabilities that are “conditioned” by some known information—can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.	Seeing, describing, and generalizing structure
Measurement	
Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombining.	Seeing, describing, and generalizing structure

Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement.	Modeling and using tools and representations
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	Seeing, describing, and generalizing structure
Transformations	
Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.	Seeing, describing, and generalizing structure
Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other.	Explaining, reasoning, and proving
Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.	Explaining, reasoning, and proving
Transformations in geometry serve as a connection with algebra, both through the concept of functions and through the analysis of graphs of functions as geometric figures.	Seeing, describing, and generalizing structure
Geometric Arguments, Reasoning, and Proof	
Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph).	Explaining, reasoning, and proving
Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures.	Explaining, reasoning, and proving
Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases, one may provide a more accessible or understandable argument than another.	Explaining, reasoning, and proving

Solving Applied Problems and Modeling in Geometry	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	Modeling and using tools and representations
Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem solving.	Modeling and using tools and representations